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Engineering

E4230
Microwave EO Instrumetation

(5 cr)

Jaan Praks
Aalto University

Can you explain?

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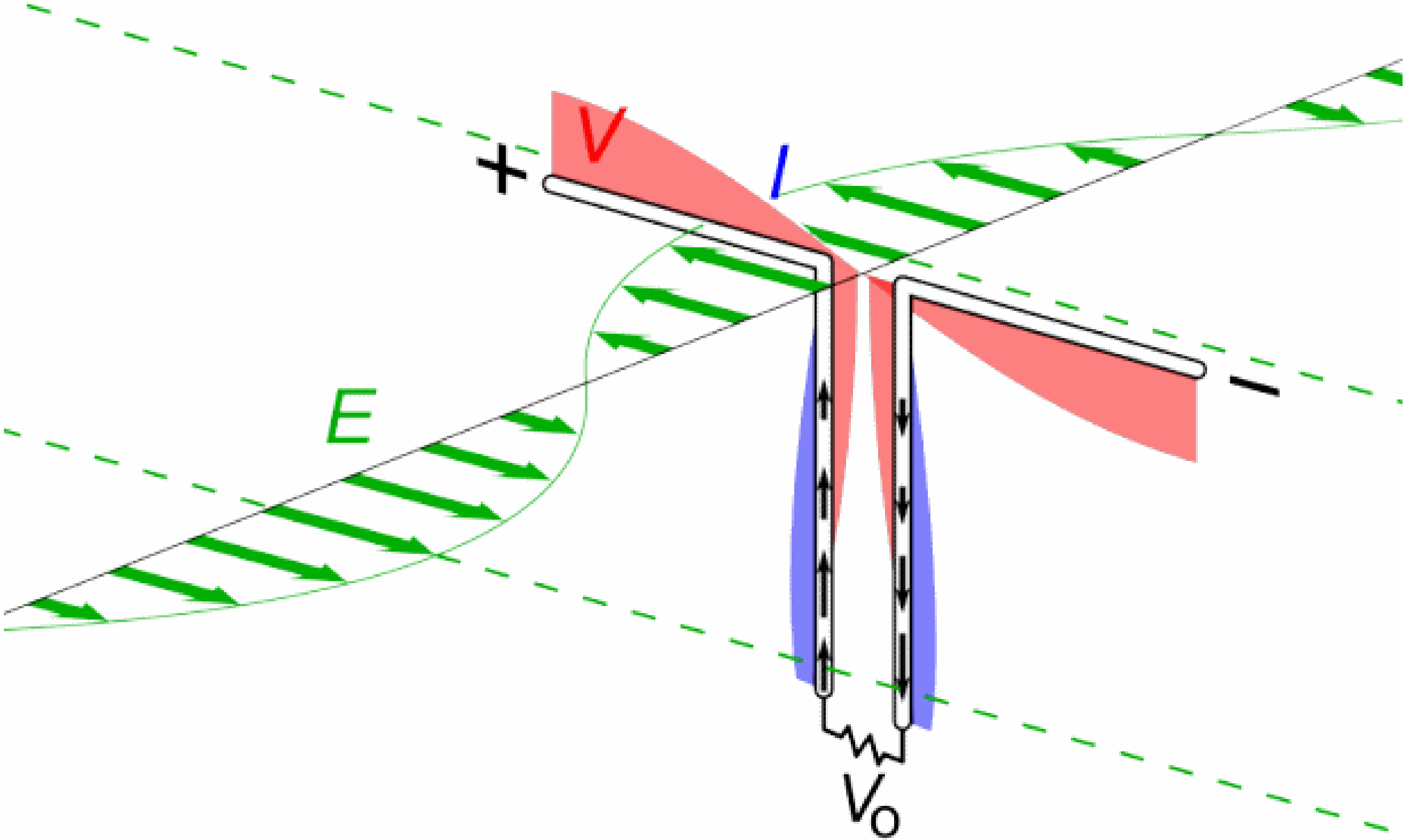

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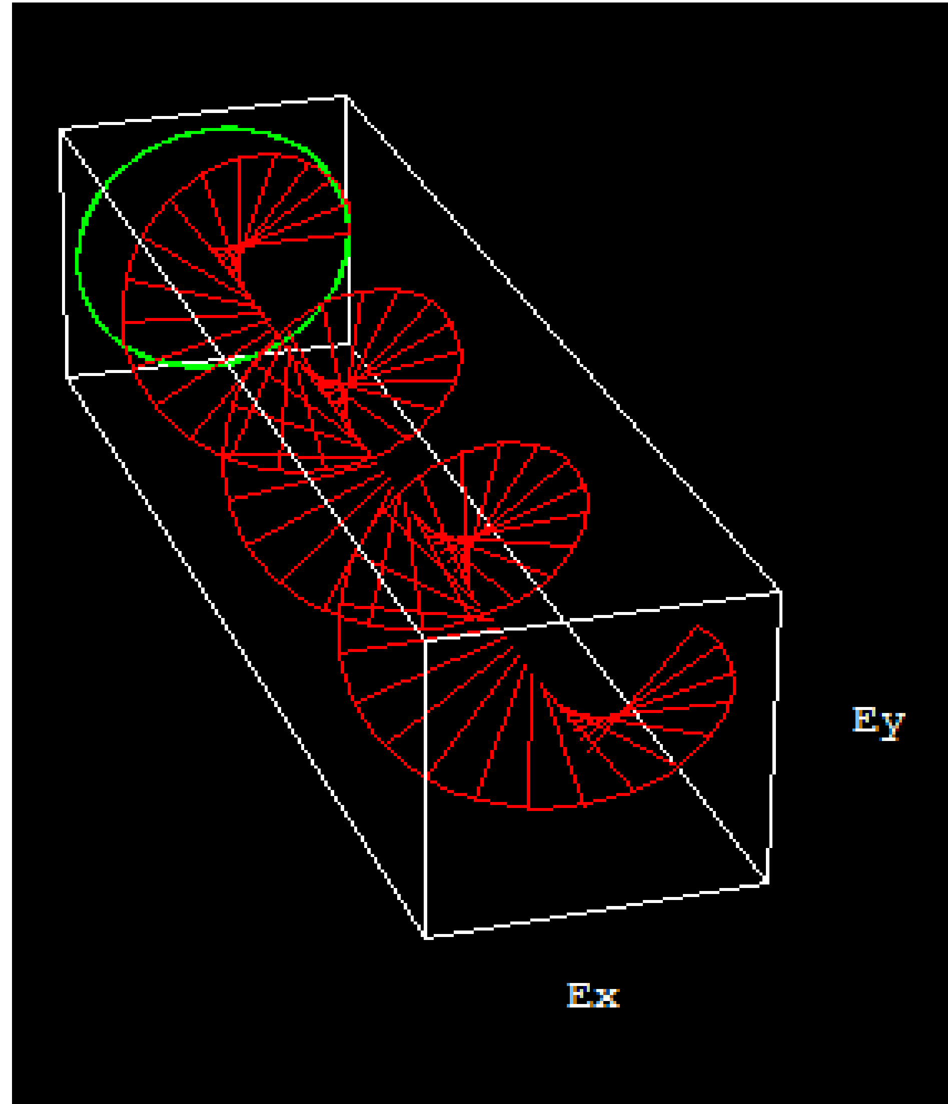
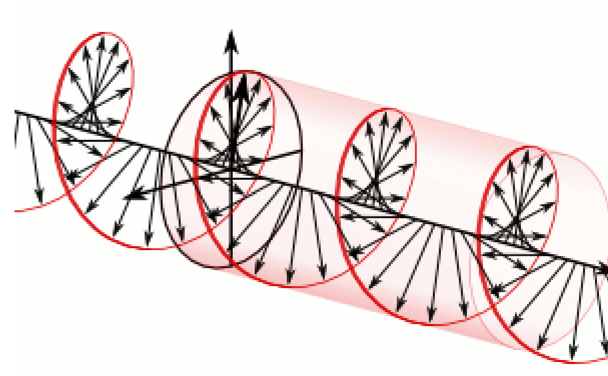
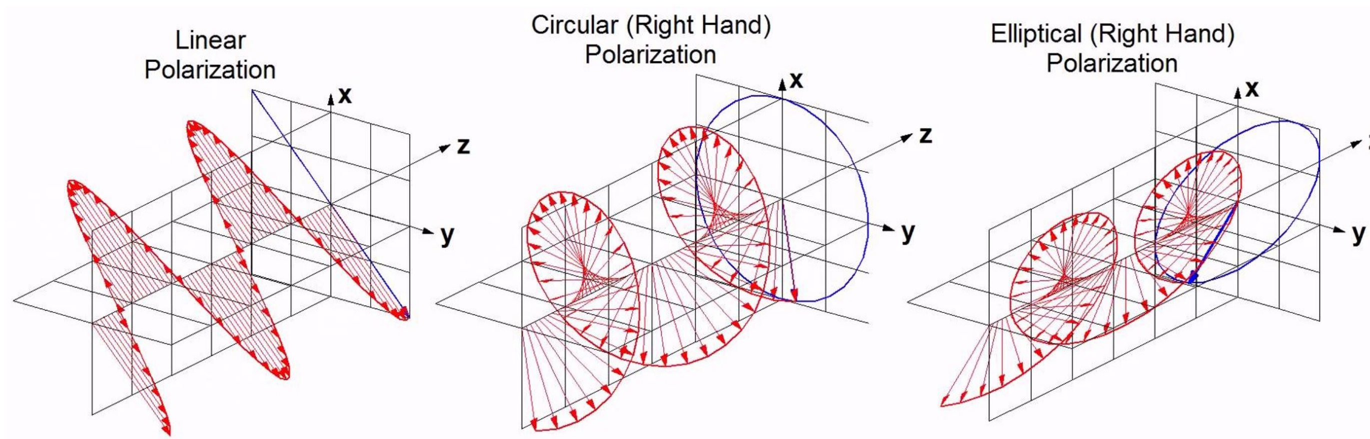
Polarization



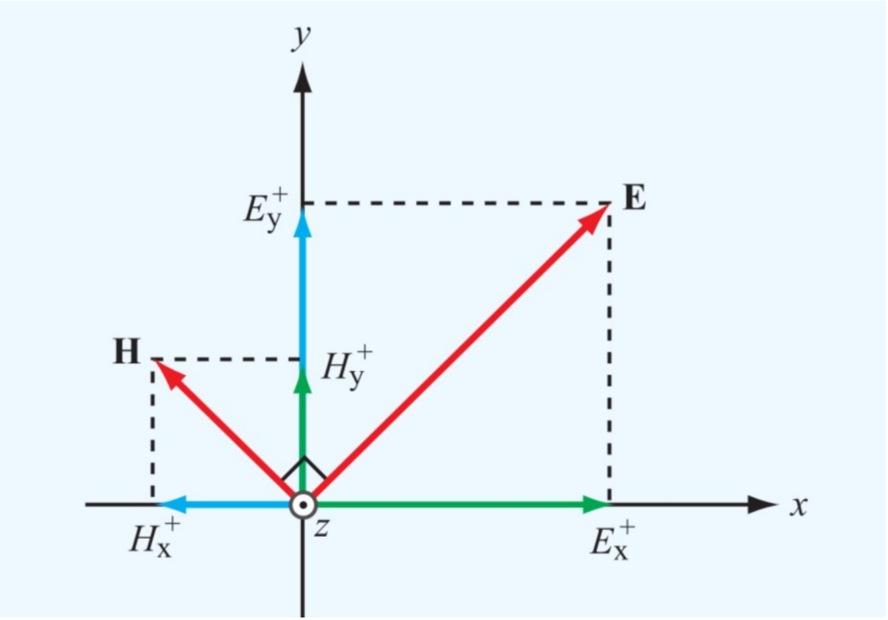
Polarization

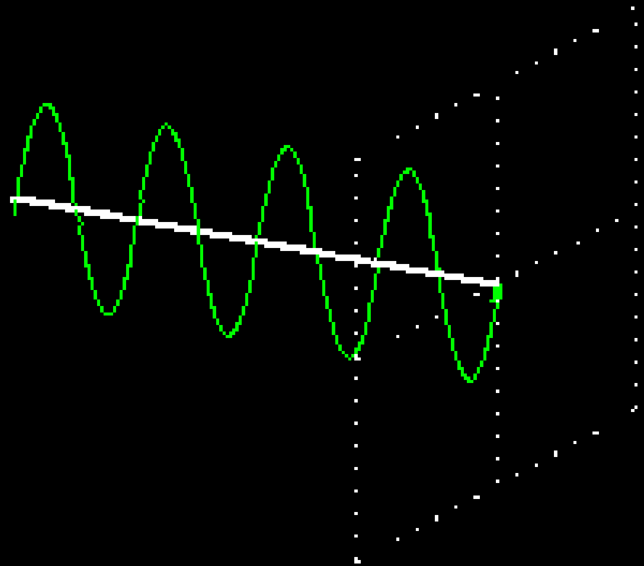
The polarization of a uniform plane wave describes the locus traced by the tip of the E vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time.

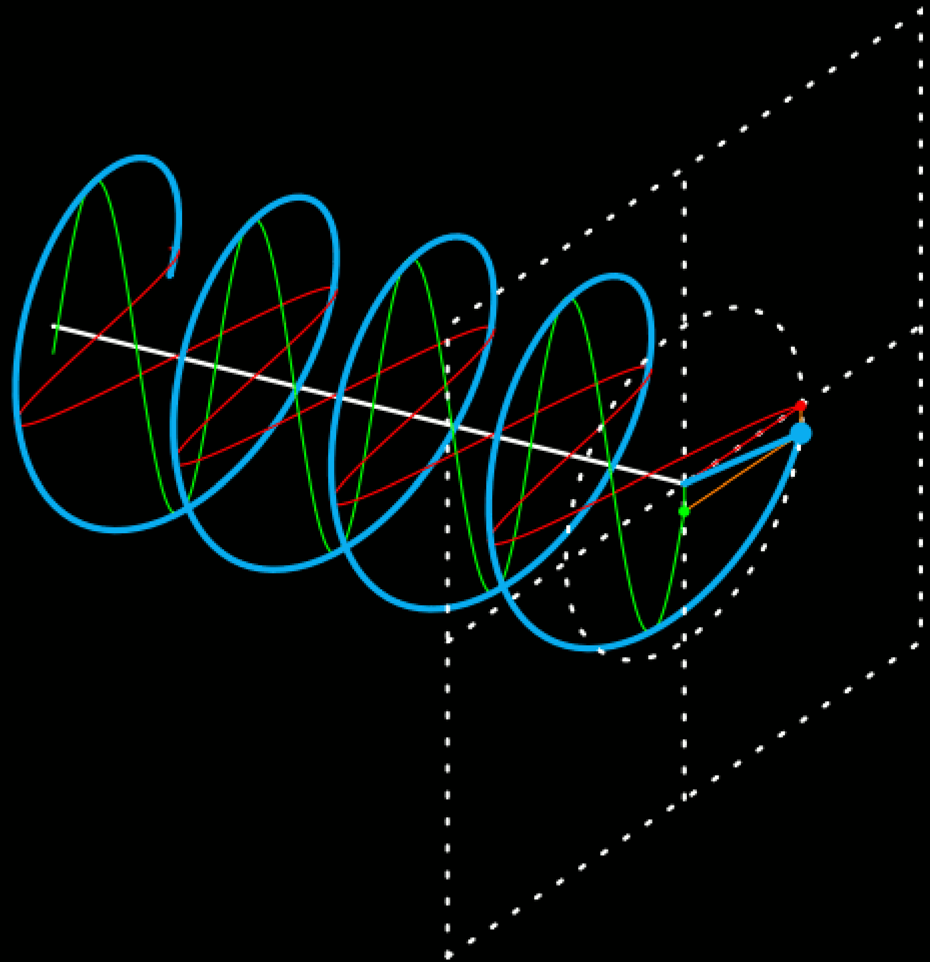


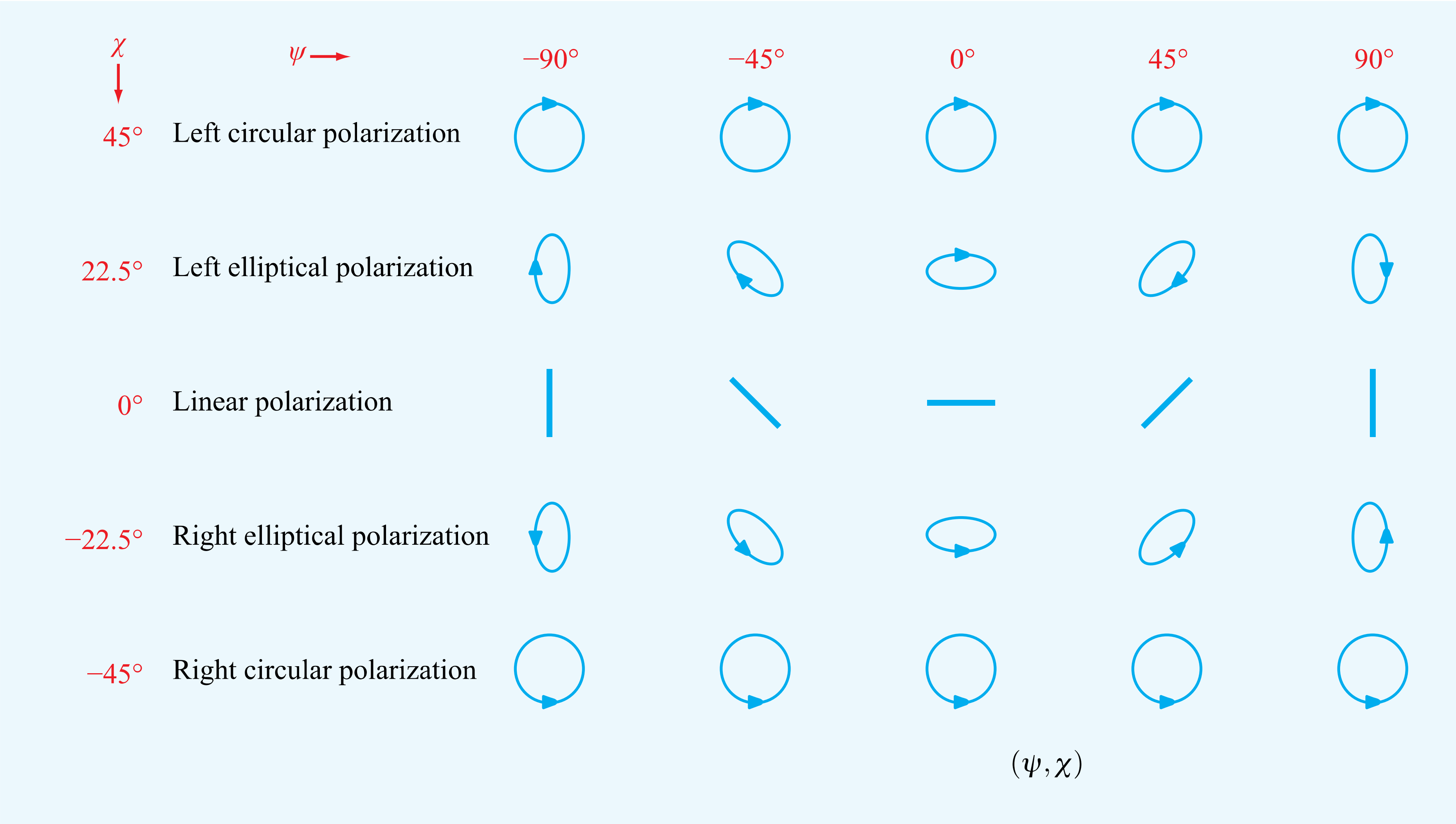


$$\mathbf{E}(z) = \hat{\mathbf{x}} E_x(z) + \hat{\mathbf{y}} E_y(z)$$








χ
ψ
−90°
−45°
0°
45°
90°
45° Left circular polarization
22.5° Left elliptical polarization
0° Linear polarization
−22.5° Right elliptical polarization
−45° Right circular polarization
(ψ, χ)

