Microeconomic Theory II
Helsinki GSE
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Problem Set 2, Due December 1, 2021

1. Consider the college admission problem where n students apply to k colleges. Students have preferences over the colleges and you may assume that their preference depends only on the college that admits them (i.e. their preferences do not depend on their peers). Colleges have similar preferences over the set of students. Assume that all colleges prefer any student to an empty slot and all students prefer all colleges to being left without a college.

   College $j \in \{1, ..., k\}$ has capacity $\overline{x}_j$. A matching of students to colleges is a function $m : \{1, ..., n\} \to \{1, ..., k\}$ such that the inverse image $m^{-1}(j)$ of j has no more than $\overline{x}_j$ elements for all j. A matching m is stable if for all $i \in \{1, ..., n\}$ and all j $\jmath \in \{1, ..., k\}$ such that $j \succ_i m(i)$ implies that $i' \succ_j i$ for all i' such that $m(i') = j$.

   Formulate a matching method that produces a stable match.

2. In a roommate problem $2m$ students must be matched in m pairs to occupy double rooms at a college. The students have rational preferences over their potential roommates. Give an example with $m = 2$ where no stable matching exists.

3. Consider the following exchange economy with two agents and three goods (real Edgeworth Box). Agent 1 has linear preferences represented by the utility function

$$u_1(x_{11}, x_{12}, x_{13}) = x_{11} + 2x_{12} + x_{13},$$

   and agent 2 has utility function

$$u_2(x_{21}, x_{22}, x_{23}) = x_{21} + 3x_{22} + 7x_{23}.$$

   (a) Let the total resources of the three goods be given by: $\overline{x}_1 = \overline{x}_2 = \overline{x}_3 = 3$. What are the Pareto-efficient allocations?

(b) Suppose that the initial endowments of the two agents are: $\omega_{11} = 2, \omega_{12} = 2, \omega_{13} = 1$ and $\omega_{21} = 1, \omega_{22} = 1, \omega_{23} = 2$. Compute the equilibrium prices and the equilibrium allocation for this economy.

4. Consider the exchange economy with two agents $i \in \{1, 2\}$ and two goods, x, y. Agent 1 has Cobb-Douglas utility function

$$u_1(x_1, y_1) = x_1^\alpha y_1^{1-\alpha},$$

and agent 2 has utility function

$$u_2(x_2, y_2) = \min\{x_2, 2y_2\}.$$

(a) Let the initial endowments be $\omega_{1x} = 2, \omega_{1y} = 3, \omega_{2x} = 4, \omega_{2y} = 1$. Draw the Edgeworth rectangle for this economy and compute its Pareto-efficient allocations.

(b) Use the first welfare theorem to find competitive equilibrium prices and allocation.

5. Assume that the agents in an exchange economy have the same differentiable, strictly increasing and strictly quasi-concave utility function that is homogenous of degree 1 and that the aggregate endowment of all goods is positive. The agents may have different initial endowments.

Assume that $u(x_1, ..., x_L) \to -\infty$ if $x_l \to 0$ for some l (so you can assume interior optimal demands).

(a) Find the Pareto-efficient allocations for this economy.

(b) Find the Walrasian equilibria for this economy.

6. Consider the exchange economy with two agents $i \in \{1, 2\}$ and two goods, x, y . Agent 1 thinks the goods are perfect substitutes

$$u_1(x_1, y_1) = x_1 + y_1,$$

and agent 2 has utility function where the goods are complements

$$u_2(x_2, y_2) = \min\{x_2, y_2\}.$$

(a) Let the initial endowments be $\omega_{1x} = 1, \omega_{1y} = 0, \omega_{2,x} = 3, \omega_{2y} = 2$. Draw the Edgeworth rectangle for this economy and compute its Pareto-efficient allocations.

(b) Show that this economy has no competitive equilibria.