Microeconomic Theory II
Helsinki GSE
Juuso Välimäki

Problem Set 4, Due December 15, 2021

1. This a modification of Problem 3 from PS 3. Rather than having linear utility functions. the agents have now Cobb-Douglas utility functions over a countably infinite number of goods $x_0, x_1, x_2, \ldots$. Let the initial endowment of both agents be constant across the goods: $\omega_{il} = 1$ for $i \in \{1, 2\}$ and all $l \in \mathbb{N}$. Let the utility functions of the two agents be given by:

$$u_1 = \sum_{l=1}^{\infty} \alpha^l \ln(x_{1l}),\; u_2 = \sum_{l=1}^{\infty} \beta^l \ln(x_{2l}),$$

   for some $\alpha, \beta > 0$.

   (a) For a given sequence p_l of prices, derive the first-order conditions for optimal consumption for the two agents for consecutive goods. (Argue that you can consider interior solutions).

   (b) Find the competitive equilibrium prices for this economy.

   (c) Interpret the model and the prices from a dynamic choice perspective.

2. Robinson Crusoe lives alone on an island and has a fixed amount of time L at his disposal. L is allocated between leisure l and work $z = L - l$. Labor produces food according to the production function $c = f(L - l)$ and RC has a utility function for leisure and consumption $u(l, c)$. Assume that the production function is strictly increasing and strictly concave and that the utility function is strictly increasing in both components and strictly convex.

   (a) Assume that RC has a split personality. On the one hand, he owns the production process and hires his own labor services at wage rate w and sells food c at prices p to maximize her profit π. On the other hand, RC also acts as a consumer/worker deciding on her optimal labor supply and consumption given the prices $p$and w so that:

$$pc \leq wz + \pi.$$

Note that the consumer RC also receives the profit of the firm as a dividend. Formulate the two problems and draw a graphical representation in the plane and display the profit of the firm in the picture. Draw a picture with an. excess demand of labor and another with equilibrium prices and wages.

(b) Solve the two problems in the case where $f(L-l) = \sqrt{L-l}$ and $u(l,c) = \ln c + \alpha \ln l$.

(c) Solve the 'planner's problem' without any prices where RC chooses optimal leisure subject to $c = f(L-l)$ and verify that you get the same solution.

3. Suppose n agents have identical preferences for present consumption c_0 and future consumption c_1:

$$u(c_0, c_1) = \ln c_0 + \alpha \ln c_1.$$

The. agents have identical initial endowments $(1,0)$ and each agent own an equal share in the sole firm operating in the economy. The firm transforms current funds x_0 to future funds x_1 according to the production function $x_1 = \beta\sqrt{x_0}$.

(a) Let $p = \frac{1}{1+r}$ be the price of period 1 funds relative to period 0 funds. Set up the firm's present value maximization problem taking r as given.

(b) Find the agents' demands for present consumption taking r as given.

(c) Solve for the equilibrium interest rate in this economy.

(d) What is the impact of a technological improvement that induces an increase in β on the interest rate?

4. An economy consists of a continuum of mass 1 of agents whose labor income y has a strictly increasing atomless cumulative distribution

function $F(y)$ on $[\underline{y}, \overline{y}]$ (with $0 < \underline{y}$). The economy has also a mass 1 of rental units available for housing. The housing units come in 3 different quality levels $q \in \{q_L, q_M, q_H\}$ with $0 < q_L < q_M < q_H$. The number of units in quality level q_k is $n_k > 0$ so that $\sum_k n_k = 1$.

(a) The agents have all the same utility function for housing quality and consumption $u(q, c)$ and their choices must be compatible with the budget constraint. Houses are indivisible and all agents must live in some house (and they get no additional utility from a second house). Formulate the consumers' optimization problem given prices p_k for houses of type k.

(b) Formulate the market clearing condition.

(c) Define a competitive equilibrium and solve for the equilibrium prices and housing allocation in the case where $u(q, c) = qc$.

(d) Can you find a utility function that is increasing in both components and that induces an equilibrium that is not positive assortative?

5. Consider a large population of identical children. A child's preferences can be represented by a utility function $U : \mathbb{R} \longrightarrow \mathbb{R}$, which takes a number of gifts received as input. Furthermore, $U' > 0$ and $U'' < 0$.

   Christmas is coming. There are two possible states of Christmas. With a probability of $\frac{1}{2}$, Christmas is happy, in which case each child receives two Christmas gifts from Santa Claus. With the remaining probability of $\frac{1}{2}$, Christmas is miserable, in which case each child receives only one Christmas gift from Santa Claus. The state of Christmas is realized on Christmas Eve.

   Children can trade three different assets. A child who holds one *risk-free asset* on Christmas Eve is entitled to one additional Christmas gift regardless of the state of Christmas. A holder of a *procyclical asset* is entitled to 2 additional Christmas gifts if Christmas is happy, and to 0 additional gifts if Christmas is miserable. A child who holds a *countercyclical asset* on Christmas Eve is entitled to 2 additional Christmas gifts if Christmas is miserable, and to 0 additional gifts if Christmas is happy. Denote the pre-Christmas prices of these assets by P^{RF}, P^{PC}, and P^{CC}, respectively (the prices are measured in terms of gifts). Short sales are allowed, and the assets must be in zero net supply.

Hint: Notice that in the equilibrium, there is no asset trade, and each child is indifferent between buying and selling any of the assets before Christmas.

(a) Determine the following three price ratios: P^{RF}/P^{PC}, P^{PC}/P^{CC}, and P^{RF}/P^{CC}. (The answer should be in terms of $U'(1)$ and $U'(2)$.) Which one of the assets is the most expensive one, and which asset is the cheapest one? Which asset has the highest expected rate of return?

(b) Consider an otherwise similar situation, but with a small difference regarding the miserable state of Christmas: if Christmas is miserable, half of the children receive 0 Christmas gifts from Santa Claus, whereas the remaining half of the children receive 2 gifts from Santa Claus. The children are still identical *ex ante*. Determine the price ratio P^{PC}/P^{CC} in this situation. If $U''' = 0$ (in the relevant part of the domain), is P^{PC}/P^{CC} equal, greater, or less than in Part (a)? If $U''' > 0$, is P^{PC}/P^{CC} equal, greater, or less than in Part (a)?