Polarization ellipse

$$\mathbf{E} = \begin{bmatrix} E_\mathrm{v} \\ E_\mathrm{h} \end{bmatrix}, \qquad (5.3)$$

$$E_\mathrm{v} = a_\mathrm{v}, \qquad (5.4a)$$

$$E_\mathrm{h} = a_\mathrm{h} e^{j\delta}, \qquad (5.4b)$$

$$\tan 2\psi = (\tan 2\alpha_0)\cos\delta = \frac{2a_\mathrm{v}a_\mathrm{h}}{a_\mathrm{v}^2 - a_\mathrm{h}^2}\cos\delta \qquad (5.5a)$$

$$(-\pi/2 \leq \psi \leq \pi/2),$$

$$\sin 2\chi = (\sin 2\alpha_0)\sin\delta = \frac{2a_\mathrm{h}a_\mathrm{v}}{a_\mathrm{h}^2 + a_\mathrm{v}^2}\sin\delta \qquad (5.5b)$$

$$(-\pi/4 \leq \chi \leq \pi/4),$$

$$\tan\alpha_0 = \frac{a_\mathrm{h}}{a_\mathrm{v}}. \qquad (5.6)$$

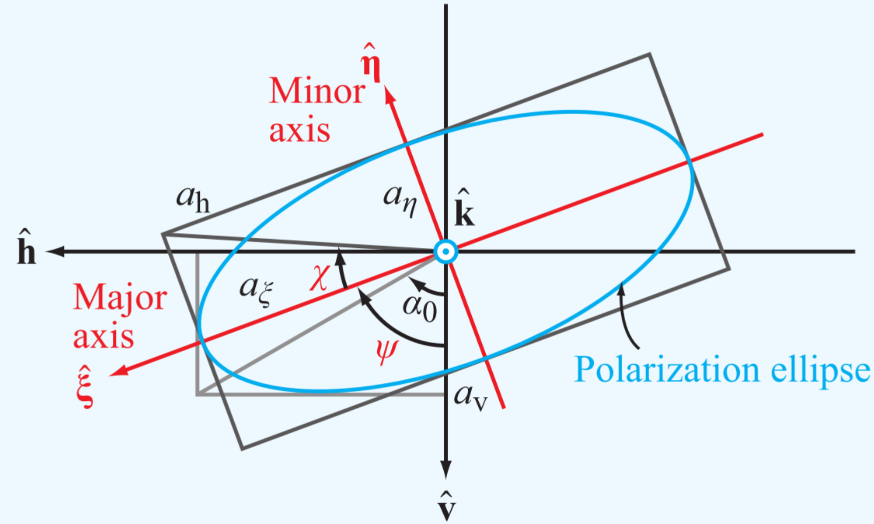


Figure 5-2: Polarization ellipse in the v–h plane for a wave traveling in the $\hat{\mathbf{k}}$ direction.

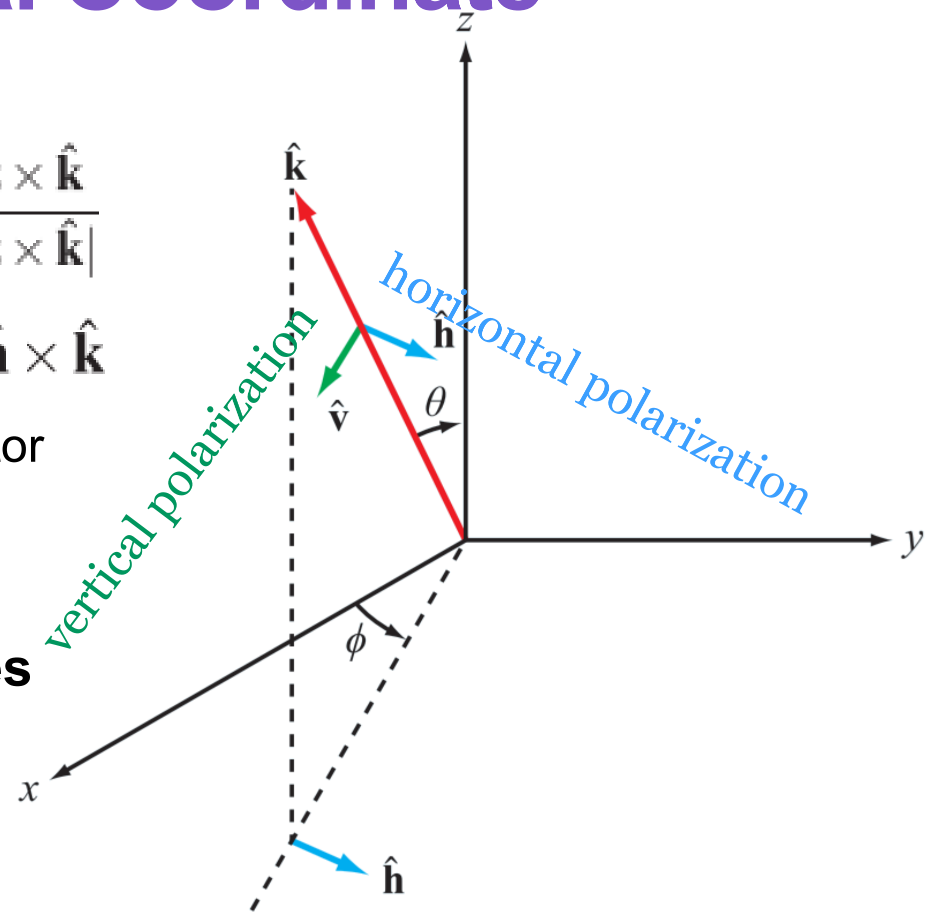
EM wave in Spherical Coordinate System

$$\hat{\mathbf{h}} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{k}}}{|\hat{\mathbf{z}} \times \hat{\mathbf{k}}|}$$

$$\mathbf{E} = (\hat{\mathbf{v}} E_{\mathrm{v}} + \hat{\mathbf{h}} E_{\mathrm{h}}) e^{-jk\hat{\mathbf{k}}\cdot\hat{\mathbf{R}}}$$

$$\hat{\mathbf{v}} = \hat{\mathbf{h}} \times \hat{\mathbf{k}}$$

propagation phase factor



$$\mathbf{E} = \begin{bmatrix} E_{\mathrm{v}} \\ E_{\mathrm{h}} \end{bmatrix}$$

Complex amplitudes

Relation between E^i and E^s

In the case of all E components

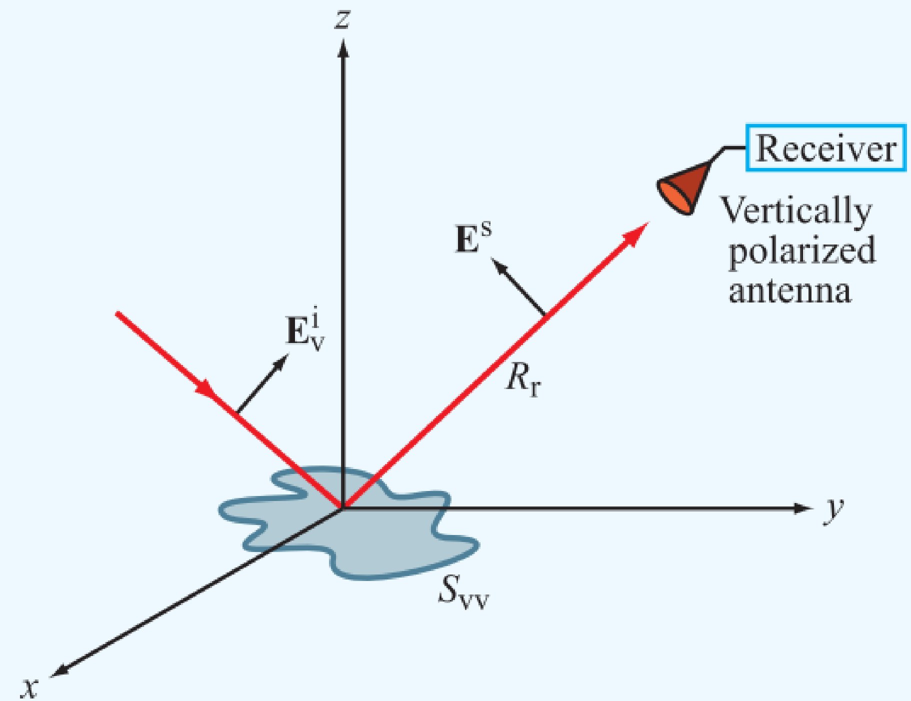
$$\mathbf{E}^{s} = \quad ? \quad \mathbf{E}^{i}$$

$$\mathbf{E}^{i} = \hat{\mathbf{v}}_{i} E_{v}^{i} + \hat{\mathbf{h}}_{i} E_{h}^{i},$$

$$\mathbf{E}^{s} = \hat{\mathbf{v}}_{s} E_{v}^{s} + \hat{\mathbf{h}}_{s} E_{h}^{s},$$

⇩

$$\begin{bmatrix} E_{v}^{s} \\ E_{h}^{s} \end{bmatrix} = \left(\frac{e^{-jkR_{r}}}{R_{r}}\right) \begin{bmatrix} \widetilde{S}_{vv} & \widetilde{S}_{vh} \\ \widetilde{S}_{hv} & \widetilde{S}_{hh} \end{bmatrix} \begin{bmatrix} E_{v}^{i} \\ E_{h}^{i} \end{bmatrix}$$



$\widetilde{S}$ is the **scattering matrix**

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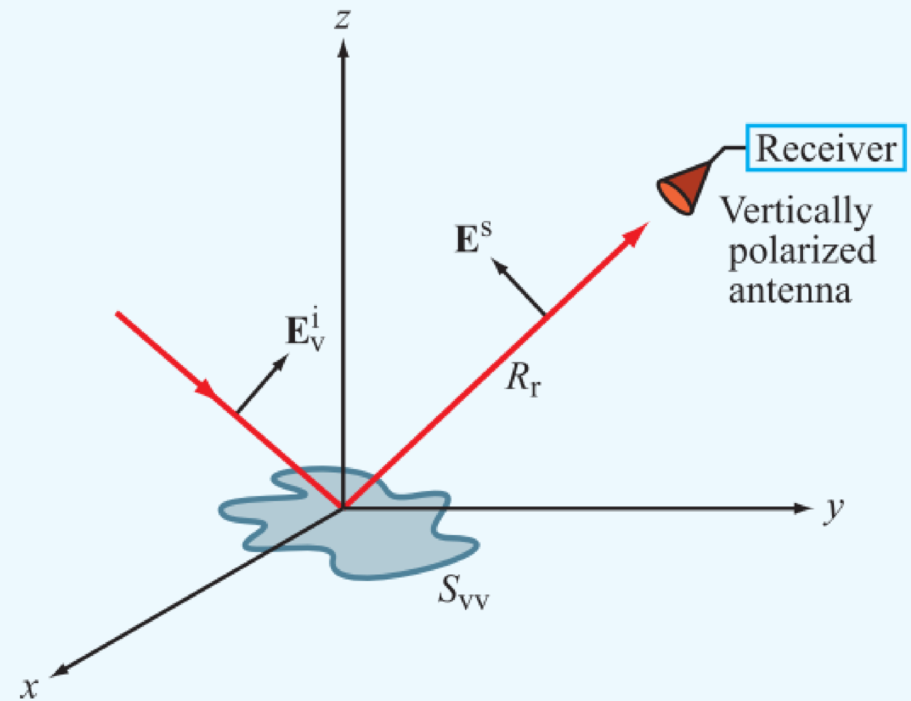
Scattering matrix in BSA

$$\mathbf{S} = \begin{pmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{pmatrix} \qquad \textbf{(BSA)}$$

$$S_{vh} = S_{hv} \qquad \textbf{(backscatter)}.$$

Because of **reciprocity theorem**



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Polarization synthesis

When scattering matrix is known, scattered wave can be calculated to ANY incident wave, with arbitrary polarization!

Calculation of response for arbitrary polarization is called polarization synthesis.

Scattering matrix can be transformed to different polarization bases.

Scattering matrix connects any arbitrary polarized incident wave to scattered wave

$$\begin{bmatrix} E_{\mathrm{v}}^{\mathrm{s}} \\ E_{\mathrm{h}}^{\mathrm{s}} \end{bmatrix} = \left(\frac{e^{-jkR_{\mathrm{r}}}}{R_{\mathrm{r}}} \right) \begin{bmatrix} \widetilde{S}_{\mathrm{vv}} & \widetilde{S}_{\mathrm{vh}} \\ \widetilde{S}_{\mathrm{hv}} & \widetilde{S}_{\mathrm{hh}} \end{bmatrix} \begin{bmatrix} E_{\mathrm{v}}^{\mathrm{i}} \\ E_{\mathrm{h}}^{\mathrm{i}} \end{bmatrix}$$

Connection between σ and S

For simple point target the relation is:

$$\sigma_{pq} = 4\pi |\widetilde{S}_{pq}|^2.$$

$$p, q = \text{v or h}$$

$$\sigma_{pq} = \lim_{R_r \to \infty} \left(4\pi R_r^2 \frac{\mathcal{S}_p^s}{\mathcal{S}_q^i} \right)$$

$$\mathcal{S}_p^s = |E_p^s|^2 / 2\eta_0$$

$$\mathcal{S}_q^i = |E_q^s|^2 / 2\eta_0$$

$$\widetilde{S}_{pq} = \widetilde{S}_{pq}(\theta_i, \phi_i;\ \theta_s, \phi_s; \theta_j, \phi_j) = \lim_{R_r \to \infty} \left[R_r e^{-jkR_r} \left(\frac{E_p^s}{E_q^i} \right) \right]$$

Polarization synthesis

Backscattering coefficient depends on antenna polarization!

$$\sigma_{0tr} = K \begin{bmatrix} E_x^r \\ E_y^t \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} = K\boldsymbol{E}^r S \boldsymbol{E}^t$$

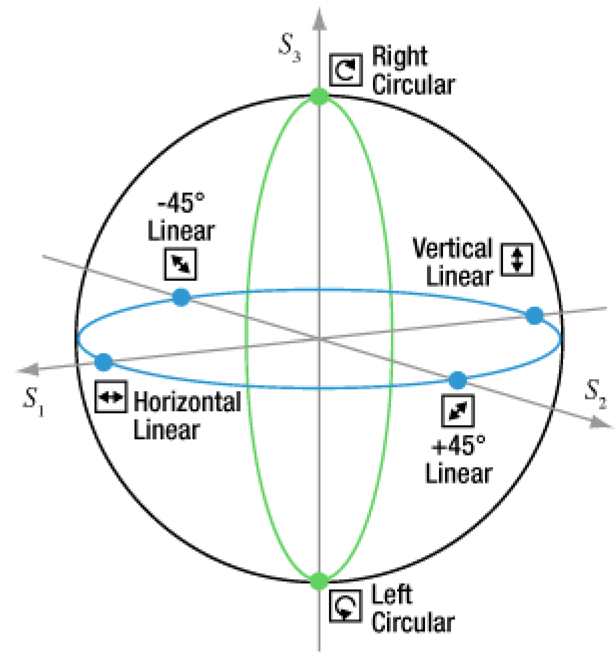
By knowing scattering matrix, backscattering is possible to calculate for ANY antenna polarization combination!

$$K = \frac{e^{-jkR_r}}{R_r}$$

Scattering matrix in different polarimetric basis

Scattering matrix can be presented in many orthogonal polarization basis systems.

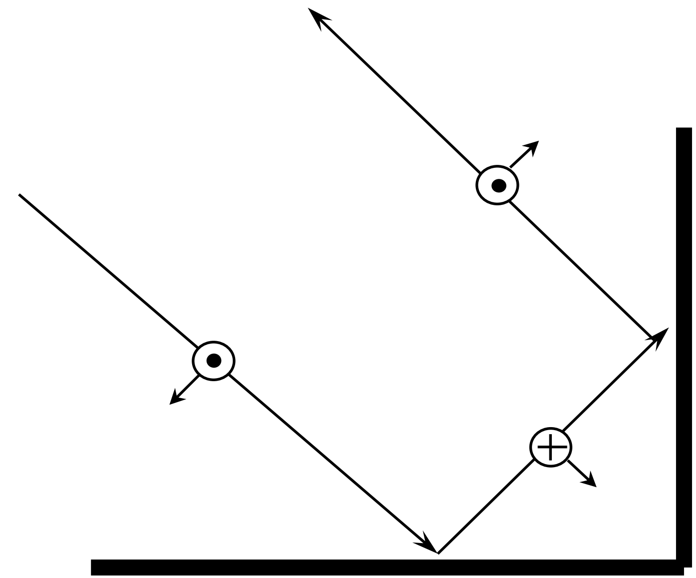
- horizontal – vertical,
- right-left circular
- -45 - +45 linear

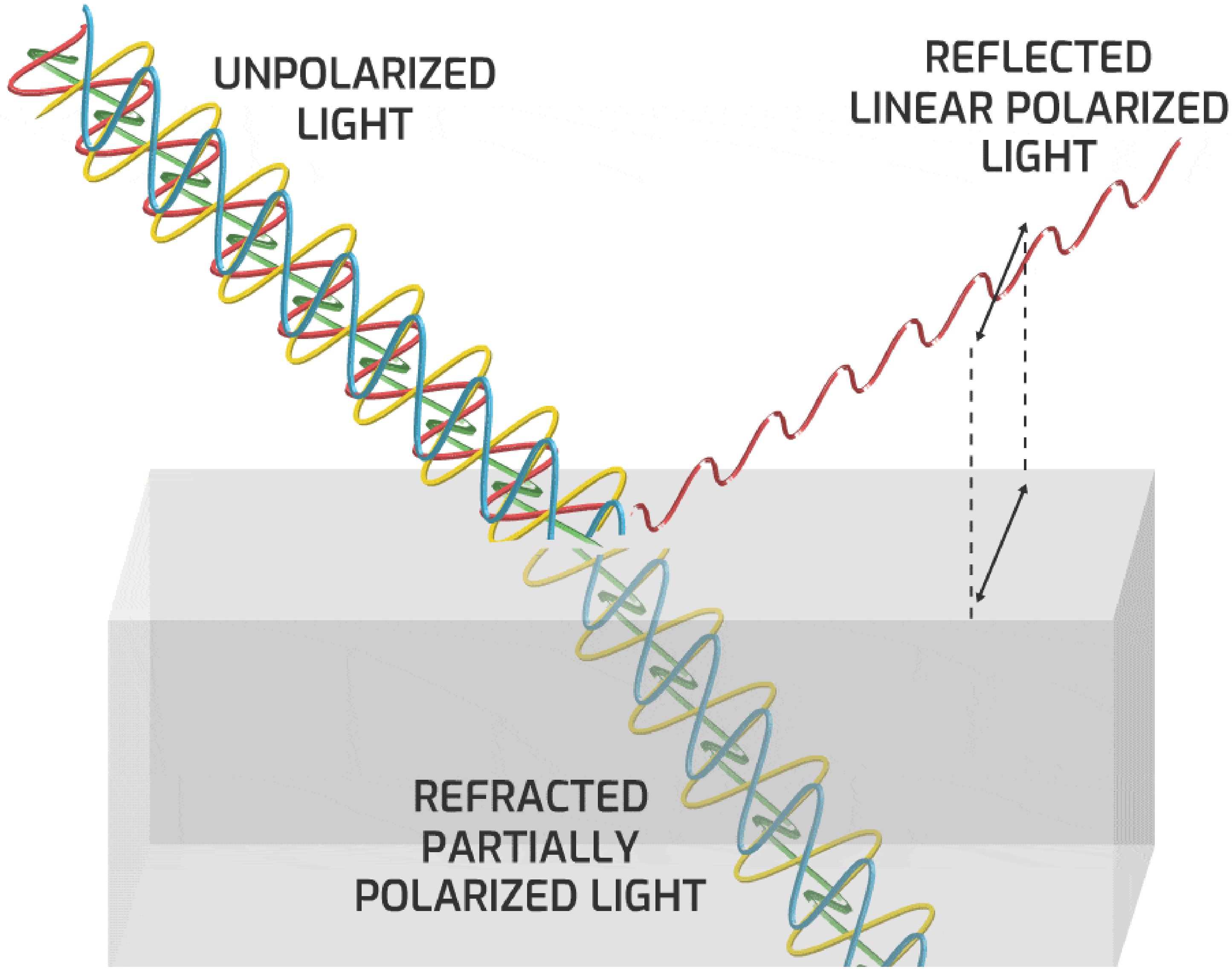


Orthogonal basis points are located on the opposite side of Poincare sphere

Why to measure polarization?

