ELEC-E5510 Speech Recognition

Hidden Markov Models

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HMM

HMM notations

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- $\pi = \{\pi_1, \ldots, \pi_N\}$: initial probability distribution

GMM-HMM parameters

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GMM

$$p(x) = \sum_{m=1}^{M} w_m N_m(x|\mu_m, \Sigma_m)$$

$$= \sum_{m=1}^{M} \frac{w_m}{\sqrt{(2\pi)^M |\Sigma_m|}} exp\left(-\frac{1}{2}(x-\mu_m)^T \Sigma_m^{-1}(x-\mu_m)\right)$$

$$\sum_{m=1}^{M} w_m = 1$$

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Markov assumption

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Output independence

$P(o_t|Q, O) = P(o_t|q_t)$

The probability of an output observation o_t depends only on the state that produced the observation (q_t) and not on any other states or any other observations

HMM operations

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Scoring

How to compute the probability of the observation sequence for a model?

HMM operations

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Decoding

How to compute the best state sequence for the observations?

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Training

How to set the model parameters to maximize the probability of the training samples?

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Scoring equation

$$
\begin{aligned}
P(O|\lambda) &= \sum_{Q} P(O|q_t,\lambda)P(Q|\lambda) \\
&= \sum_{Q} \pi_{q_1} * b_{q_1}(o_1) * a_{q_1 q_2} b_{q_2}(o_2) * \dots * a_{q_{T-1} q_T} b_{q_T}(o_T)
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- NO, the problem is with the $\sum_Q$
- It is not feasible to consider all possible state sequences separately (for N states and T observation we have $O(2T * N^T)$ sequences)
- We need a better way of handling the state sequences.

Scoring using a search network

- Let's create a search graph!

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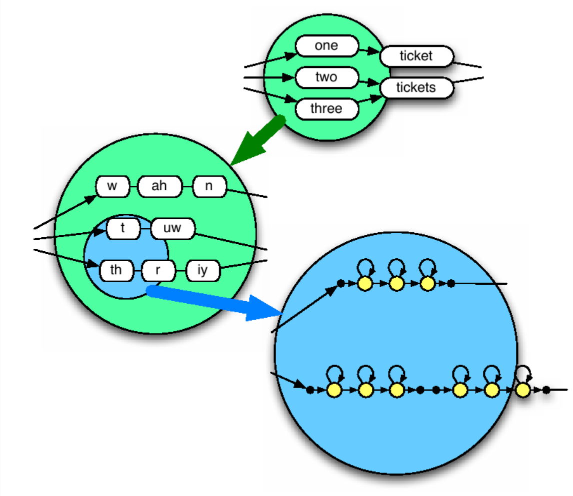

(Picture by S.Renals)

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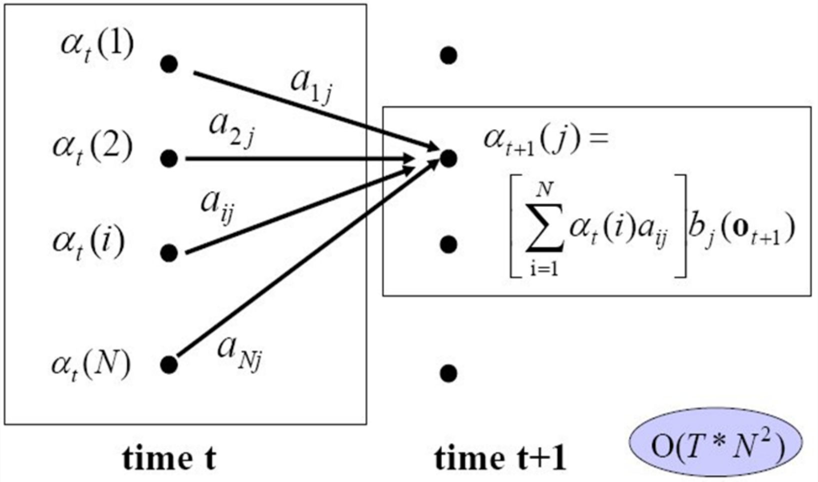
Forward algorithm



