

1. **(Just for fun)** (3p)

During the lectures we found that the effect of collisions is to give the plasma finite resistivity,  $\eta$ . Using the expression for this specific resistivity on slide 11, calculate its value (assume  $\ln \Lambda = 16 = \text{const.}$ ) for

- (a) a typical tokamak research plasma,  $T_e = 1 \text{ keV}$ .
- (b) a high-performance fusion plasma,  $T_e = 20 \text{ keV}$ .
- (c) Compare these to typical resistivities of some metals (copper:  $\eta = 2 \times 10^{-8} \Omega \text{ m}$ , stainless steel:  $\eta = 7 \times 10^{-7} \Omega \text{ m}$ ). How would you rate these plasmas as conductors?

2. **(Juggling units)** (3p)

During lectures we got two different expressions for the diffusion coefficient: the theoretically derived classical diffusion coefficient,

$$D_{\text{cl}} = \frac{\eta_{\perp} n T}{B^2},$$

and the semi-empirical Bohm diffusion coefficient,

$$D_{\text{B}} = \frac{1}{16} \frac{T_e}{e B}.$$

Show that despite their very different looks both give the diffusion coefficient in the units of  $\text{m}^2 \text{ s}^{-1}$ .

3. **(Plasma current in a tokamak)** (3p)

A tokamak is a toroidal plasma container where a strong toroidal magnetic field is generated by large coils. Furthermore, an electric current is driven in the fully ionized plasma by an electric field applied along the toroidal magnetic field. How many  $\text{V m}^{-1}$  must be applied to drive a total current of 200 kA in a plasma with  $T_e = 500 \text{ eV}$ ,  $\ln \Lambda = 16 = \text{const}$  and a cross-sectional area of  $75 \text{ cm}^2$ ?

4. **(More algebraic gymnastics)** (6p)

A cylindrical column of plasma in a uniform magnetic field  $\mathbf{B}_t = B_0 \hat{z}$  carries a uniform current density  $\mathbf{j} = j_0 \hat{z}$ , where  $\hat{z}$  is a unit vector parallel to the axis of the cylinder.

- (a) Calculate the magnetic field  $\mathbf{B}_p(r)$  produced inside the plasma column by the plasma current.
- (b) Write an expression for the  $\nabla B$  drift of a charged particle in terms of  $B_0$ ,  $j_0$ ,  $r$ ,  $v_{\perp}$ ,  $q$ , and  $m$ . You may assume that the field calculated in (a) is small compared to  $B_0$  (but not zero), and that all particle energy is in the perpendicular motion.
- (c) If the plasma has electrical resistivity, there is also an electric field  $E = E_0 \hat{z}$ . Calculate the drift due to this field, taking into account the helicity of the magnetic field.

5. **(Food for thought: About auroras)**

Consider the phenomena called aurora borealis. What do they have to do with plasma physics? Use all knowledge gained during this course (and other courses, and the internet, if need be) to explain what they are, why they appear geographically where they do, what fuels them, and why they are not “on” all the time. Return your short write-up in MyCourses before the next lecture.