- When the wave is reflected the polarization of reflected wave depends on the polarization of incident wave and the structure and dielectric constant of the reflected objects

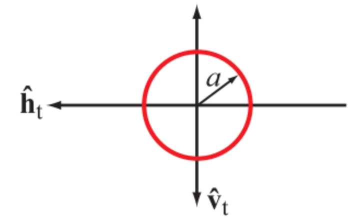



UNPOLARIZED
LIGHT
REFLECTED
LINEAR POLARIZED
LIGHT
REFRACTED
PARTIALLY
POLARIZED LIGHT

Basic backscattering mechanisms

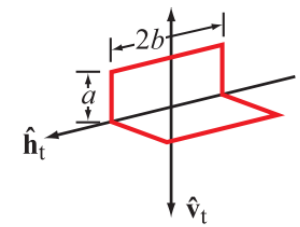
- Single reflection

$$\mathbf{S} = \frac{a}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \textbf{(metal sphere)}. \qquad (5.139)$$



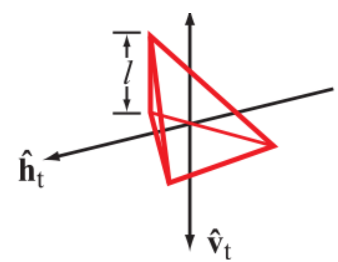
- Double reflection

$$\mathbf{S} = \frac{k_0 ab}{\pi}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (5.140)$$



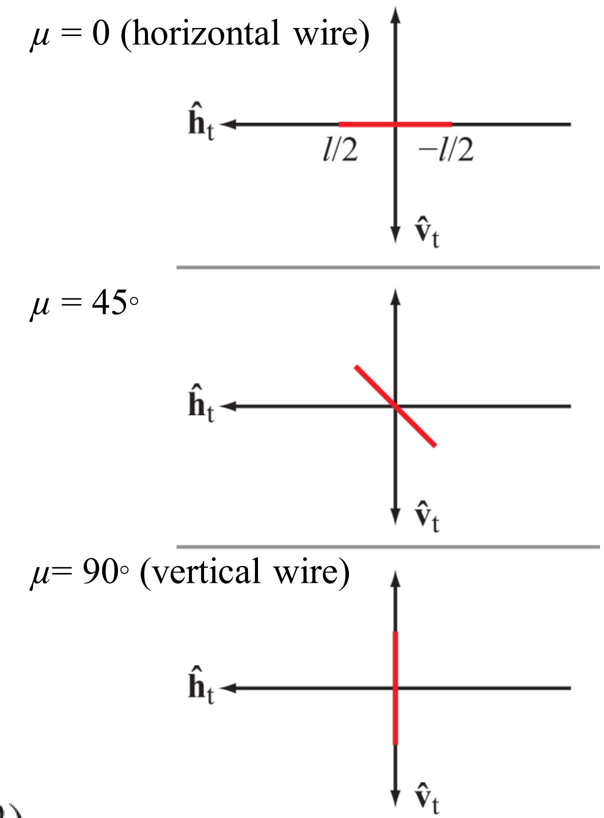
- Trihedral reflector

$$\mathbf{S} = \frac{k_0 l^2}{\sqrt{2}\,\pi}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (5.141)$$



Scattering matrix of thin cylinder target

Polarization response can be estimated and for many target polarization change is possible to calculate analytically.



Scattering matrix for conducting cylinder

$$\mathbf{S} = \frac{k_0^2 l^3}{3[\ln(4l/a) - 1]} \begin{pmatrix} \sin^2\mu & -\sin\mu\cos\mu \\ -\sin\mu\cos\mu & \cos^2\mu \end{pmatrix}. \quad (5.142)$$

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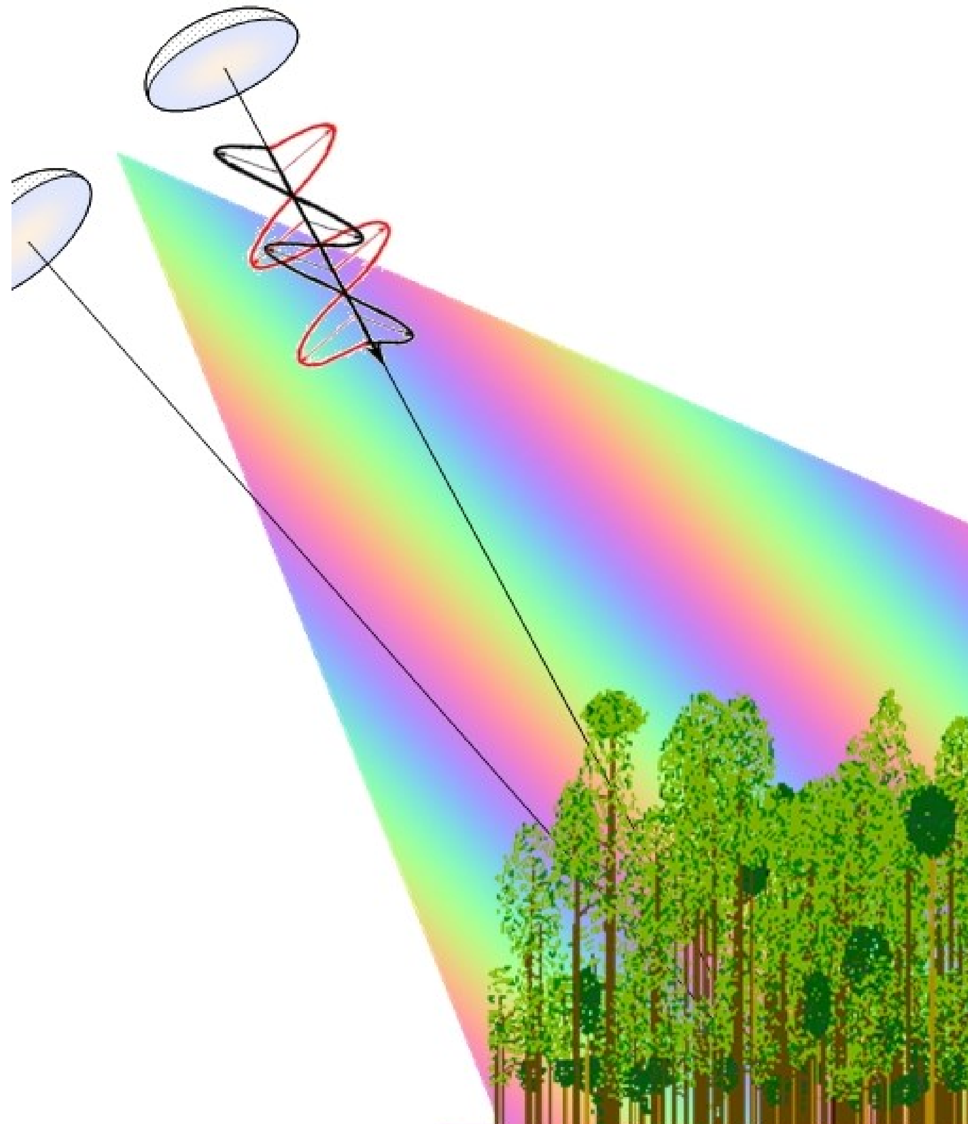
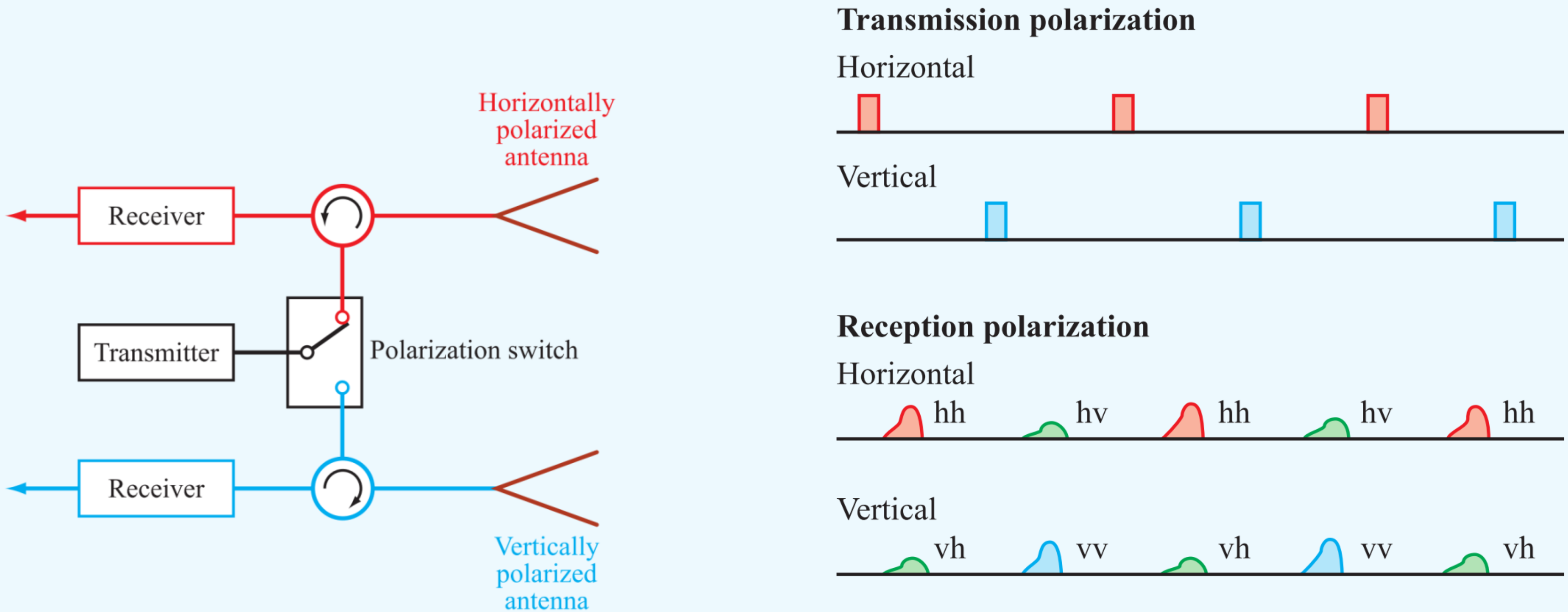
Polarimetric radar





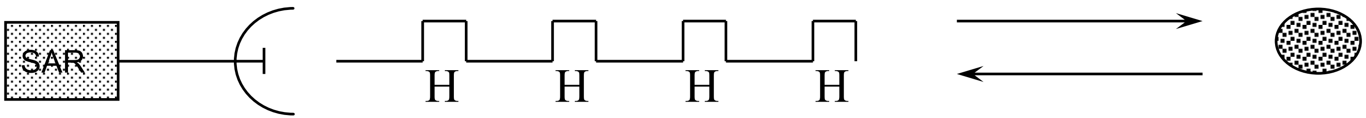
(a) Block diagram

(b) Timing diagram

Figure 5-36: Calibration of polarimetric radar. A polarimetric radar is implemented by alternately transmitting signals out of horizontally and vertically polarized antennas and receiving at both polarizations simultaneously. Two pulses are needed to measure all the elements in the scattering matrix [van Zyl and Kim,

Polarimetric Measurement

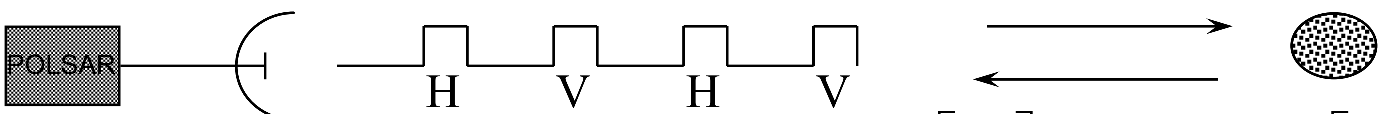
Measuring one polarization



$$E_x^r = K\left[S_{hh}\right]E_x^t$$

Polarimertic SAR measures scattering matrix

for most media Shv=Svh (reciprocity)



$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$

Backscattering for arbitrary polarization !

$$\sigma_{0tr} = K \begin{bmatrix} E_x^r \\ E_y^t \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} = KE^r SE^t$$

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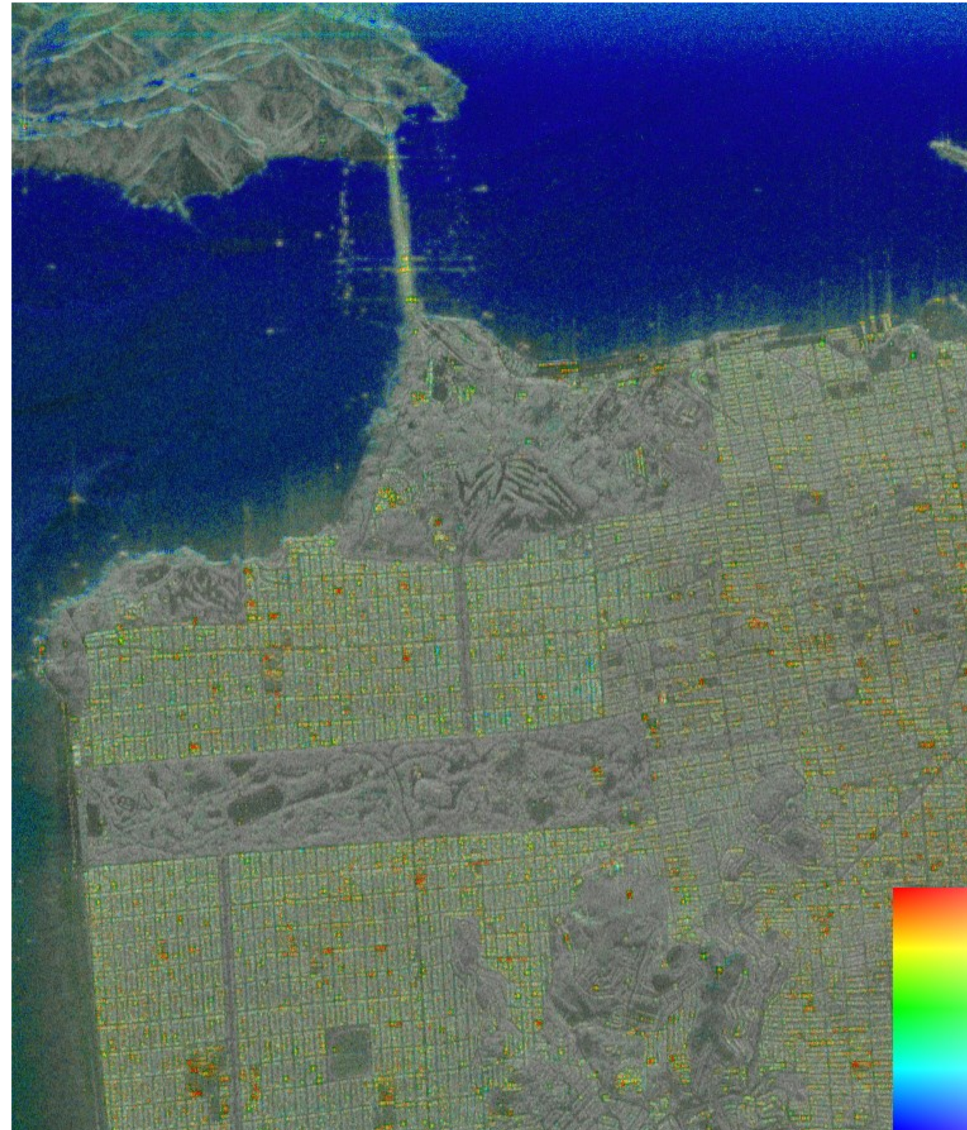
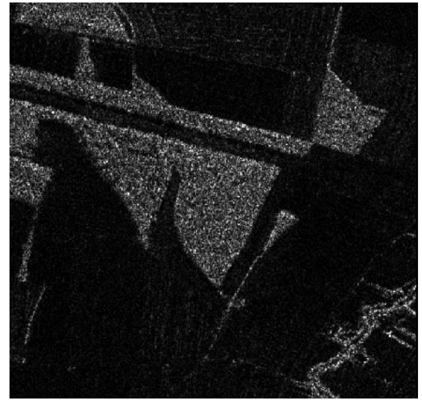
Polarimetric image