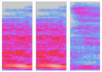
Picture by B. Pellom

Forward algorithm, exercise 1

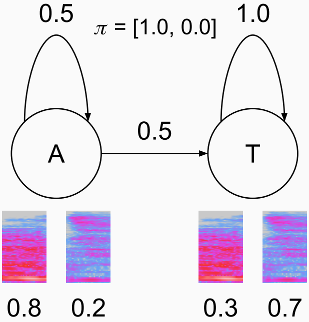
Given an HMM, and the initial probabilities:

$\pi = [\underbrace{1.0}_{A}, \underbrace{0.0}_{T}]$

What is the probability of observing

$O =$ 

$P(O|\lambda) = ?$



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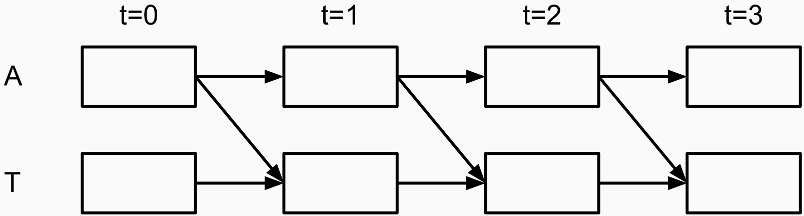

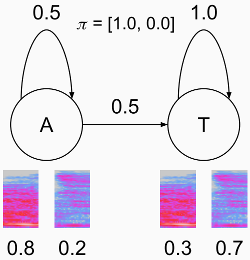

Answer

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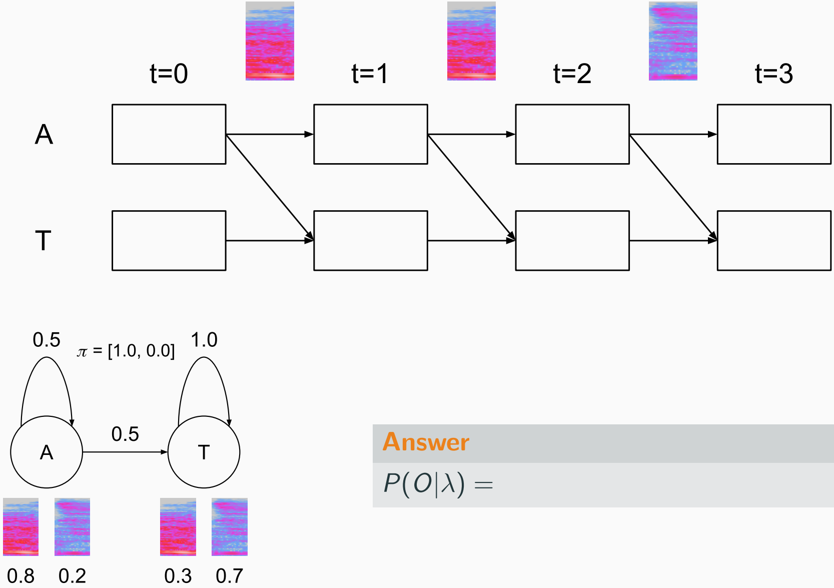

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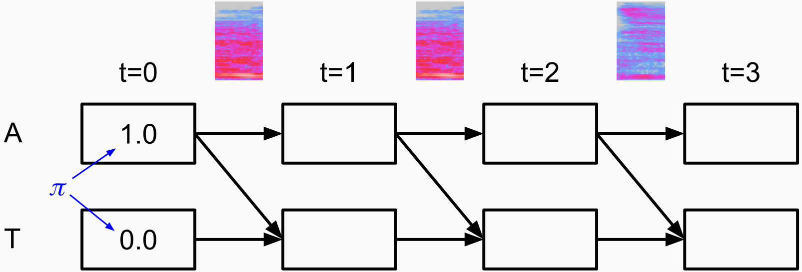

