PHYS-C0252 - Quantum Mechanics Part 2
Section 5.3

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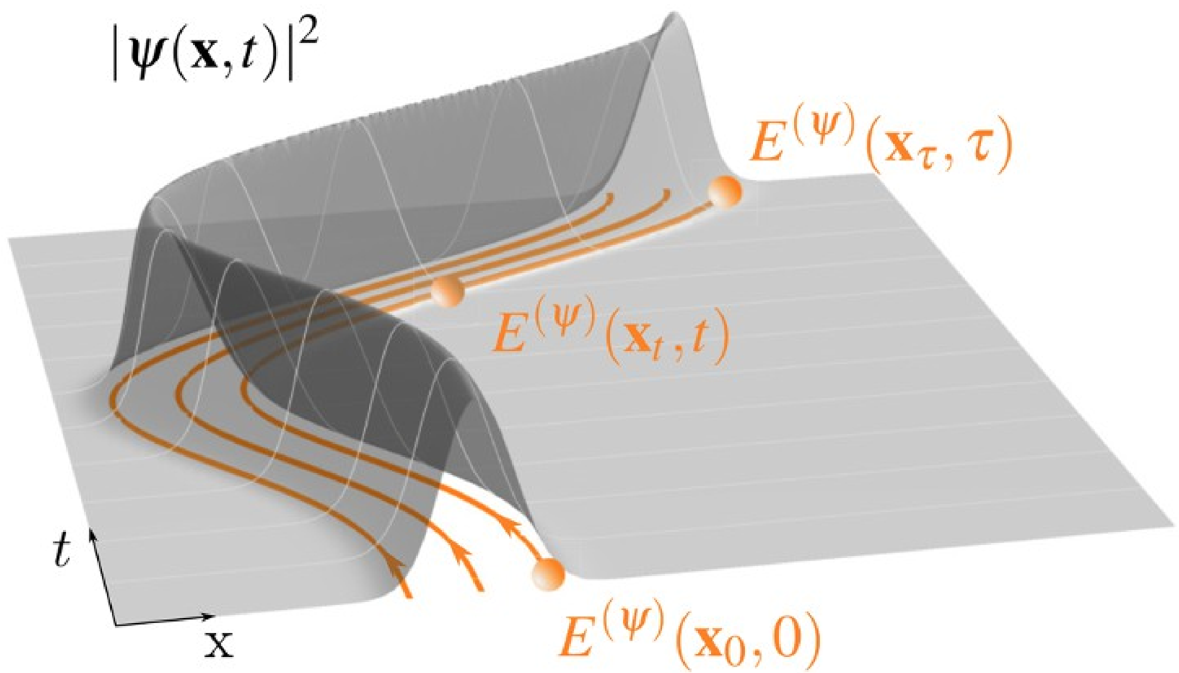

5.3 Annihilation and Creation Operators

- Consider first the case of fermions (antisymmetric WFs). The creation operator $C_\alpha^\dagger$ is defined by the relations

$$C_\alpha{}^\dagger|0\rangle = |\alpha\rangle \equiv |\phi_\alpha\rangle\,,$$

$$C_\alpha{}^\dagger|\beta\rangle = C_\alpha{}^\dagger\, C_\beta{}^\dagger|0\rangle = |\alpha\beta\rangle = -|\beta\alpha\rangle\,,$$

$$C_\alpha{}^\dagger|\beta\gamma\rangle = C_\alpha{}^\dagger\, C_\beta{}^\dagger\, C_\gamma{}^\dagger|0\rangle = |\alpha\beta\gamma\rangle\,,$$

$$\text{etc}\,.$$

- The Pauli exclusion principle requires that

$$C_\alpha{}^\dagger|\alpha\cdots\rangle = 0$$

- The adjoint operator $C_\alpha \equiv (C_\alpha^\dagger)^\dagger$ is defines the annihilation operator

$$C_\alpha|\alpha\rangle = |0\rangle$$

$$C_\alpha|0\rangle = 0$$

- It is easy to show (homework?) that these fermionic operators obey an *anticommutation* relation

$$\{C_\alpha, C_\beta^\dagger\} \equiv C_\alpha C_\beta^\dagger + C_\beta^\dagger C_\alpha = \delta_{\alpha\beta} I$$

and the number operator $N = \sum_\alpha {C_\alpha}^\dagger \, C_\alpha$

- Similarly, for the case of bosons (symmetric WFs) the creation operator $a_\alpha^\dagger$ is defined by the relations

$$a_\alpha{}^\dagger|0\rangle = |\phi_\alpha\rangle = |0,0,\ldots,n_\alpha=1,0,\ldots\rangle\,,$$

$$a_\alpha{}^\dagger|n_1,n_2,\ldots,n_\alpha,\ldots\rangle \;\propto\; |n_1,n_2,\ldots,n_\alpha+1,\ldots\rangle\,.$$

and the annihilation operator $a_\alpha \equiv (a_\alpha^\dagger)^\dagger$

$$a_\alpha|\phi_\alpha\rangle = |0\rangle\,,$$

$$a_\alpha|n_1,n_2,\ldots,n_\alpha,\ldots\rangle \;\propto\; |n_1,n_2,\ldots,n_\alpha-1,\cdots\rangle\,,\;(n_\alpha>0)\,,$$

$$a_\alpha|n_1,n_2,\ldots,n_\alpha=0,\ldots\rangle = 0\,.$$

- The number operator is given by

$$N = \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$$

and

$$a_{\alpha} |n_1, n_2, \ldots, n_{\alpha}, \ldots\rangle = (n_{\alpha})^{1/2} \, |n_1, n_2, \ldots, n_{\alpha} - 1, \ldots\rangle$$

$$a_{\alpha}{}^{\dagger} |n_1, n_2, \ldots, n_{\alpha}, \ldots\rangle = (n_{\alpha} + 1)^{1/2} \, |n_1, n_2, \ldots, n_{\alpha} + 1, \ldots\rangle$$

- These were proven for the QHO already. The bosonic operators obey a *commutation* relation

$$\left[a_{\alpha}, a_{\beta}^{\dagger}\right] \equiv a_{\alpha} a_{\beta}^{\dagger} - a_{\beta}^{\dagger} a_{\alpha} = \delta_{\alpha\beta} I$$