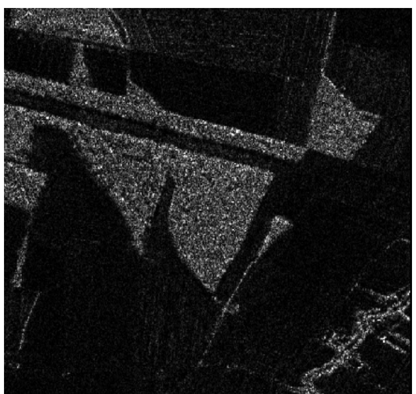


S_{hh}

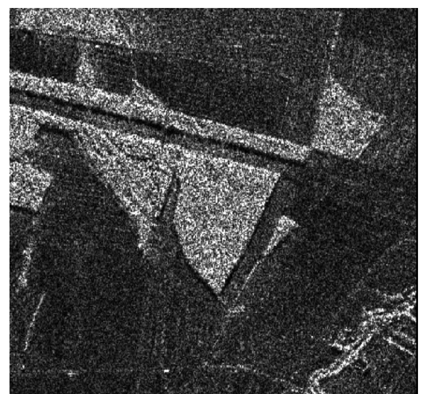


S_{hv}

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$



S_{vh}



S_{vv}

Shh

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$

Svv

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$

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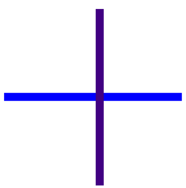
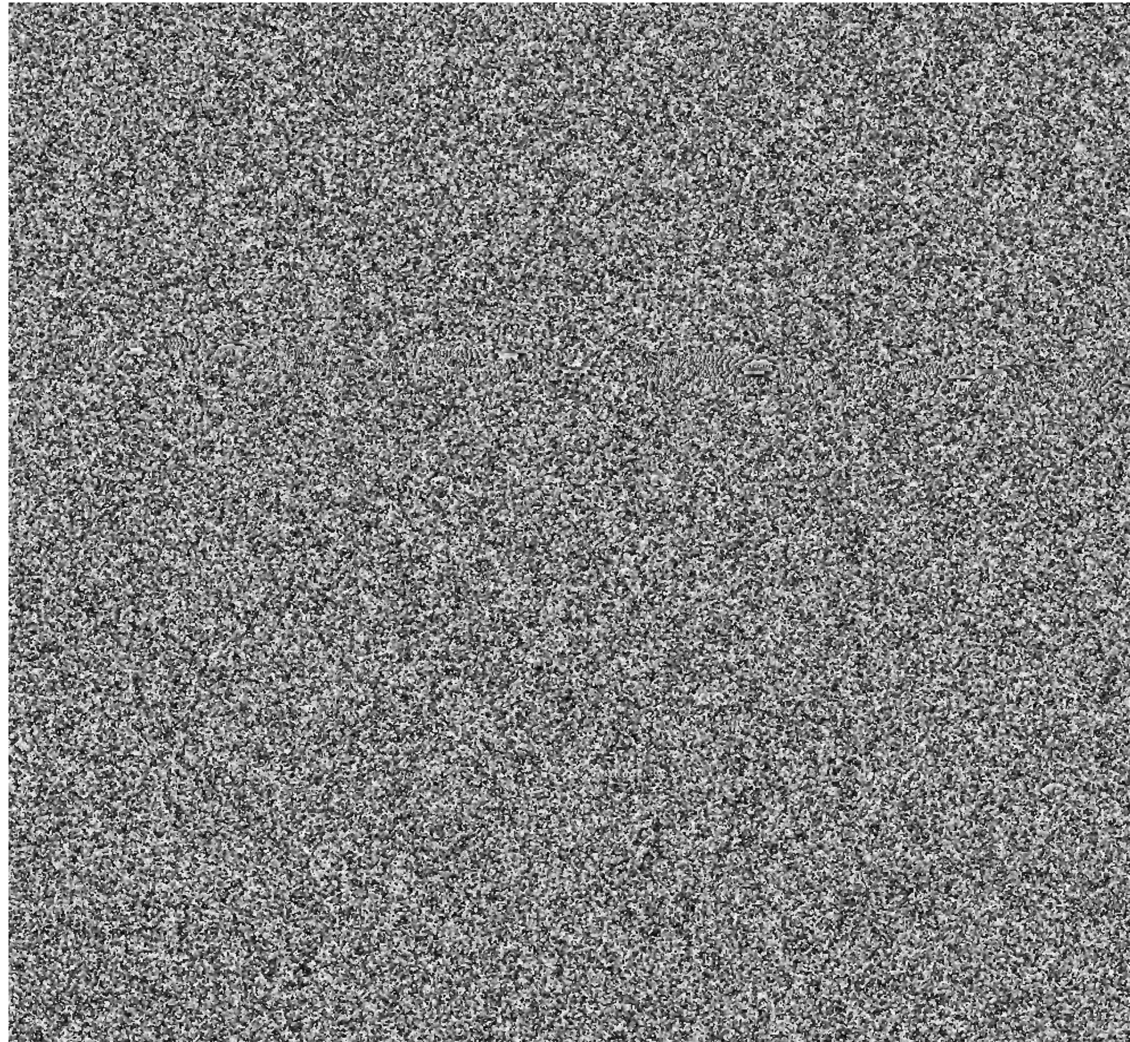
Shv

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$



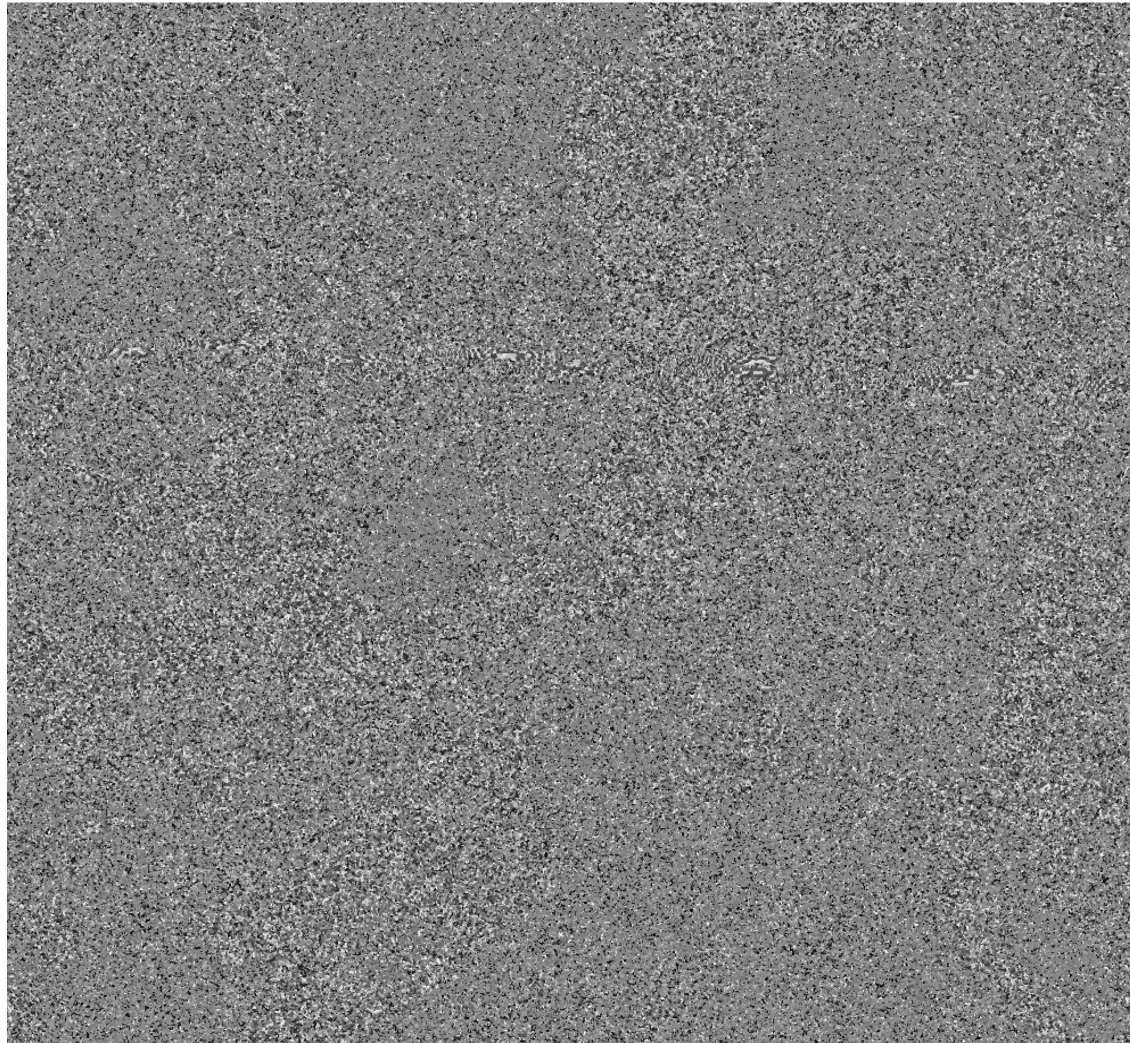
Phase Shh

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$



Phase
Shh-Svv

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$



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Span

Rotating antennas

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Visualizing Polarimetric image

Pauli RGB

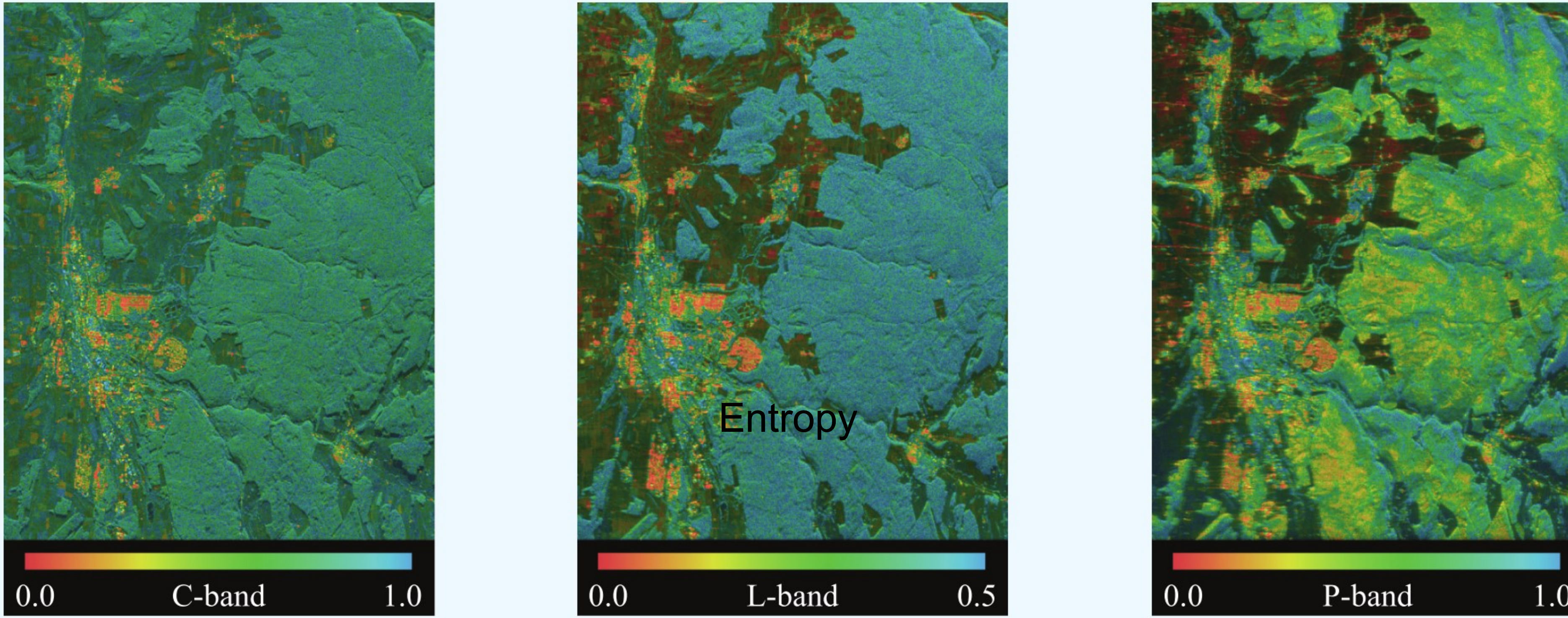
$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = K \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix}$$

Polarimetric Indices

Polarization channels allow calculation of various linear combinations of channels with special interpretation:

$$RVI = \frac{\frac{1}{8\sigma HV}}{\frac{1}{\sigma HH} + \frac{1}{\sigma VV} + \frac{1}{2\sigma HV}}$$

Radar Vegetation Index



(a) C-band (b) L-band (c) P-band

Figure 5-48: RVI images of the area shown in Fig. 5-47 at three different frequencies. The RVI is scaled from 0 (black) to 1 (white). Note that the L-band RVI in the forested area is higher than the C-band RVI, while the C-band RVI is higher than the others in the agricultural areas..

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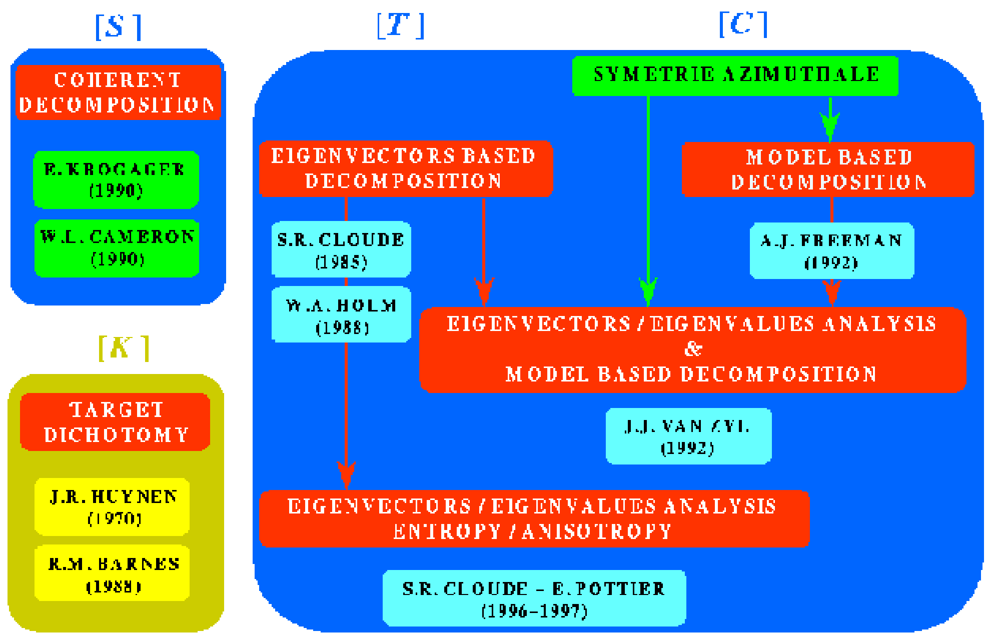
Decompositions

Various decompositions schemes present measured target as sum of simple known targets



Overview of Decomposition Theorems (by Eric Pottier)

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Point target coherent decompositi ons

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Pauli Decomposition

Pauli spin matrixes

Pauli matrices are a set of three 2 × 2 complex matrices which are Hermitian and unitary.

Any 2x2 complex matrix can be represented as a sum of unitary matrix and Pauli matrices:

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Wolfgang Pauli (1900–1958)

$$\mathbf{S} = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \delta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Polarimetric Pauli Decomposition

Scattering matrix can be presented as a sum of Pauli matrixes (in Pauli basis).

$$[S] = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix} = \alpha [S]_a + \beta [S]_b + \gamma [S]_c$$

where:

$$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}}$$

$$\beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}}$$

$$\gamma = \sqrt{2} S_{hv}$$

$$[S]_a = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[S]_b = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

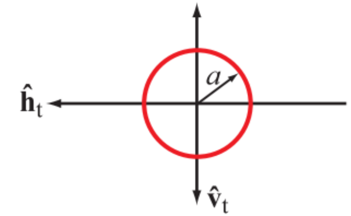
$$[S]_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$[S]_d = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

What color?

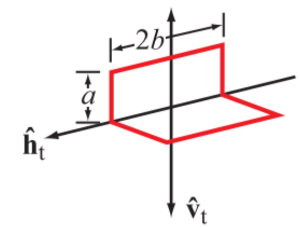
- Single reflection

$$\mathbf{S} = \frac{a}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \textbf{(metal sphere)}. \qquad (5.139)$$



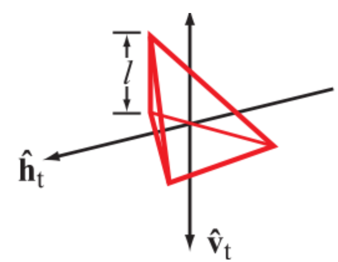
- Double reflection

$$\mathbf{S} = \frac{k_0 ab}{\pi}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (5.140)$$



- Trihedral reflector

$$\mathbf{S} = \frac{k_0 l^2}{\sqrt{2}\,\pi}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (5.141)$$



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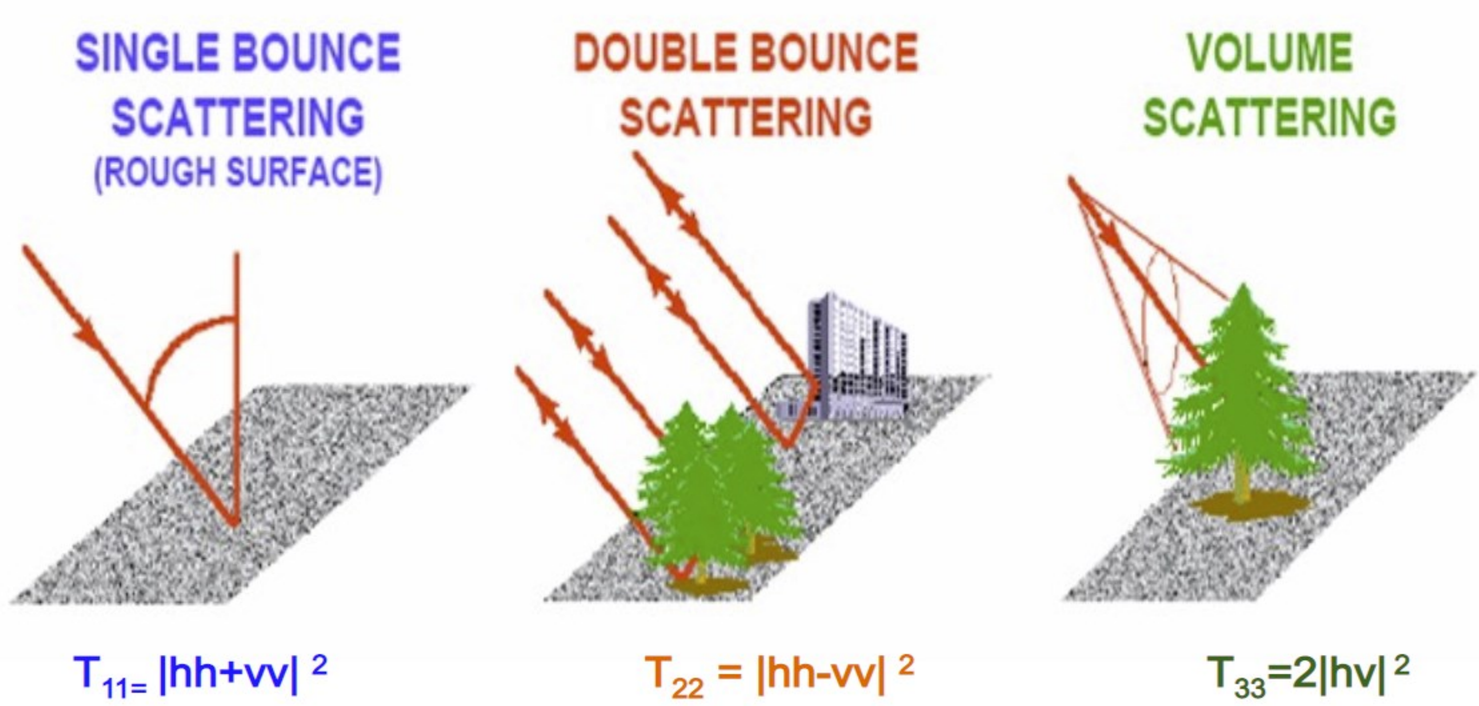
SINGLE BOUNCE SCATTERING (ROUGH SURFACE)
DOUBLE BOUNCE SCATTERING
VOLUME SCATTERING
$T_{11} = |hh+vv|^2$
$T_{22} = |hh-vv|^2$
$T_{33} = 2|hv|^2$

