Aalto University
School of Electrical
Engineering

ELEC-E8125 Reinforcement learning
Partially observable Markov decision processes

Joni Pajarinen

16.11.2021

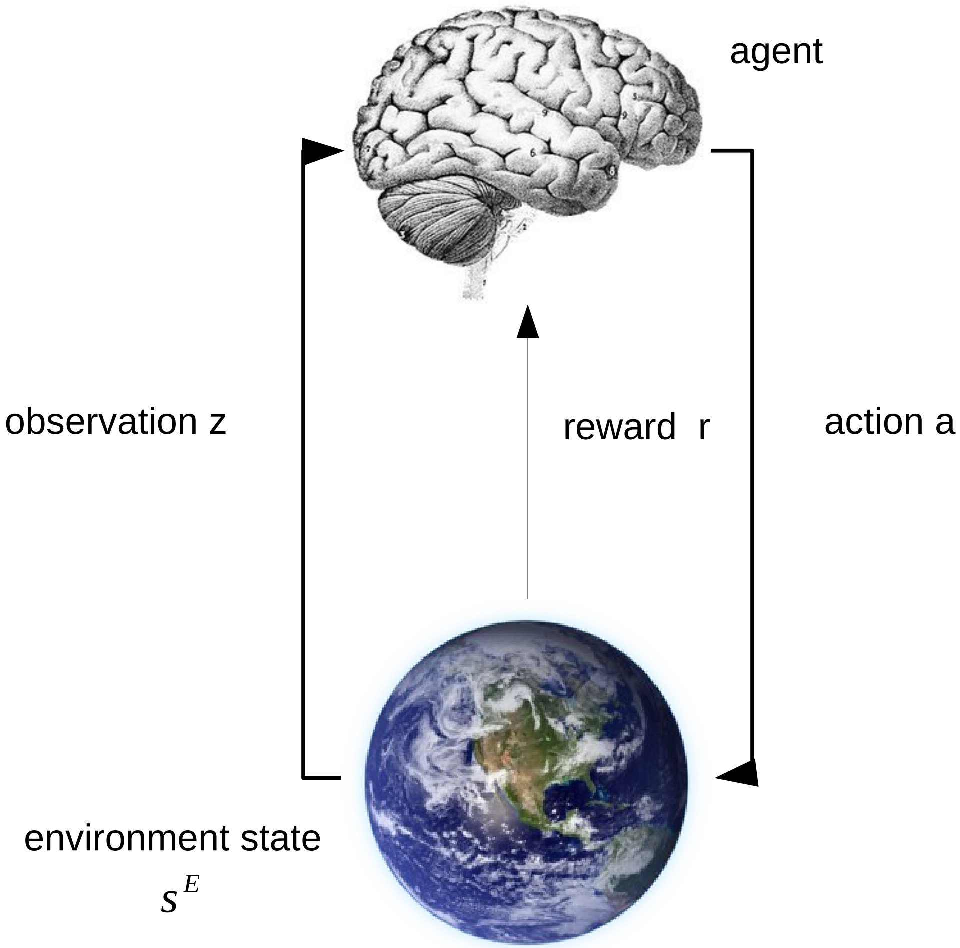
Today

- Partially observable Markov decision processes

Learning goals

- Understand POMDPs and related concepts.
- Be able to explain why solving POMDPs is difficult.

Partially observable MDP (POMDP)



POMDP
Environment not directly observable

Defined by dynamics
$P(s^E_{t+1} | s^E_t, a_t)$

Reward function
$r_t = r(s_{t+1}, s_t)$

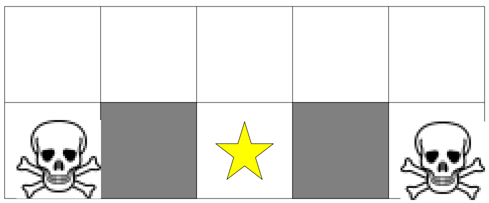
Observation model
$P(z_t | s^E_t, a_t)$

Solution similar, eg.
$a^*_{1,\ldots,T} = max_{a_1,\ldots,a_T} E\left[\sum_{t=1}^{T} r_t\right]$

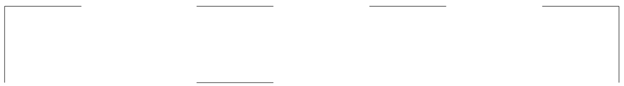
Agent does not directly observe environment state!

Partial observability example

- Observe only adjacent walls.
- Starting state unknown, in upper row of grid.
- Assume perfect actions.