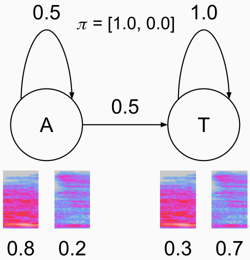

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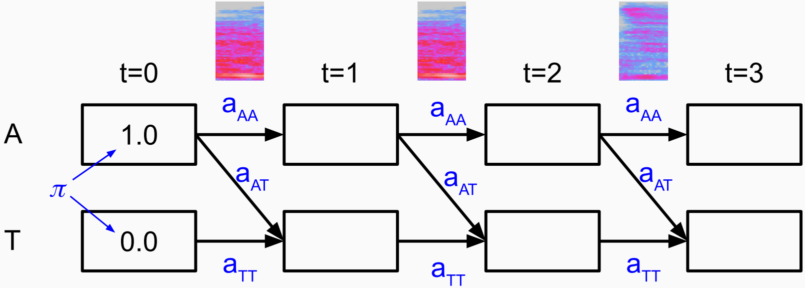

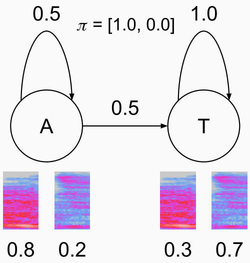

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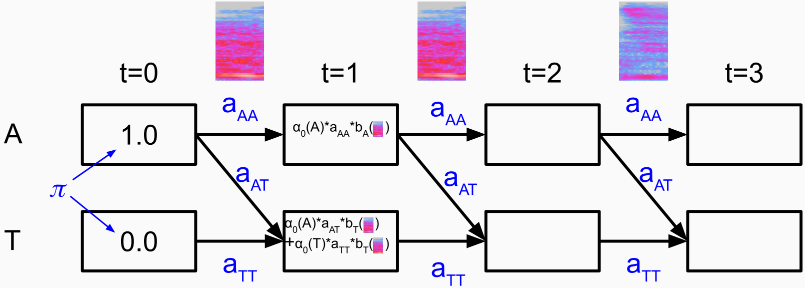

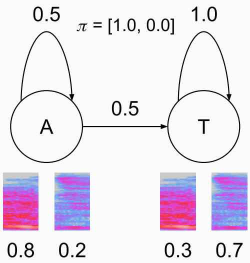

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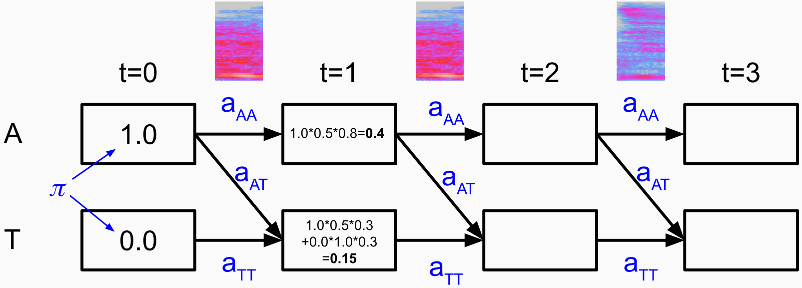

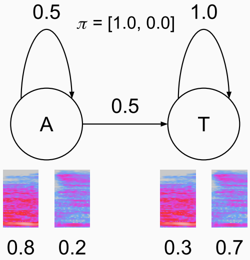

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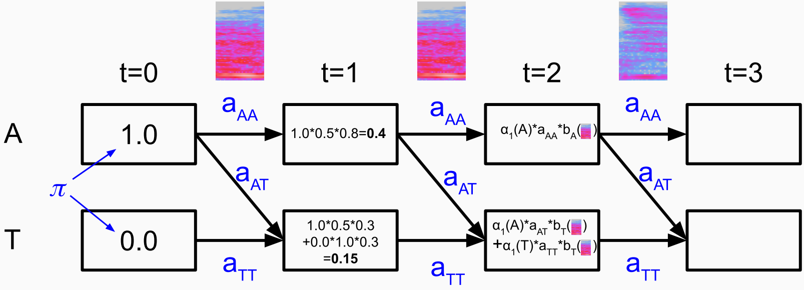