Pauli decomposition

R=Shh-Svv
G=Shv+Svh
B=Shh+Svv

Shh-Svv is low for single reflection

Shh+Svv is low for double reflection

Pauli decomposition

Boxcar average

R=Shh-Svv
G=Shv
B=Shh+Svv

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Pauli decomposition visualisation

Benefits:

- Simple interpretation
- Easy to calculate

Drawbacks:

- Color scheme is always different

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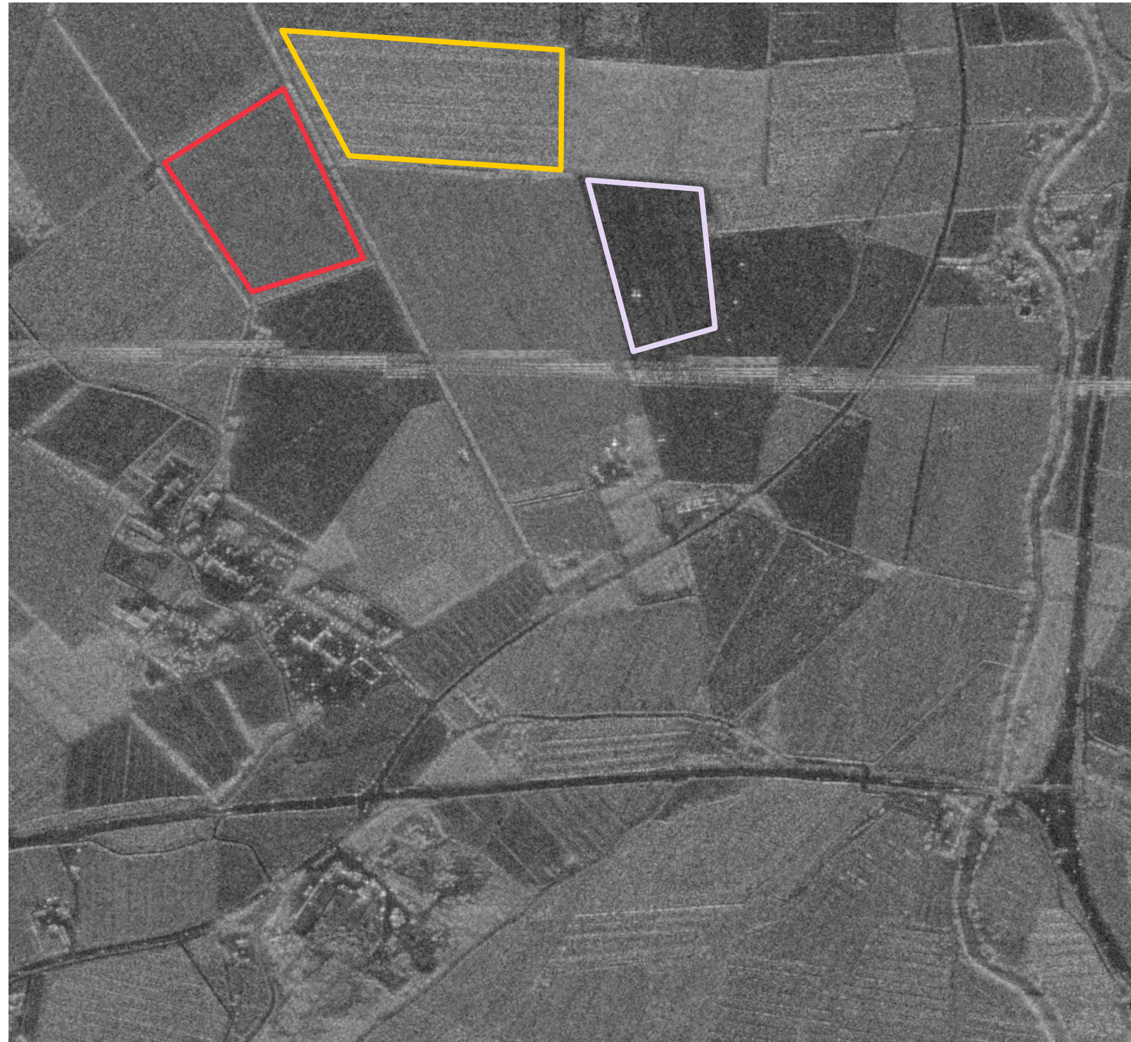
Distributed targets and polarization

Distributed targets

We are interested in areas.

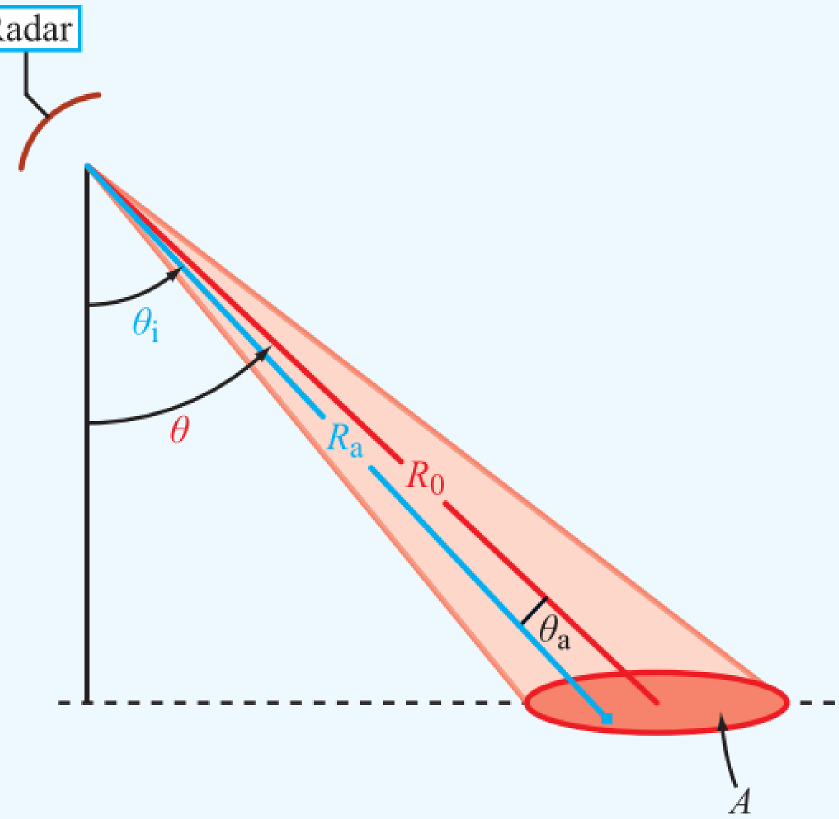
Areas have statistical properties.



When imaging land, the target is not point target!

$$P_p^{\mathrm{r}}(\theta) = \iint_A \frac{P_q^{\mathrm{t}}\, G^2(\theta_{\mathrm{a}}, \phi_{\mathrm{a}})\, \lambda^2}{(4\pi)^3 R_{\mathrm{a}}^4} \cdot \sigma_{pq}^0 \, dA$$

$$\sigma_{pq}^0 = \sigma_{pq}/A$$



backscattering cross section per unit area
backscattering coefficient
radar reflectivity
are the same parameter

From: Microwave Radar and Radiometric Remote Sensing, by Ulaby and Long, 2014, with permission.

Distributed target

For a distributed target occupying Nc cells each of area A, its polarization synthesis equation is given by

$$\sigma^0_{\mathrm{rt}}(\psi_\mathrm{r}, \chi_\mathrm{r}; \psi_\mathrm{t}, \chi_\mathrm{t}) = \frac{4\pi}{AN_\mathrm{c}} \sum_{i=1}^{N_\mathrm{c}} |\mathbf{p}^\mathrm{r} \cdot \mathbf{S}_i \mathbf{p}^\mathrm{t}|^2$$

$$= \frac{4\pi}{A} \left\langle |\mathbf{P}^\mathrm{r} \cdot \mathbf{S}\mathbf{p}^\mathrm{t}|^2 \right\rangle, \dagger \qquad (5.143)$$

(distributed target)

Covariance matrix preserves statistical relationships between scattering matrix elements!

$$\vec{k}_B = [S_{HH}, S_{HV}, S_{VH}, S_{VV}]^T$$

$$[C]_{4\times4} = \langle \vec{k}_B \vec{k}_B^{t} \rangle = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & \langle S_{HH} S_{HV}^* \rangle & \langle S_{HH} S_{VH}^* \rangle & \langle S_{HH} S_{VV}^* \rangle \\ \langle S_{HV} S_{HH}^* \rangle & \langle |S_{HV}|^2 \rangle & \langle S_{HV} S_{VH}^* \rangle & \langle S_{HV} S_{VV}^* \rangle \\ \langle S_{VH} S_{HH}^* \rangle & \langle S_{VH} S_{HV}^* \rangle & \langle |S_{VH}|^2 \rangle & \langle S_{VH} S_{VV}^* \rangle \\ \langle S_{VV} S_{HH}^* \rangle & \langle S_{VV} S_{HV}^* \rangle & \langle S_{VV} S_{VH}^* \rangle & \langle |S_{VV}|^2 \rangle \end{bmatrix}$$

Concept of covariance matrix

$$\Sigma = \begin{bmatrix} \mathrm{Var}(X) & \mathrm{Cov}(X,Y) & \mathrm{Cov}(X,Z) \\ \mathrm{Cov}(X,Y) & \mathrm{Var}(Y) & \mathrm{Cov}(Y,Z) \\ \mathrm{Cov}(X,Z) & \mathrm{Cov}(Y,Z) & \mathrm{Var}(Z) \end{bmatrix}$$

Polarimetic measurement; statistical point of view

- **Target is described by set of scattering matrices, each are random measurement from certain distribution function, specific for the target.**
- **Signal is complex zero-mean Gaussian if number of scatterers is large in resolution cell (applies for many SAR measurements, does not apply always for longer wavelength and high resolution)**
- **All the information about Gaussian-distributed target carried by covariance matrix.**
- **Statistical model describes the actual information content of the data**

- Measurement vector

$$S = \begin{pmatrix} S_{hh} \\ S_{hv} \\ S_{vv} \end{pmatrix} \equiv \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

- Covariance matrix

$$C_{kl} = \left\langle S_k S_l^* \right\rangle$$

$$C = \begin{pmatrix} \mathrm{var}(S_1) & \mathrm{cov}(S_1 S_2) & \mathrm{cov}(S_1 S_3) \\ \mathrm{cov}(S_1 S_2) & \mathrm{var}(S_2) & \mathrm{cov}(S_2 S_3) \\ \mathrm{cov}(S_3 S_1) & \mathrm{cov}(S_3 S_2) & \mathrm{var}(S_3) \end{pmatrix}$$

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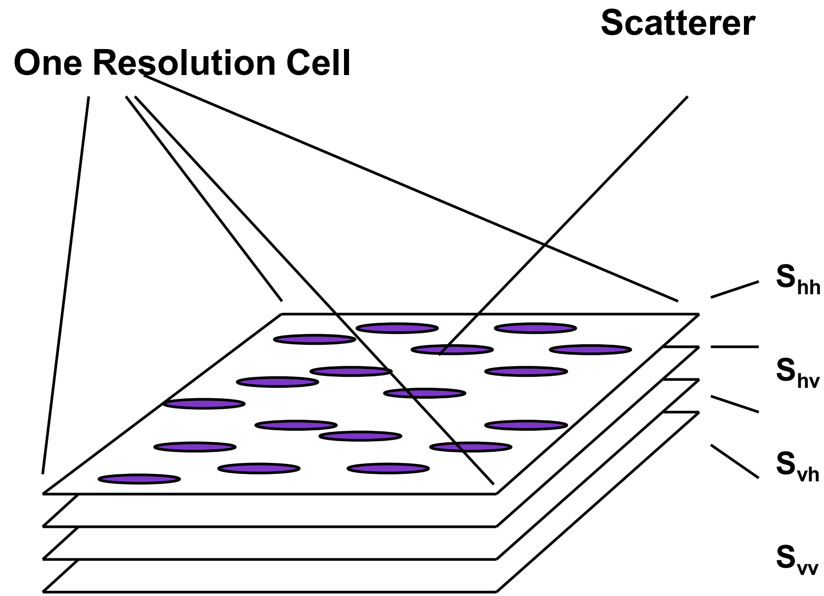
Working with polarimetric data

Polarimetric image

- Polarimetric SAR measures multidimensional information (five dimensions for single scatterer and nine dimensions for distributed scatterer)
- It is difficult to visualize all parameters of scatterer on one image



Polarimetric data:

- Polarimetric data is complex
- Polarimetric data has more parameters than can be visualized in RGB
- Polarimetric properties are fully preserved only for SLC slant range data

Usually polarimetric images are presented in Pauli Basis or presented by using Decomposition techniques which provide easily interpretable parameters

Different representations in polarimetry

Allthough, large variety of matrices are used in polarimetry, there are only two fundamental matrices:

Scattering matrix S

- Contains information about single pixel, single coherent wave or measurement.
- Can not be averaged (expectation value is zero)
- Pauli matrices
- Sinclair matrices
- etc.

Covariance matrix C

- Contains statistical information about ensemble of S matrices or multiple measurements
- Can be averaged
- Stokes matrix
- Muller matrix
- Coherency matrix
- Etc

Covariance matrix is matrix which contains information about S matrix distribution and its moments.

From S matrix is possible to calculate always C matrix, but not always in other way round.

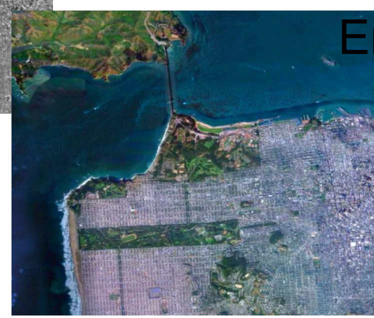
Different matrices in the same matrix class are just the same information in different basis.

Polarimetric SAR image pixel is a complex matrix describing target's properties

Optical B&W image

Pixel=(I)

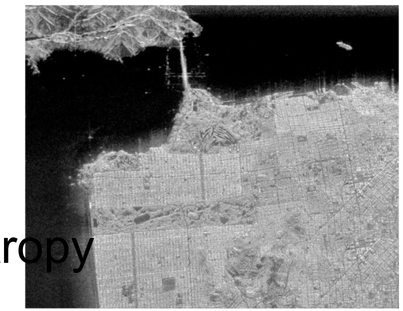
Color image



Pixel=(R,G,B)

Entropy

PolSAR image

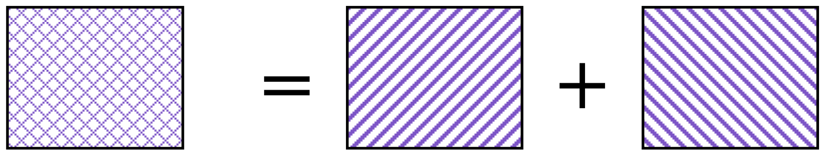


$$\text{Pixel}=\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

$$\text{Averaged pixel}=\begin{bmatrix} \langle |S_{HH}|^2 \rangle & \langle S_{HH}S_{HV}^* \rangle & \langle S_{HH}S_{VH}^* \rangle & \langle S_{HH}S_{VV}^* \rangle \\ \langle S_{HV}S_{HH}^* \rangle & \langle |S_{HV}|^2 \rangle & \langle S_{HV}S_{VH}^* \rangle & \langle S_{HV}S_{VV}^* \rangle \\ \langle S_{VH}S_{HH}^* \rangle & \langle S_{VH}S_{HV}^* \rangle & \langle |S_{VH}|^2 \rangle & \langle S_{VH}S_{VV}^* \rangle \\ \langle S_{VV}S_{HH}^* \rangle & \langle S_{VV}S_{HV}^* \rangle & \langle S_{VV}S_{VH}^* \rangle & \langle |S_{VV}|^2 \rangle \end{bmatrix}$$

Radar target decompositions

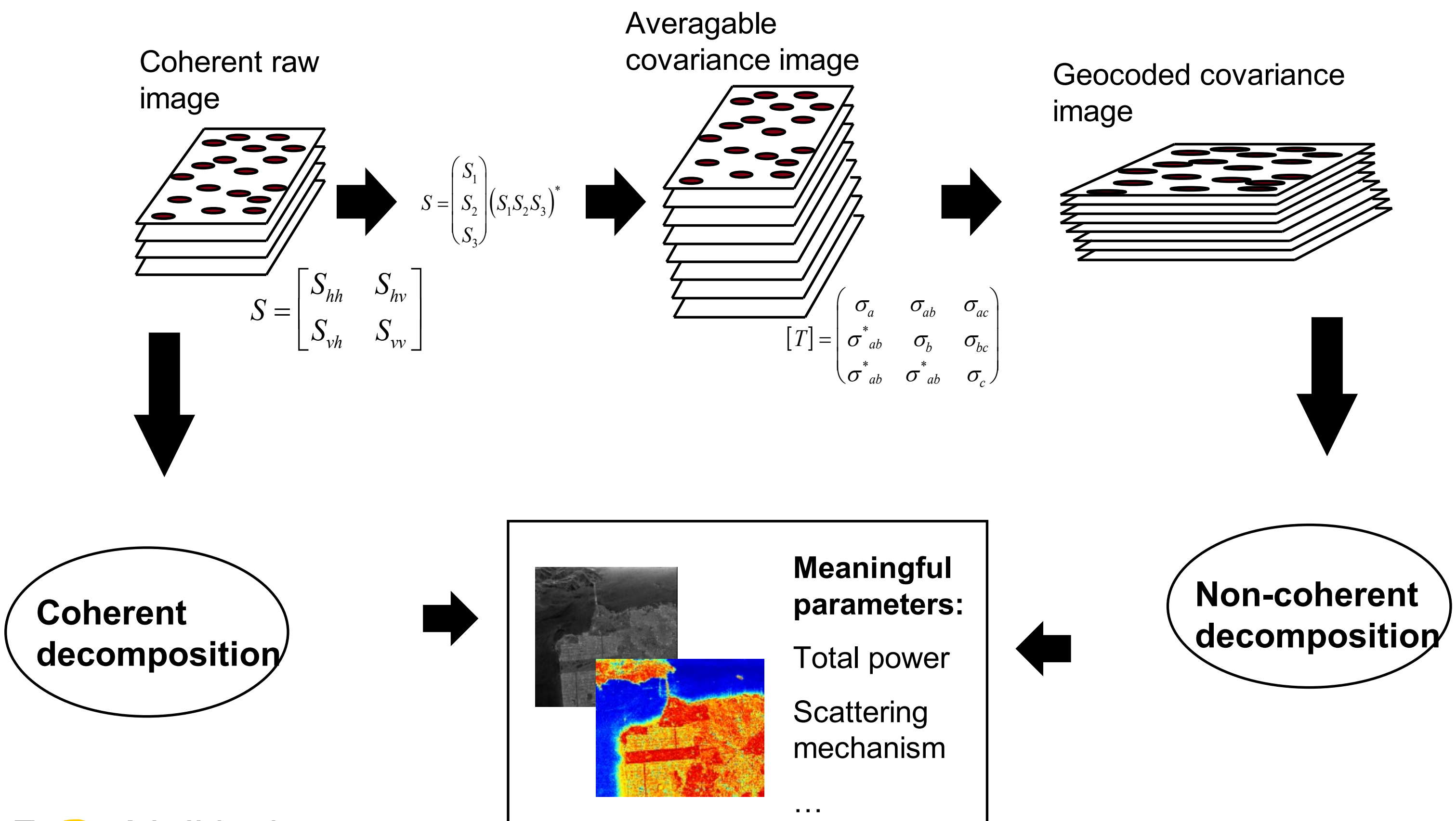
- The Idea of target decomposition is to present target scattering properties as a sum of simple scatterers.