Observations:



- Give a policy as function of observations!

- Any problems?

Can you present a (time-dependent) optimal policy as a tree?

History and information state

- *History* (= Information state) is the sequence of actions and observations until time t.

- Information state is Markovian, i.e.,

$$P_I(I_{t+1}|a_t, I_t) = P_I(I_{t+1}|a_t, I_t, I_{t-1}, \ldots, I_0)$$

- POMDP thus corresponds to an Information state MDP.

What is the largest feline in the world?

Example: Tiger problem

r=10

r=-100

A = {open right, open left, listen}

P(HL|TL)=0.85
P(HR|TL)=0.15
P(HL|TR)=0.15
P(HR|TR)=0.85

?

What kind of policy would be reasonable?

Policy depends on history of observations and actions = information state.

Belief state, belief space MDP

- Belief state = distribution over states.
  - Compresses information state.

- Belief $b_t(s) \equiv p(s_t = s | I_t)$ ← Can be represented as a vector $\boldsymbol{b} = (b(s_1), b(s_2), \ldots)$

- POMDP corresponds to belief space MDP.
- POMDP solution can be structured as
  - State estimation (of belief state) +
  - Policy on belief state.

Belief update

Similar to state estimator, e.g. Kalman filter, particle filter:

= state estimation

“measurement update” “prediction”

$$b_z^a(s) = b_{t+1} = \frac{\boxed{P(z|s,a)}\boxed{\sum_{s'} P(s|s',a)\,b_t(s')}}{\boxed{\sum_{s',s''} P(s''|s',a)\,P(z|s'',a)\,b_t(s')}}$$

Normalization factor

Single step policies

- Value of belief state for a particular single step policy

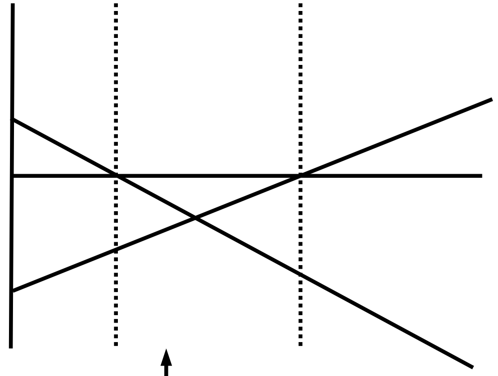
$$V_\pi(\boldsymbol{b}) = \sum_s b(s) V_\pi(s)$$

- Can be represented as *alpha vector* (consisting of values for each state)

$$V_\pi(\boldsymbol{b}) = \boldsymbol{\alpha}^T \boldsymbol{b}$$

- Value of optimal policy is then

$$V^*(\boldsymbol{b}) = max_i \, \boldsymbol{\alpha}_i^T \boldsymbol{b}$$



Maximum over all actions

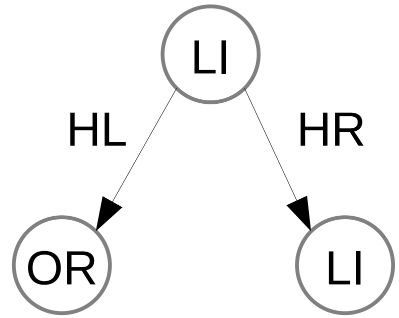
Piecewise linear and convex (PWLC)

Conditional plans and policy trees

- Similar to single step policies, value functions of multi-step policies can be represented as alpha vectors.

- Best policy for a particular belief is then again

$$V^*(\boldsymbol{b}) = max_i \, \boldsymbol{\alpha}_i^T \boldsymbol{b}$$



What's the α-vector of this policy?

