Microeconomic Theory II
Helsinki GSE
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Problem Set 3, Due December 8, 2021

1. Draw the Production Edgeworth-Box for the following two production technologies and find the Production efficient input allocations for the two cases below:

   (a) Goods 1 and 2 is produced according to:

   $$q_1 = k_1^{\frac{1}{2}} l_1^{\frac{1}{2}},\ q_2 = k_2^{\frac{1}{3}} l_2^{\frac{2}{3}},$$

   and the total quantities of the factors are : $\overline{k} = 3,\ \overline{l} = 2$.

   (b) Goods 1 and 2 is produced according to:

   $$q_1 = \min\{2k_1, l_1\},\ q_2 = k_2 + 3l_2,$$

   and the total quantities of the factors are : $\overline{k} = 2,\ \overline{l} = 5$.

2. Consider the constant returns to scale Cobb-Douglas production technology, where output q is produced from m inputs z_j for $j \in \{1, ..., m\}$ according to:

   $$q = z_1^{\beta_1} z_2^{\beta_2} \dots z_m^{\beta_m},$$

   where $\beta_j > 0$ for all j, and $\sum_{j=1}^{m} \beta_j = 1$. Show that for positive production of q in a competitive equilibrium at output price p and input prices w_j for $j \in \{1, ..., m\}$, the prices must satisfy:

   $$p = (\frac{w_1}{\beta_1})^{\beta_1} (\frac{w_2}{\beta_2})^{\beta_2} \dots (\frac{w_m}{\beta_m})^{\beta_m}.$$

3. Consider an exchange economy with two agents and a countably infinite number of goods $x_0, x_1, x_2, ...$ Let the initial endowment of both agents be constant across the goods: $\omega_{il} = 1$ for $i \in \{1, 2\}$ and all $l \in \mathbb{N}$. Let the utility functions of the two agents be given by:

   $$u_1 = \sum_{l=1}^{\infty} \alpha^l x_{1l},\ u_2 = \sum_{l=1}^{\infty} \beta^l x_{2l},$$

   for some $\alpha, \beta > 0$.

(a) For a given sequence p_l of prices, derive the first-order conditions for optimal consumption for the two agents for consecutive goods.

(b) Find the competitive equilibrium prices for this economy.

(c) Interpret the model and the prices from a dynamic choice perspective.

4. Consider an economy with three consumers and two identical firms producing a homogenous indivisible good. The consumers have unit demand: each consumer values the first unit of the good at 7 and further units bring no additional value to them. The firms' marginal production costs are: 6 for the first unit, 3 for the second unit and 5 for the third unit.

   An allocation in the economy specifies $q_{ij} \in \{0, 1\}$ and $p_{ij} \in \mathbb{R}_+$, where $q_i j = 1$ if consumer i buys from from firm j and 0 otherwise. p_{ij} is the payment from consumer i to firm j. Assume that all participants in the market have payoffs that are quasilinear in money.

   (a) Describe the Pareto-Efficient allocations for this economy.

   (b) Show that the core of this economy is empty.

5. (Harder, Crossover to Macroeconomics) A single, representative agent lives forever and consumes (and holds assets) optimally given the market prices. The preferences are represented by:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

   where the per period utility from consumption c_t in period t is given by $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ (CRRA).

   Endowments: The agent is endowed at $t = 0$ with 1 tree. In each period, the tree yields stochastic consumption (dividend) d_t that cannot be stored. The dividend d_t evolves as follows:

   - If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ for all $t' > t$.
   - If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability π and $d_{t+1} = d_t$ with probability $1 - \pi$, and $\gamma > 1$.

In words: d grows at rate $\gamma - 1$ until some random event occurs (with probability $1 - \pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (choose this as numeraire) and trees (price p_t). Assume that p_t is cum dividend, meaning that d_t accrues to the household who buys the tree in t and holds it into $t + 1$.

(a) State the agent's optimization problem paying particular attention to the intertemporal budget constraint:

$$p_{t+1}(k_{t+1} - k_t) + c_t = d_t k_{t+1},$$

where k_{t+1} denotes the holdings of trees from t to $t + 1$. Derive the first-order condition characterizing the optimal consumptions in consecutive periods .

(b) Define a competitive equilibrium.

(c) Characterize the stochastic process of p_t. Is p_t a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that $\frac{p_t}{d_t}$ is constant during the phase with growth.

(d) What happens to the stock market (i.e. p_t) when the economy stops growing? Does it crash (does the price decline)? Under what condition?