- Reasonable decomposition should be unique, invariant under reasonable transformations and stable under small perturbations.

- Different presentations of polarimetric data leads to different decomposition techniques, but all of them are based on matrix algebra.

Typical Polarimetric SAR image processing



Freeman-Durden decomposiution

Model based decomposition

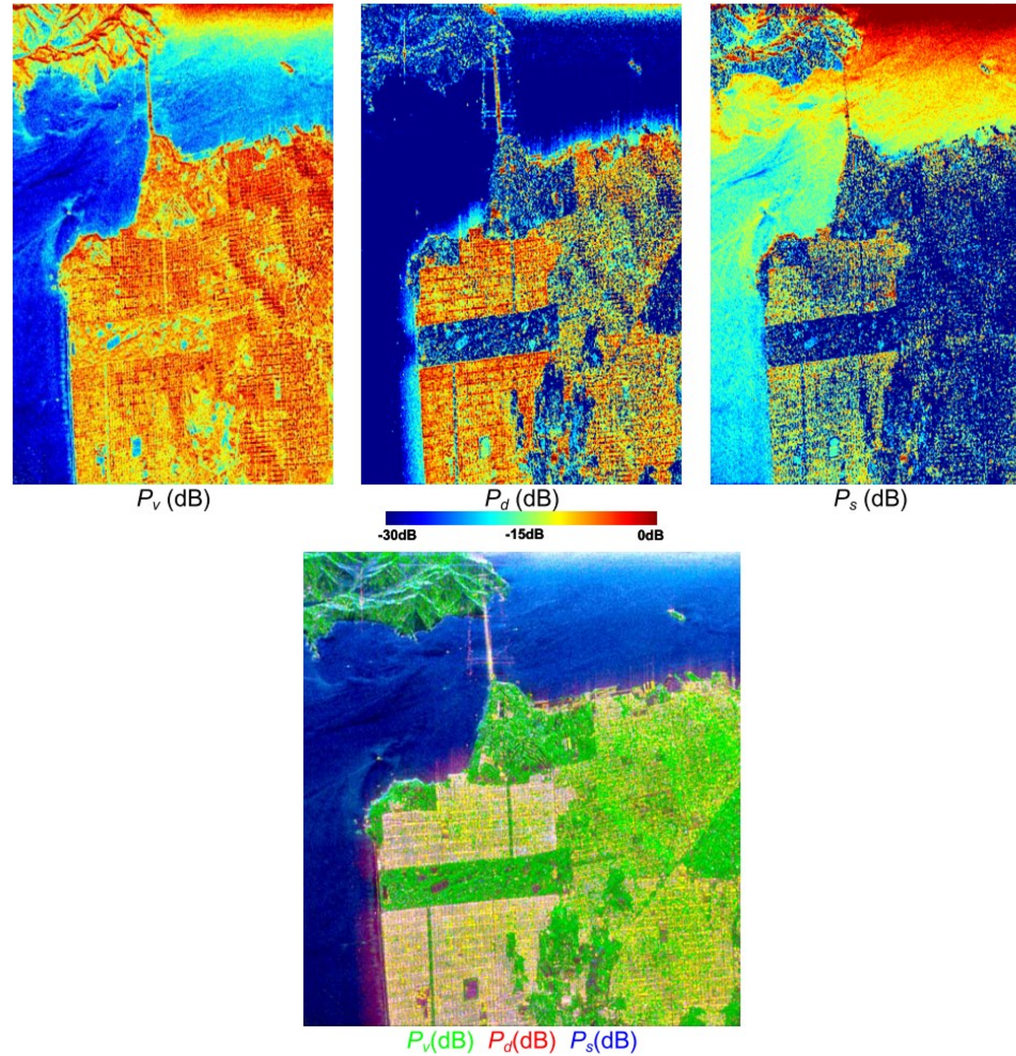


Figure 6 Intensities corresponding to the Freeman decomposition P_v, P_d and P_s and the combination of them in an RGB image. Images are shown in a dB scale.

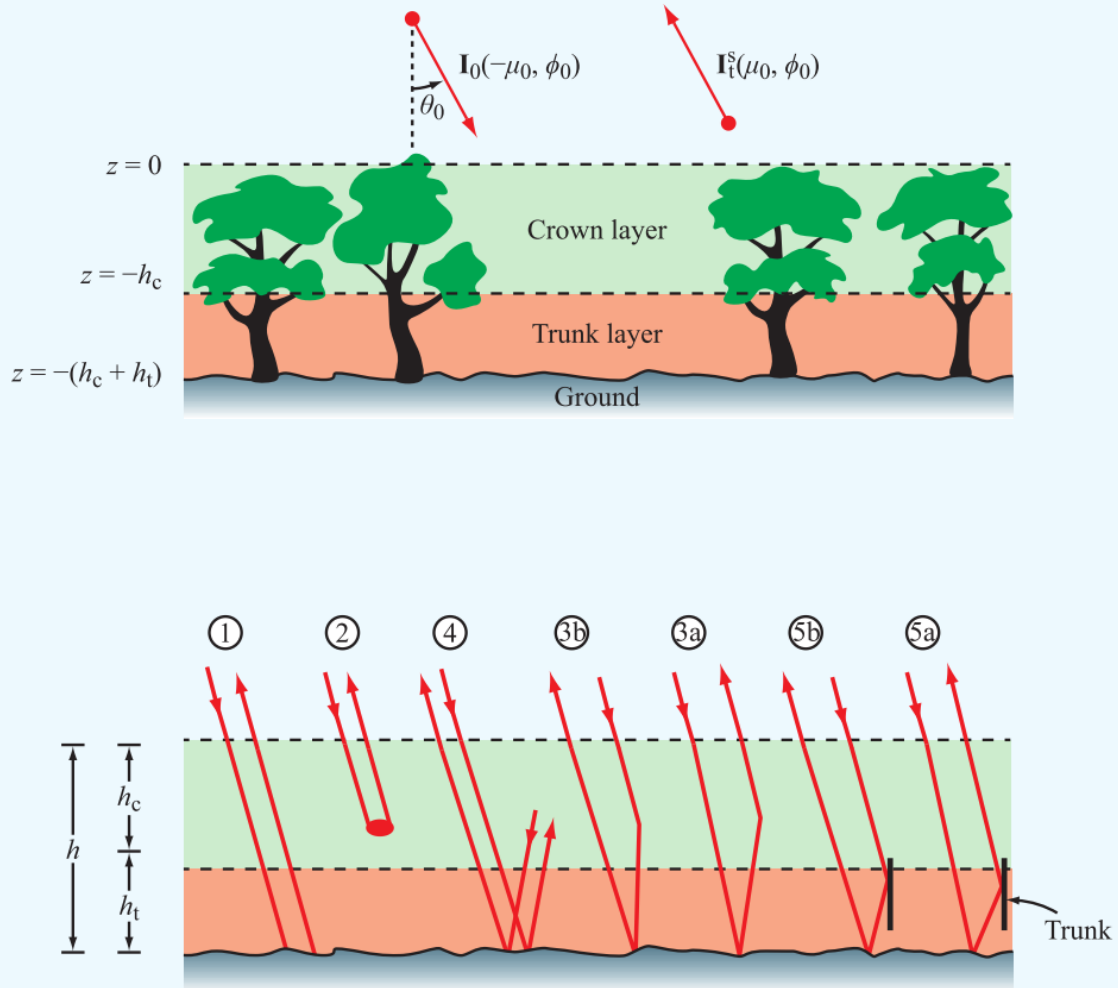


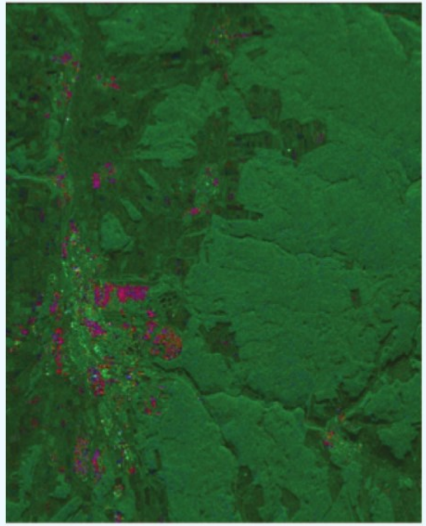
Figure 5-49: Scattering mechanisms for a forest canopy.

From: Microwave Radar and Radiometric Remote Sensing, by Ulaby and Long, 2014, with permission.

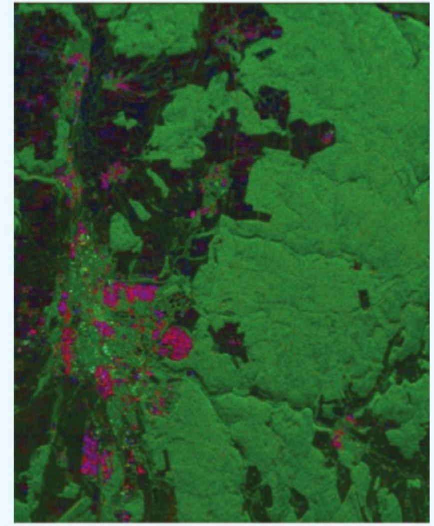
$$
<[T]> = \frac{f_s}{1+|\alpha|^2}\begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{f_d}{1+|\alpha|^2}\begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{f_v}{4}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{f_c}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm j \\ 0 & \mp j & 1 \end{bmatrix}
$$

↑ **Surface** ↑ **Double bounce** ↑ **Volume** ↑ **Helix**

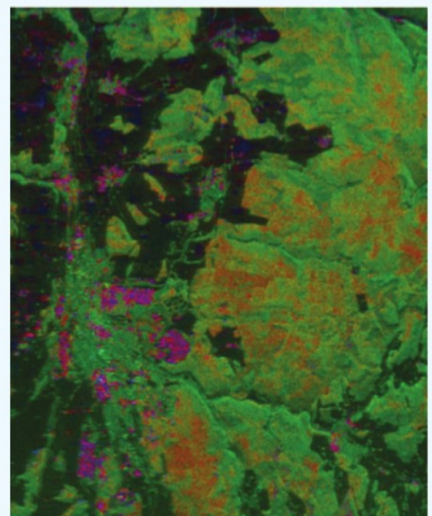
From: Microwave Radar and Radiometric Remote Sensing, by Ulaby and Long, 2014, with permission.



(a) C-band



(b) L-band



(c) P-band

Figure 5-50: Results of the Freeman-Durden decomposition for the Black Forest image at (a) C-band, (b) L-band, and (c) P-band. Surface, dihedral-corner, and volume scattering components are displayed in blue, red, and green colors, respectively.

From: Microwave Radar and F

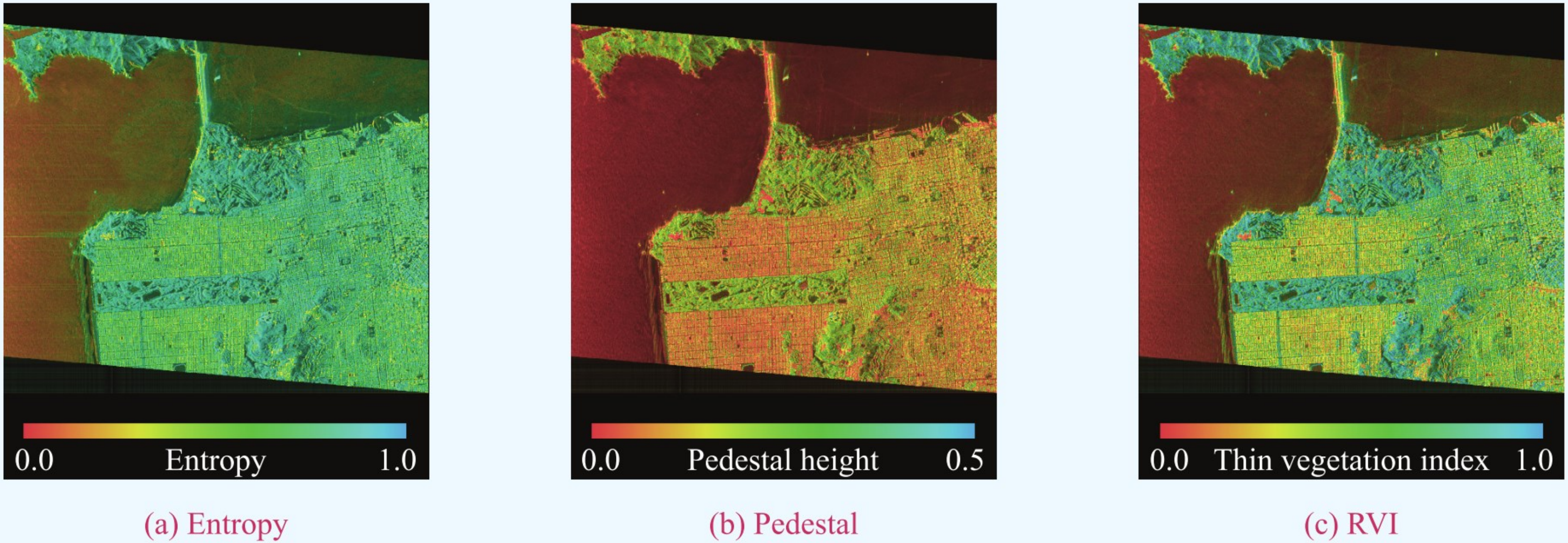


Figure 5-46: Radar images of San Francisco showing the three measures of scattering randomness: (a) entropy scaled from 0 (black) to 1 (white); (b) pedestal height scaled from 0 (black) to 0.5 (white); and (c) the radar vegetation index scaled from 0 (black) to 1 (white).

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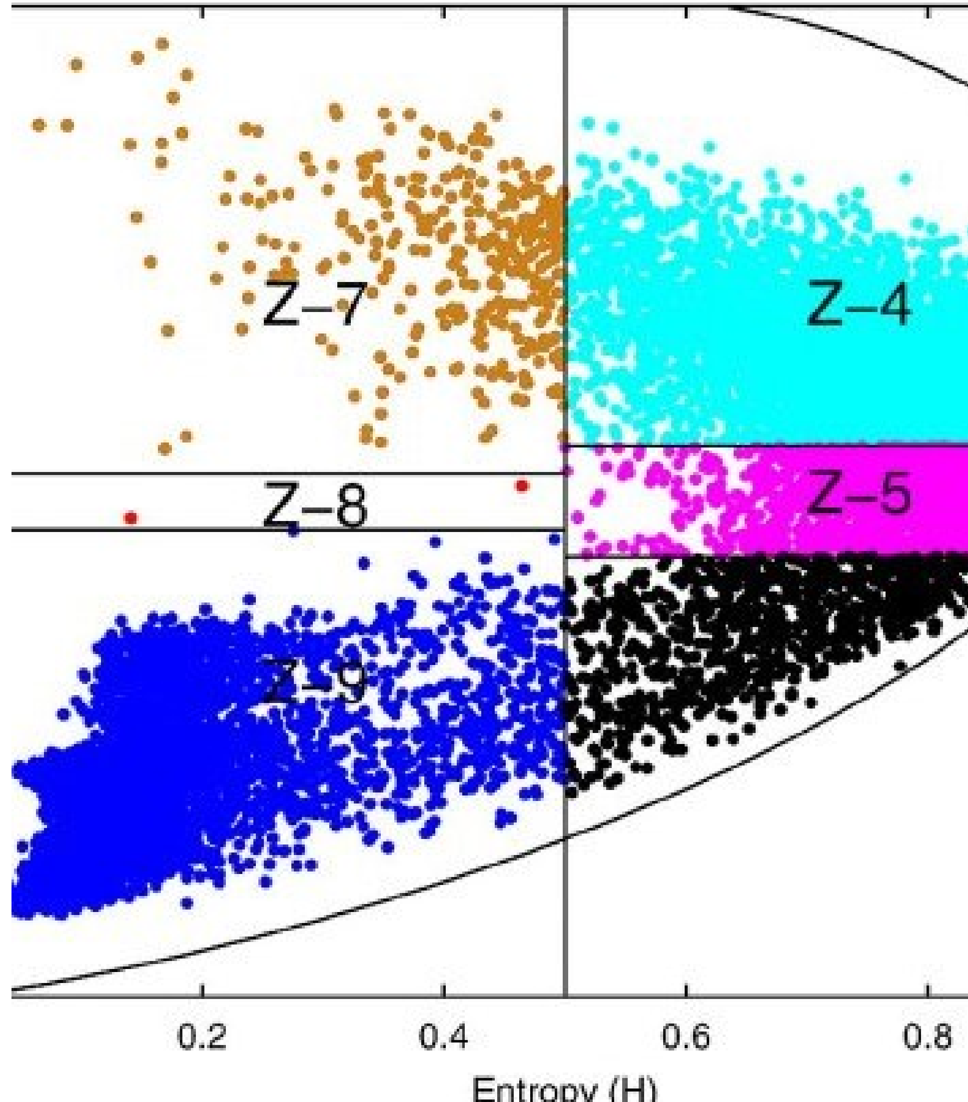
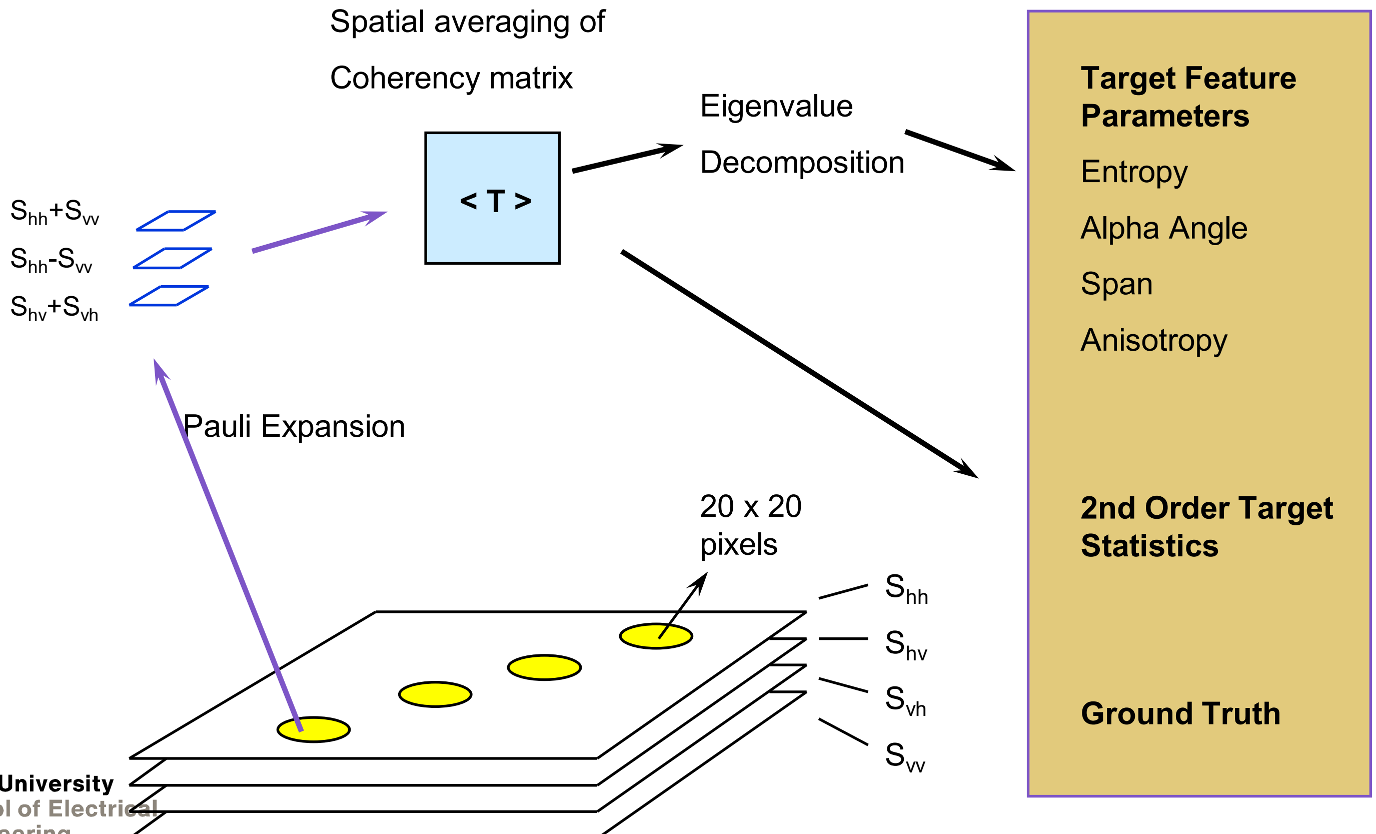
Alpha-entropy or Cloude-Pottier decomposition (eigenvalue decomposition)