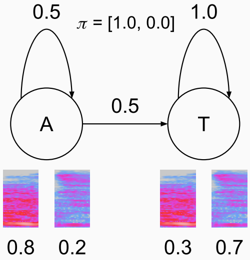

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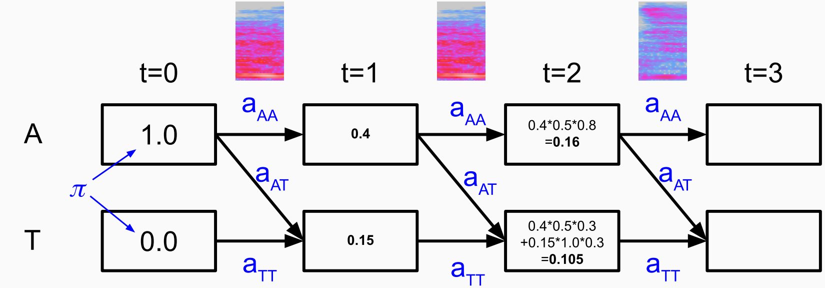

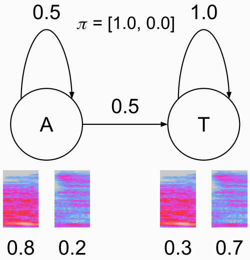

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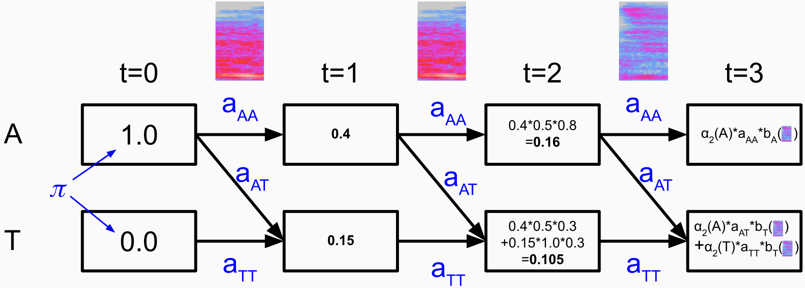

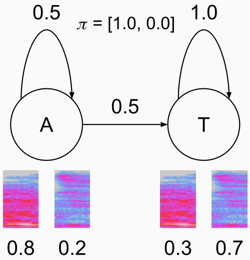

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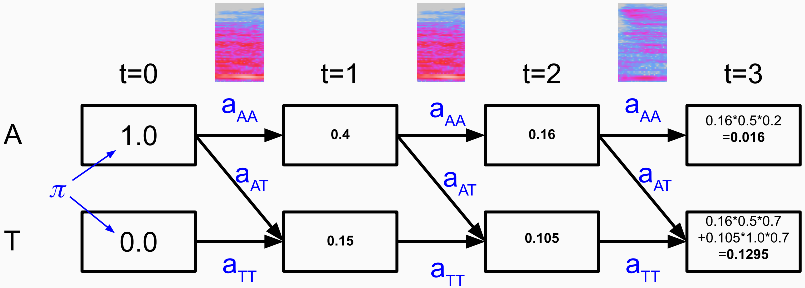

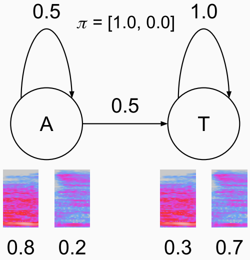

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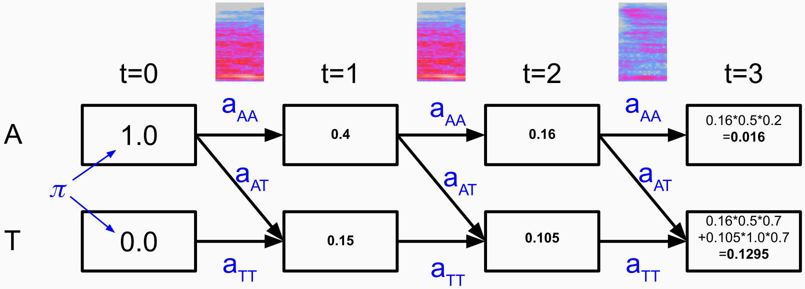

