3a) Non-convex LP model for CHP plant

Minimize fuel costs given power and heat production

Variables	**Unit**	**Description**
x(j)	1	Weights for extreme characteristic operating points, j=1,…9
y(k)	1	Binary variables determining in which sub-area plant operates
Parameter	**Unit**	**Description**
P(j)	MWh/h	Power production at characteristic points
Q(j)	MWh/h	Heat production at characteristic points
F(j)	MWh/h	Fuel consumption at characteristic points, Fj=RPj*Pj+RQj*Qj
P	MWh/h	Power demand
Q	MWh/h	Heat demand
CF	EUR/MWh	Fuel price (value does not affect optimization as long as CF>0)
A(j,k)	1	Matrix telling if characteristic point j belongs to area k

Objective function

min Sum(F(j)*x(j))	EUR	Minimize fuel consumption

Constraints

sum(P(j)*x(j) = P	Power production must be P
sum(Q(j)*x(j) = Q	Heat production must be Q
sum(x(j)) = 1	Sum of weights must be 1
x(j) >= 0	Weights must be non-negative
x(j)<=sum(A(j,k)*y(k))	Disable points not in current area
y(k) binary	

Configuration for Excel Solver

CF	20	EUR/MWh	You can use any fuel price, or just minimize fuel consumption

Characteristic operating points

Point j	**P(j)**	**Q(j)**	**RP(j)**	**RQ(j)**	**F(j)**	**Efficiency**
1	20	20	1,2	1,1	46	87,0%
2	100	90	1,2	1,1	219	86,8%
3	70	40	2	1,1	184	59,8%
4	20	0	3	1,1	60	33,3%
5	50	0	2,6	1,1	130	38,5%
6	70	15	2,7	1,1	205,5	41,4%
7	100	65	2,5	1,1	321,5	51,3%
8	40	100	1,2	1,01	149	94,0%
9	80	100	1,2	1,07	203	88,7%
Cases	Copy each of these in turn into Demand and optimize					
P	40	40	40	80	80	80
Q	20	35	60	50	70	100

Convex areas

A1	**A2**	**A2**	**A(j,k)*y(k)**
1	1	0	1
1	0	1	1
1	1	1	1
0	1	0	0
0	1	0	0
0	1	1	0
0	0	1	0
1	0	0	1
1	0	0	1

Demand
80
70

	x(j),y(k)	**Table of results**		Copy optimization results into this table			
x1	0,2105	0,5	0,7105	0,48	6E-17	0,2105	0
x2	0,6842	2E-16	0,1842	0,16	0,3333	0,6842	0
x3	0,1053	0,25	0,1053	0	0,4	0,1053	0

x4	3E-17		0	4E-17	0	4E-17	3E-17	0
x5	2E-17		0,25	4E-17	0	0	2E-17	0
x6	3E-17		0	3E-17	0	0,2667	3E-17	0
x7	0		0	0	0	0	0	0
x8	0		0	0	0,36	0	0	6E-17
x9	0		0	0	0	0	0	1
y1	1		0	1	1	0	1	1
y2	0		1	0	0	0	0	0
y3	0		0	0	0	1	0	0
Sum(x)	1		1	1	1	1	1	1
Sum(y)	1		1	1	1	1	1	1
P	80		40	40	40	80	80	80
Q	70		20	35	60	50	70	100
F	178,89		101,5	92,395	110,76	201,4	178,89	203
Efficiency	83,8%		59,1%	81,2%	90,3%	64,5%	83,8%	88,7%
Obj	3577,9		2030	1847,9	2215,2	4028	3577,9	4060

Comparison with convex model

F	96,486	90,284	110,76	191	176,78	203
Efficienc	62,2%	83,1%	90,3%	68,1%	84,8%	88,7%
Obj	1929,7	1805,7	2215,2	3820	3535,7	4060

Convex model gives often a little higher fuel consumption and a little higher costs, because it represents more accurately the non-convex shape of the characteristic.