SAR Data Analysis example using decomposition



Cloude-Pottier eigenvalue decomposition

Target scattering vector in Pauli basis

Coherency Matrix:

$$\bar{k} = [a,b,c]^T = \frac{1}{\sqrt{2}}[S_{hh}+S_{vv}, S_{hh}-S_{vv}, S_{hv}+S_{vh}]^T$$

$$\langle \mathbf{T} \rangle = \langle \vec{k}\vec{k}^{\dagger} \rangle$$

$$\langle \mathbf{T} \rangle = \lambda_1 \vec{k}_1 \vec{k}_1^{\dagger} + \lambda_2 \vec{k}_2 \vec{k}_2^{\dagger} + \lambda_3 \vec{k}_3 \vec{k}_3^{\dagger}$$

$$\mathbf{T} = \mathbf{U_3} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{U_3^{\dagger}}$$

$$\mathbf{U_3} = [\, \vec{k}_1 \quad \vec{k}_2 \quad \vec{k}_3 \,]$$

Classification parameters:

Entropy

$$H = -\sum_{i=1}^{4} p_i \log p_i$$

Average Alpha (scattering mechanism)

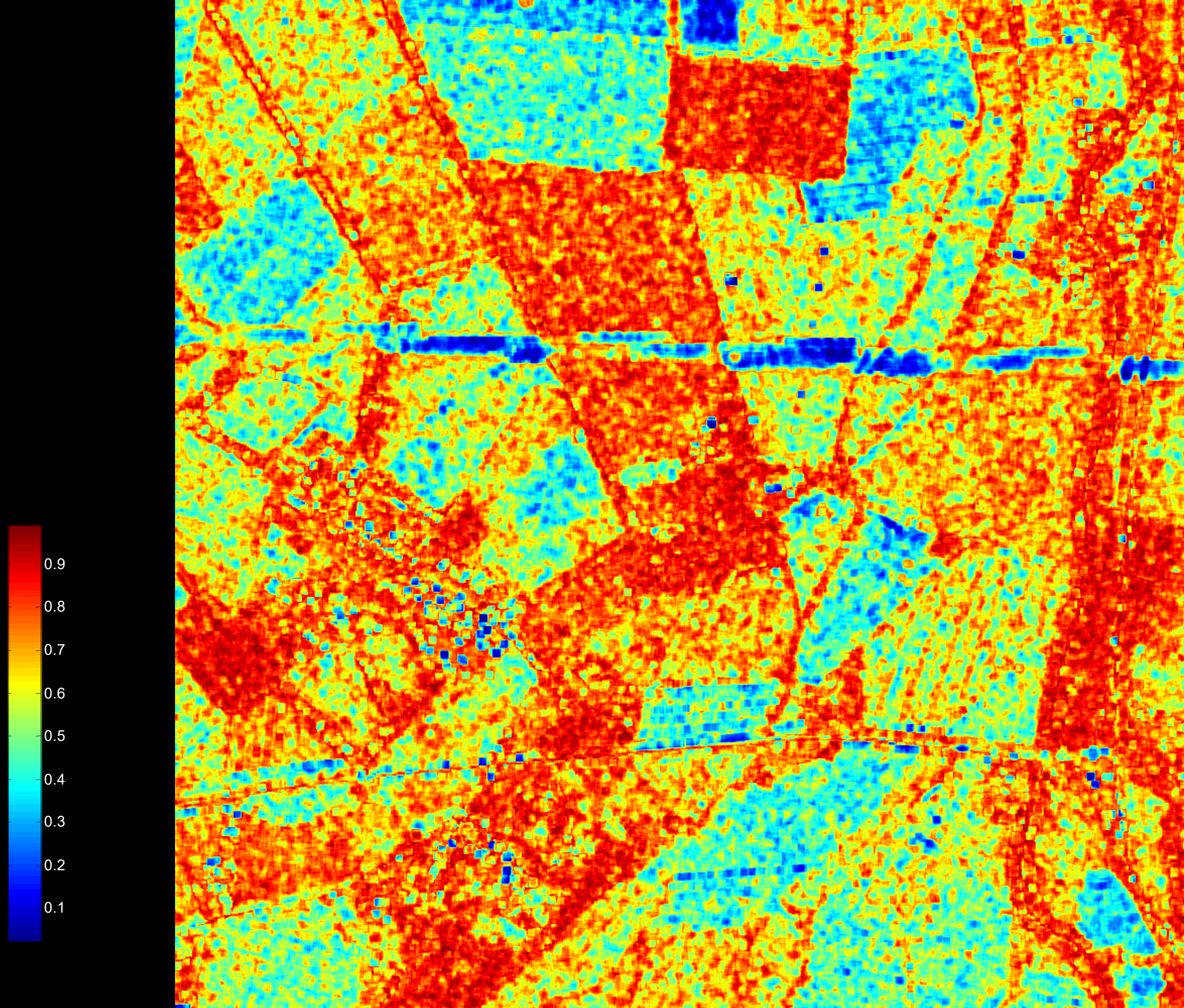
$$\langle \alpha \rangle = \sum_{i=1}^{4} p_i \alpha_i$$

p_i eigenvalue parameter

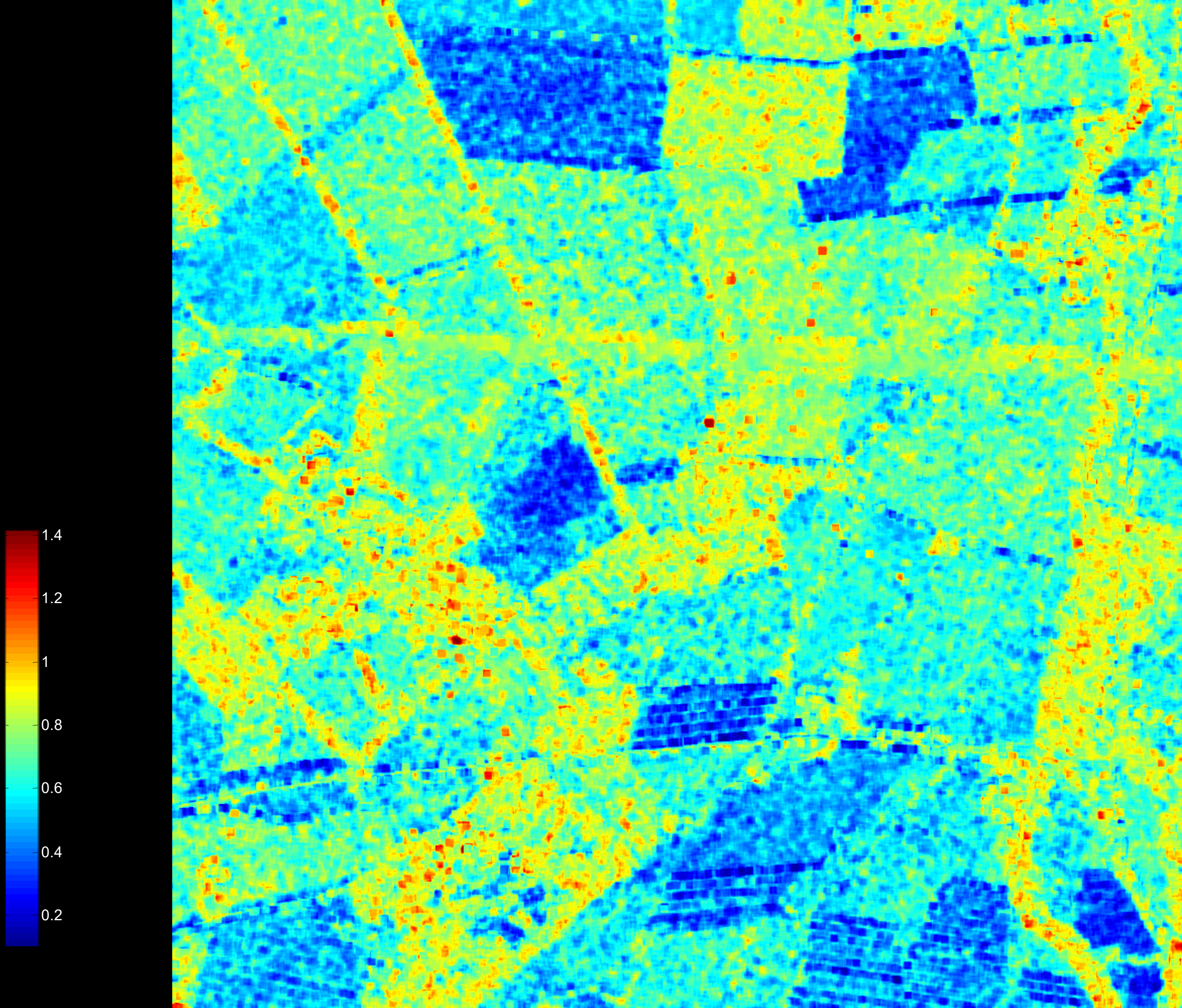
α_i eigenvector orientation

$$p_i = \frac{\lambda_i}{\sum_{i=1}^{3} \lambda_i}.$$

Entropy



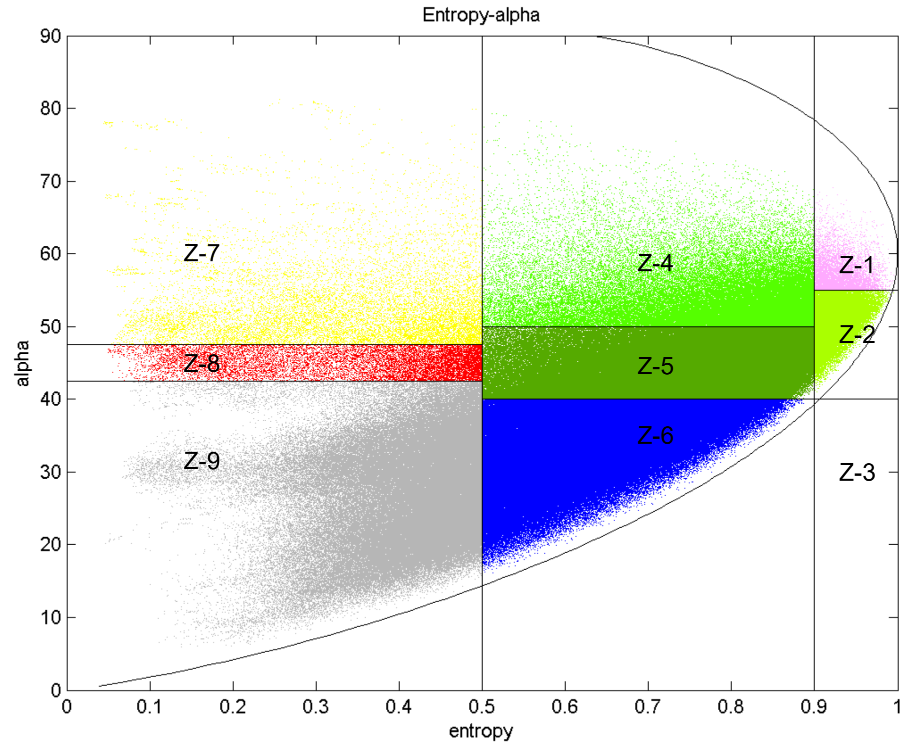
Alpha



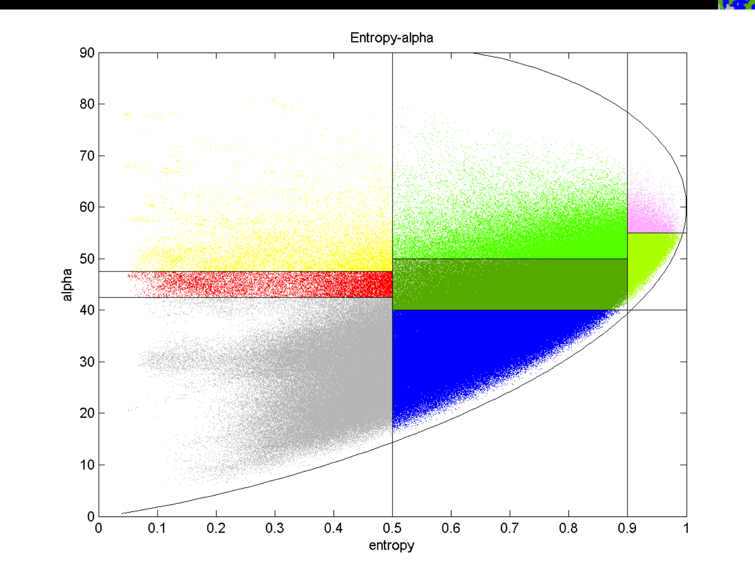
Entropy-alpha classification space

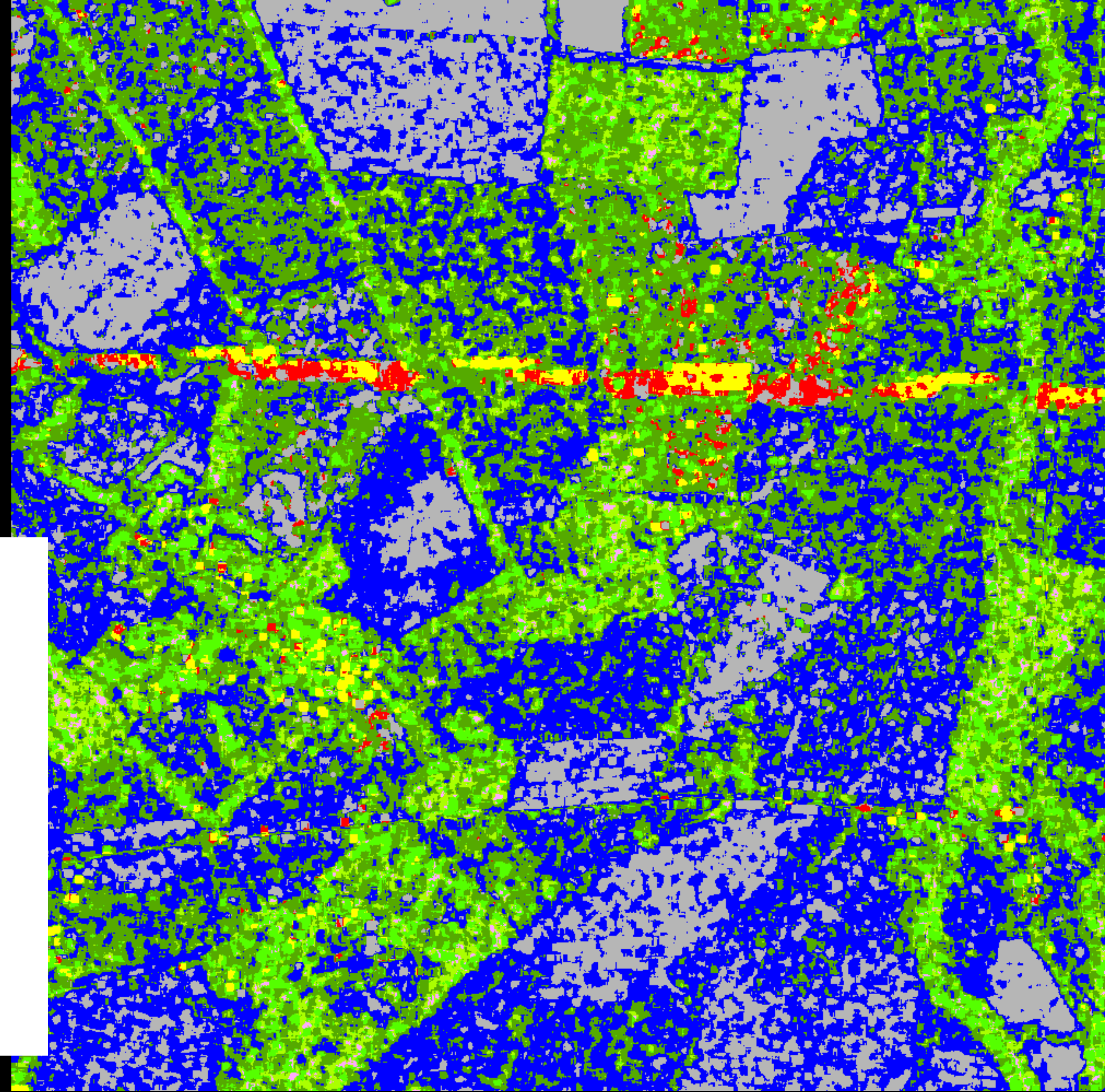
Classes:

Zone 9: Low Entropy Surface Scatter,
Zone 8: Low Entropy Dipole Scattering,
Zone 7: Low Entropy Multiple Scattering
Zone 6: Medium Entropy Surface Scatter,
Zone 5: Medium Entropy Vegetation Scattering
Zone 4: Medium Entropy Multiple Scattering
Zone 3: High Entropy Surface Scatter
Zone 2: High Entropy Vegetation Scattering,
Zone 1: High Entropy Multiple Scattering.