Value iteration on belief states

- Bellman equation

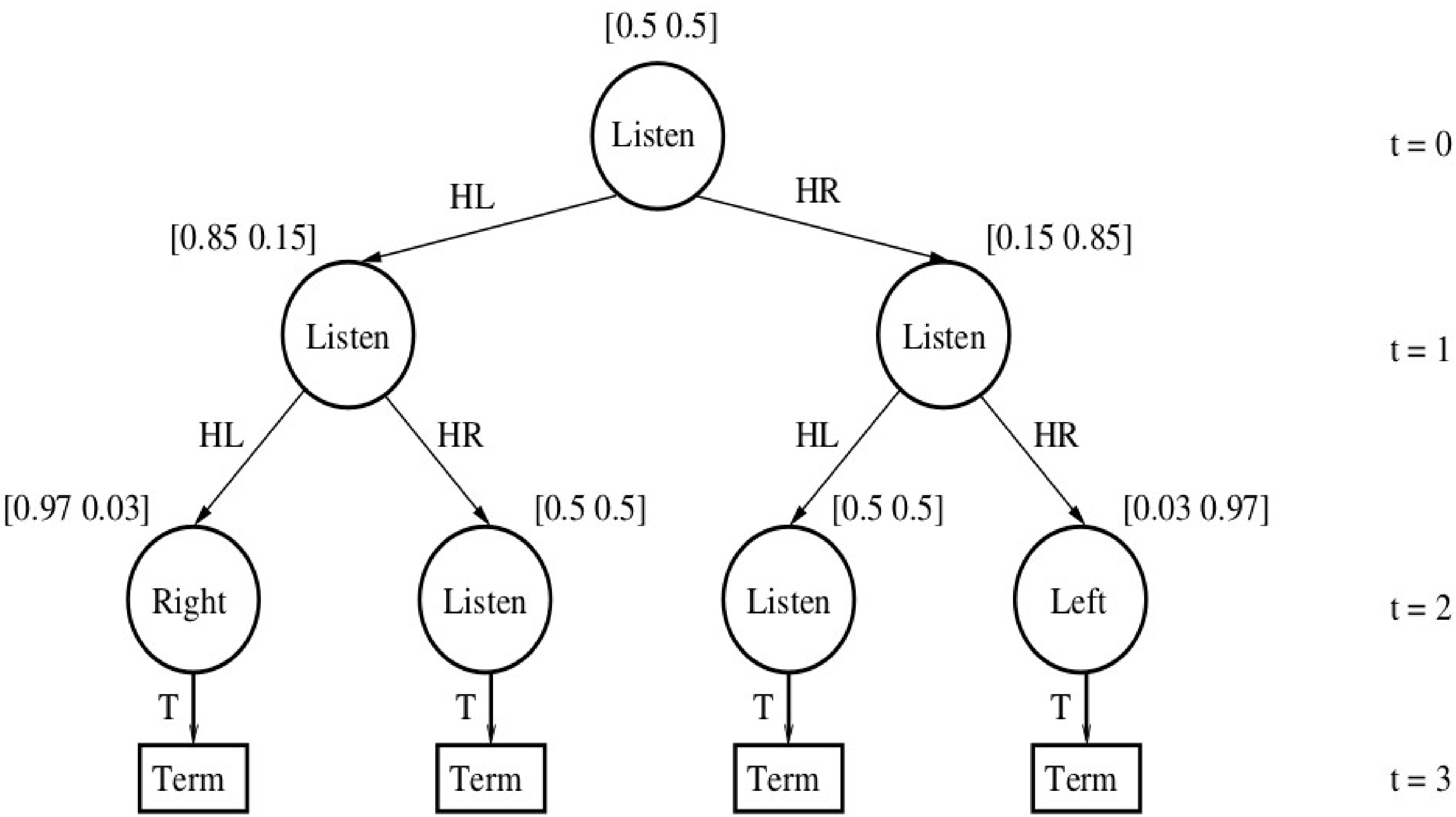
$$V^*_{n+1}(b) = max_a \left[\sum_s b(s) r(s,a) + \gamma \sum_z \sum_{s'} P(z|s',a) \sum_s P(s'|s,a) b(s) V^*_n(b^a_z) \right]$$

- No trivial closed form solution (similar to MDP tabulation) because *V(b)* is a function of a continuous variable.
- At each iteration, each plan of previous iteration is combined with each possible action/observation pair to generate plans of length *n*+1.
  - At each iteration number of conditional plans increases by

$$|V_{n+1}| = |A| |V_n|^{|Z|}$$

- Some conditional plans often not optimal for any belief.
  - Corresponding alpha-vectors never dominant.
  - Alpha-vectors (/conditional plans) can be pruned at each iteration.

Starting from known belief state



Computational complexity

- For a known starting belief state and horizon *H,* the size of a full policy tree is $\left(|A||Z|\right)^{H}$

- Infinite horizon POMDPs thus not possible to construct in general.

- Note: Linear systems with Gaussian uncertainty optimally solvable by Kalman filter + optimal control.

Summary

- Partially observable MDPs are MDPs with observations that depend stochastically on state.
- POMDP integrates optimal information gathering to optimal decision making.
- POMDP = belief-state estimation + belief-state MDP.
- POMDPs computationally intractable in general situations.
  - Approximations are needed for larger than toy problems.

Next week: Larger POMDPs