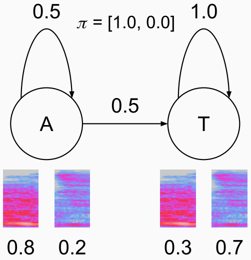

Answer

$P(O|\lambda) =$
$\sum_{i=1}^{N} \alpha_T(i) = 0.016 + 0.1295 = 0.1455$

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Decoding

- Given a sequence of observations $O = o_1, o_2, \ldots, o_T$
- What is the sequence of hidden states $Q = q_1, q_2, \ldots, q_T = ?$
- That maximizes $P(O, Q|\lambda)$

Viterbi algorithm

Initialization

$$\delta_1(i) = \pi_i b_i(o_1)$$

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Recursion

$$\delta_t(i) = \max_{1 \leq j \leq N} [\delta_{t-i}(j) a_{ji}] b_i(o_t)$$

$$\psi_t(i) = \operatorname{argmax}_{1 \leq j \leq N} [\delta_{t-i}(j) a_{ji}]$$

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$$P^* = \max_{1 \le i \le N} \delta_T(i)$$

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Backtrace

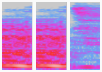
$$q_t^* = \psi_{t+1}(q_{t+1}^*)$$

Viterbi algorithm, exercise 2

Given an HMM, the initial probabilities:

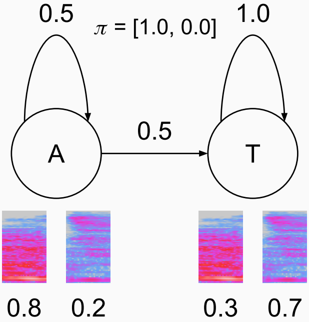
$\pi = [\underbrace{1.0}_{A}, \underbrace{0.0}_{T}]$

And the observation sequence

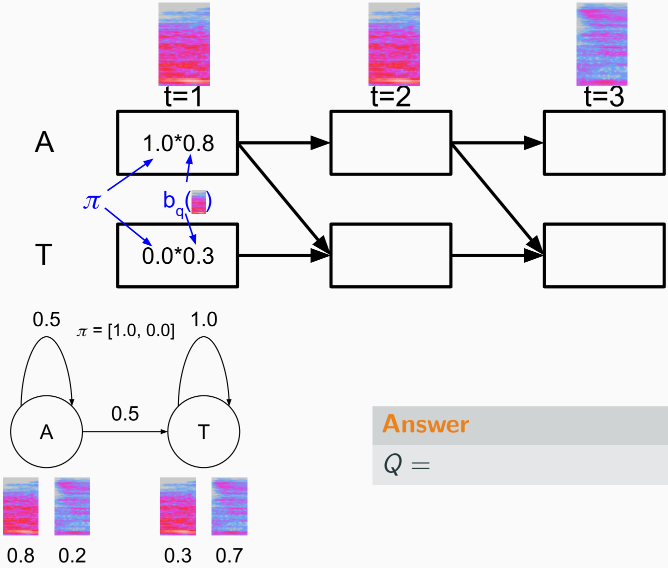
$O =$ 

What is the most probable state-sequence (Q)?

$argmax_Q P(O, Q|\lambda) = ?$



Viterbi algorithm, exercise 2 solution



Answer

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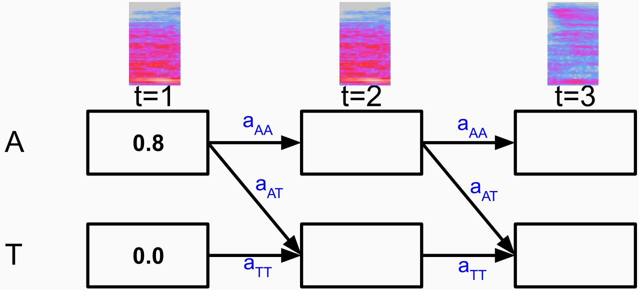