Entropy Alpha classification



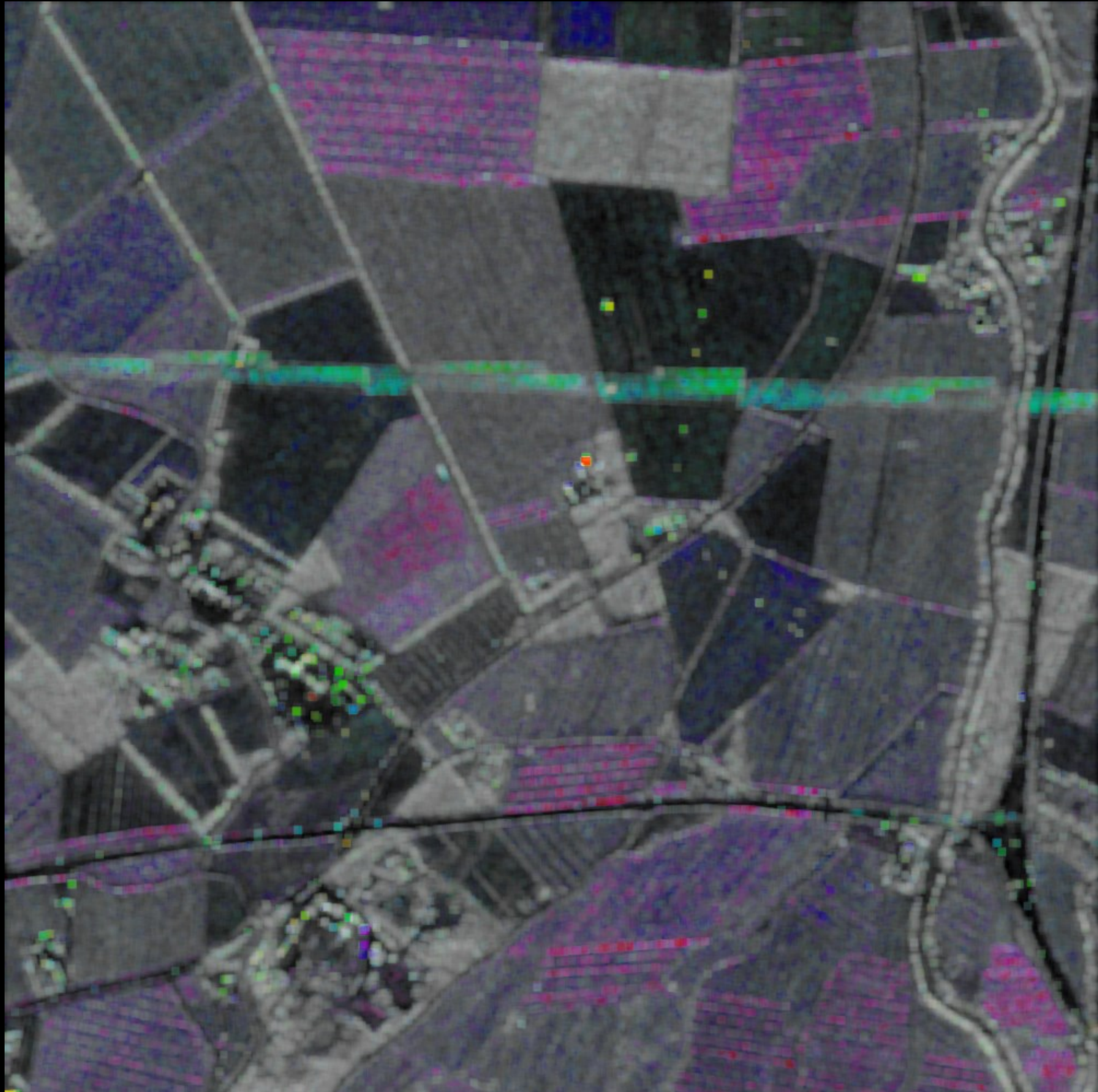


HSI

H=alpha

S=1-Entropy

I=Span



Combined hi-res HSI

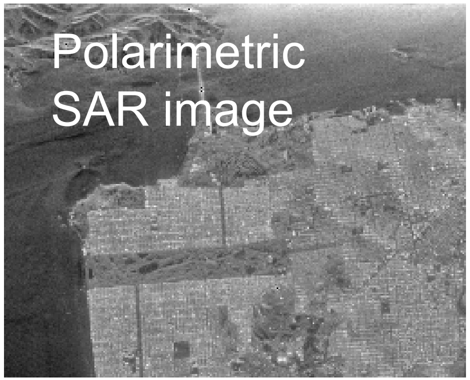
H=alpha

S=1-Entropy

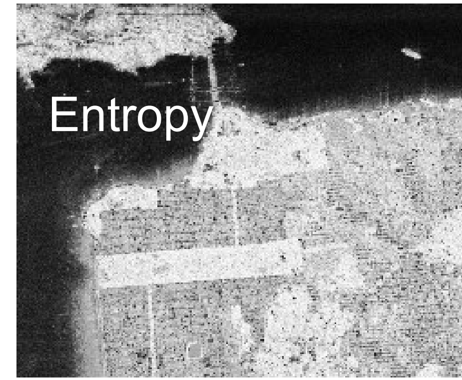
I=Span

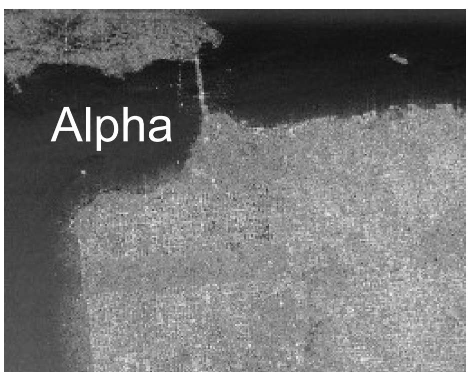
Google

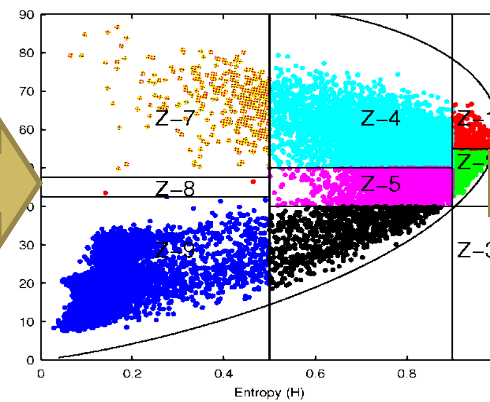
Polarimetric eigenvalue decomposition

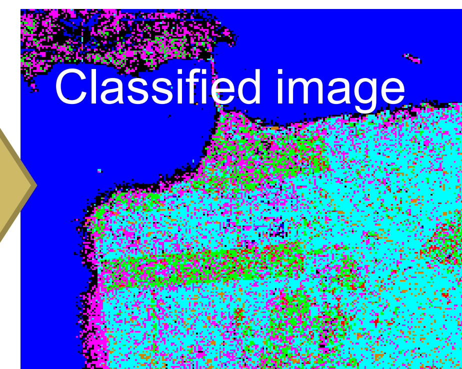


Eigenvalue and eigenvector analysis for each pixel

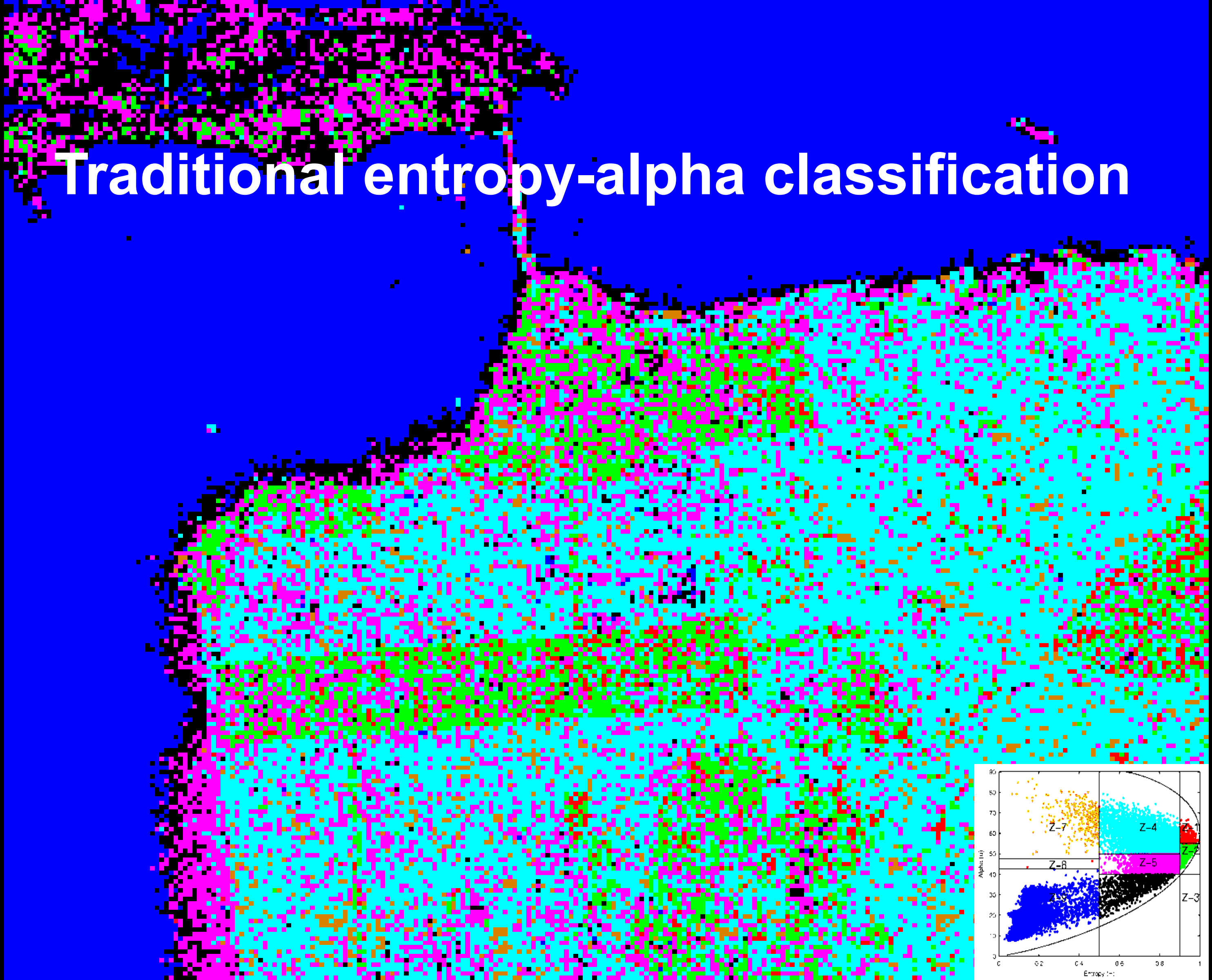




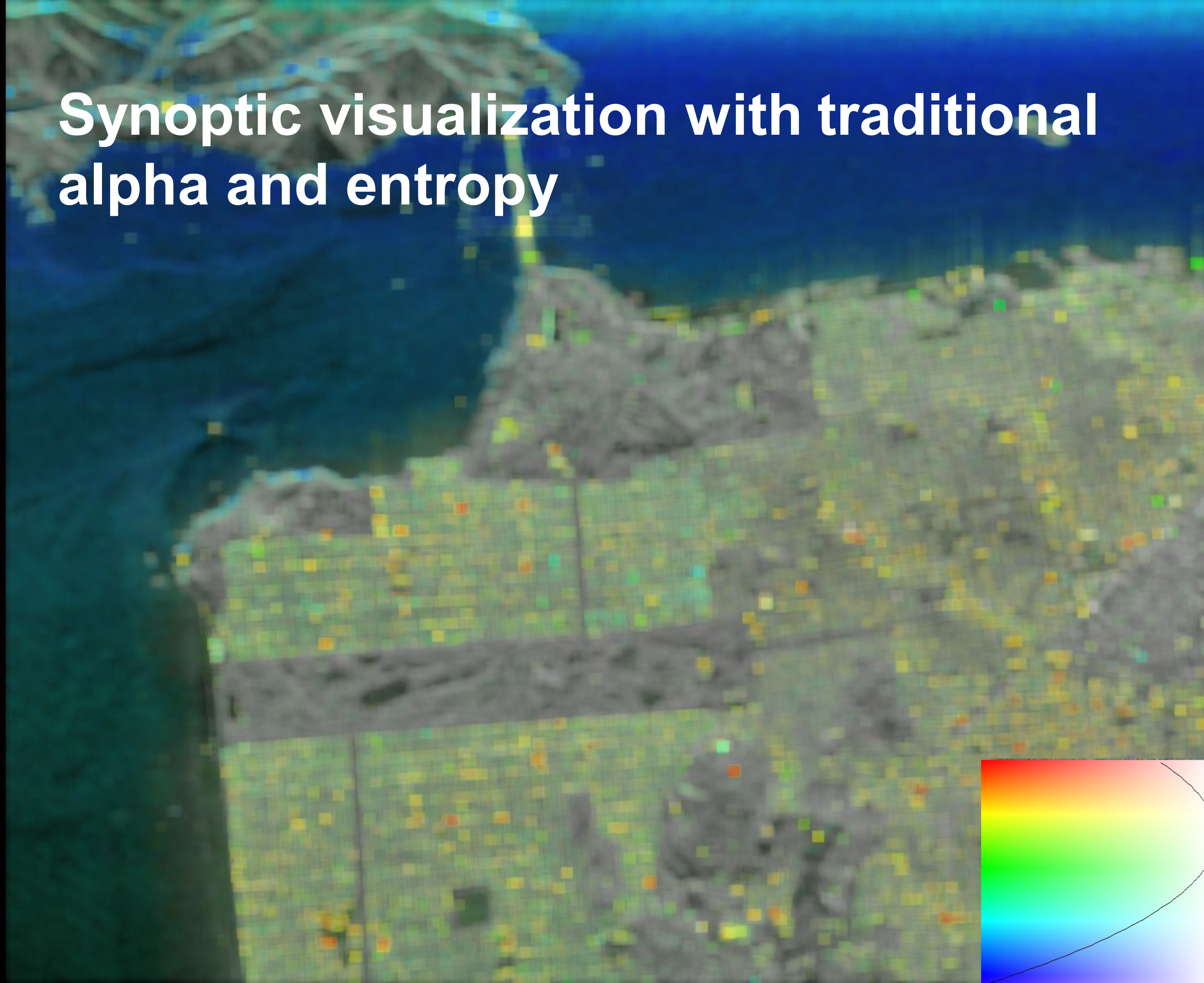




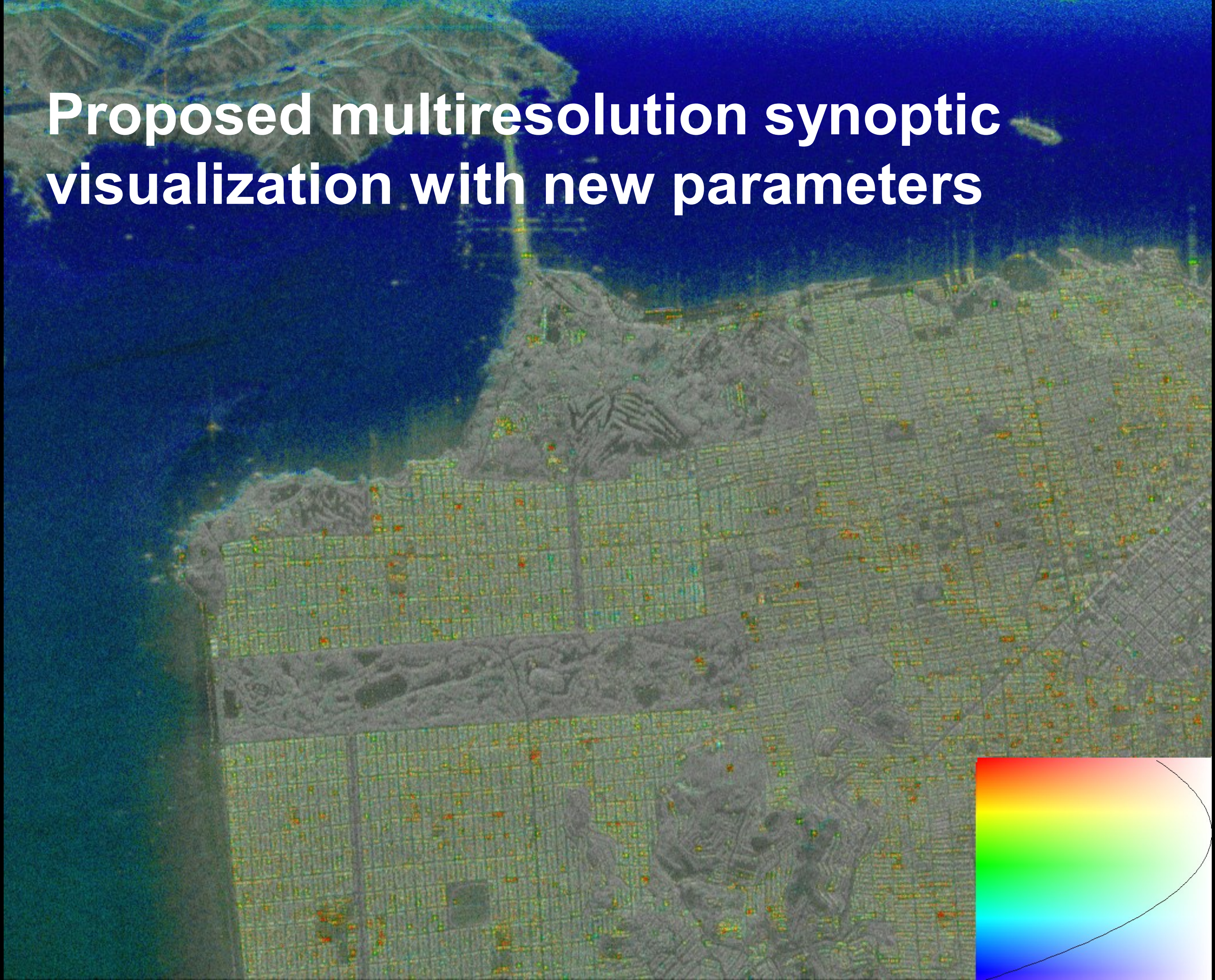
Traditional entropy-alpha classification



Synoptic visualization with traditional alpha and entropy



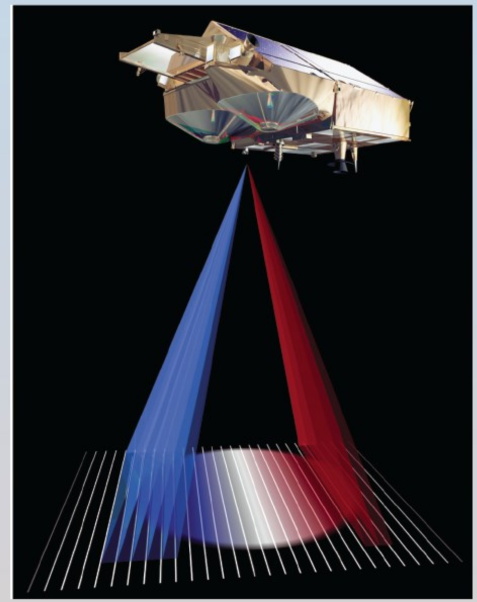
Proposed multiresolution synoptic visualization with new parameters




Jaan Praks
Full-res alpha
Entropy
Full-res span

- Ulaby
- Long
- Blackwell
- Elachi
- Fung
- Ruf
- Sarabandi
- Zebker
- Van Zyl

Microwave Radar and Radiometric Remote Sensing



END