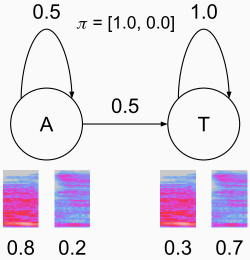

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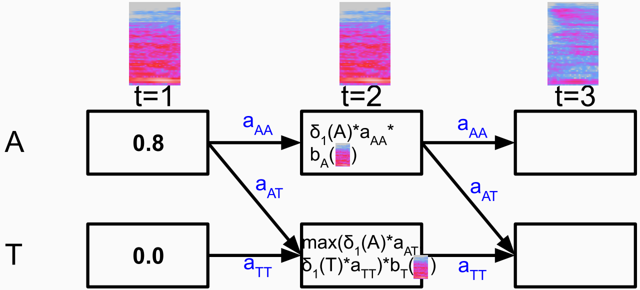

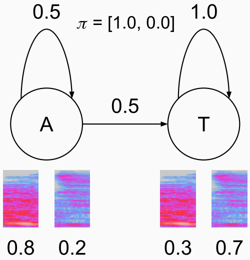

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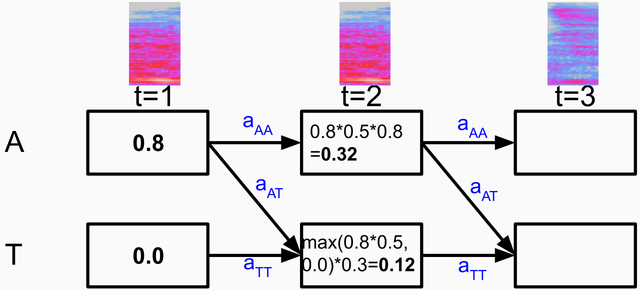

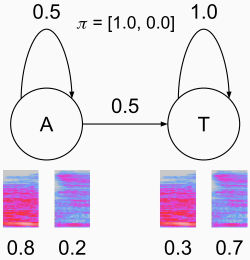

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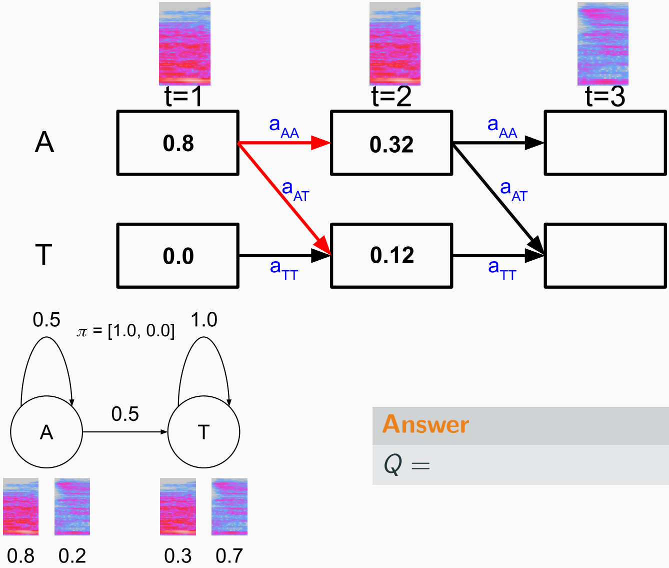

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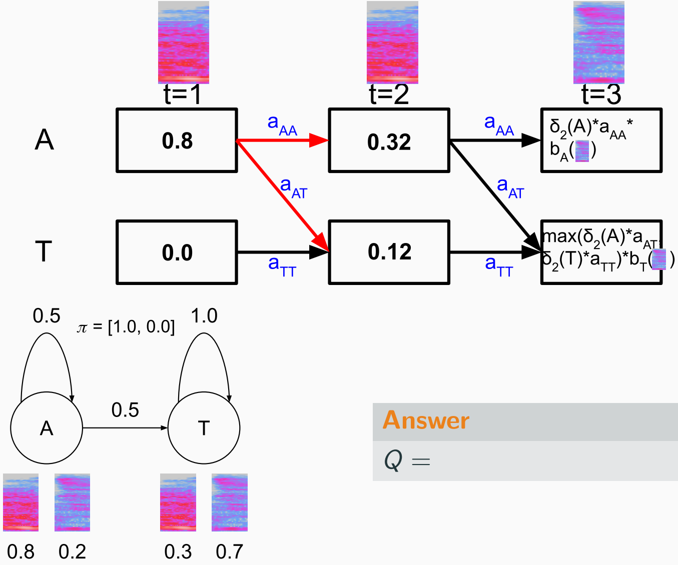

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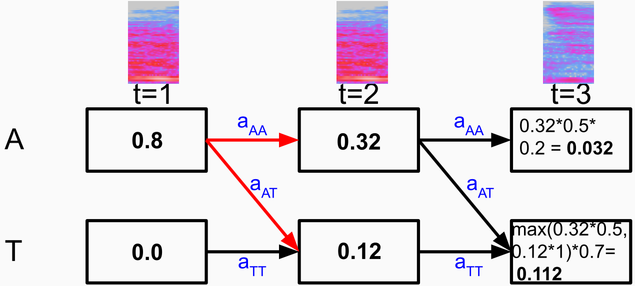

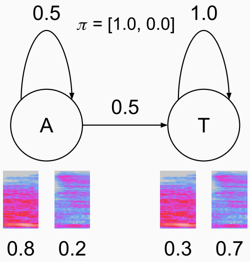

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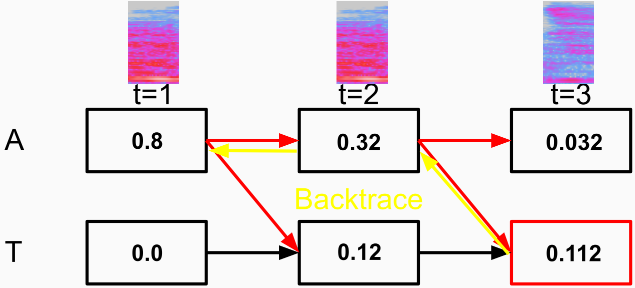

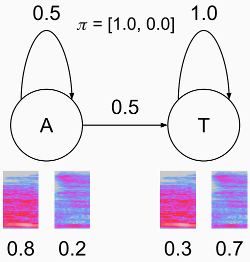

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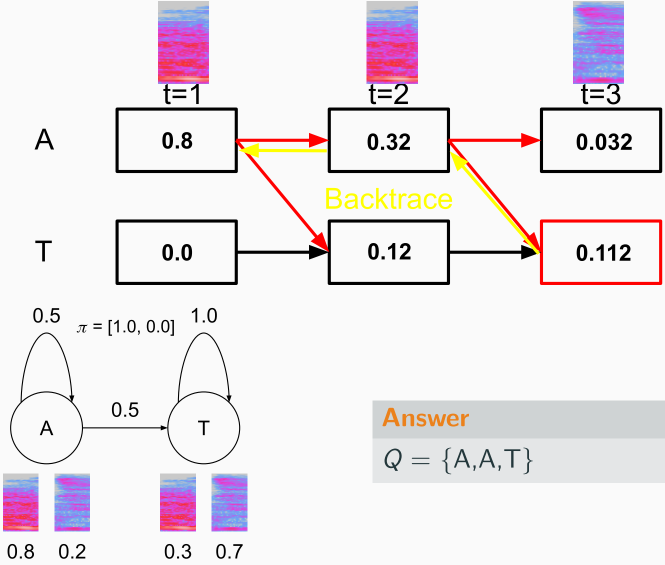

Answer

$Q = \{\text{A,A,T}\}$

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It's equivalent to stopping and restarting the HM on each frame.

Training

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Forward-Backward algorithm

1. Initialize the model parameters (A, B)

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3. Update the model parameters using $P(q_t = i|A, B, O)$
4. Iterate from 2.

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- Technically uses "Hard alignment" vs the "soft alignment" in Forward-Backward
- Simpler, but converges likewise to the (local) optimum

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