**Game Theory Final Exam**

March 8th, 2021

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*The exam is 4 hours and has a total of 120 points. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about a question or statement, please explain clearly what you assume when answering. Point totals reflect the difficulty of the problem and give a rough estimate for how long the question should take.*

1. This question concerns a modification of the bargaining model we saw in class. Suppose two players are bargaining over a dollar. Let the ordered pair $(x_1, x_2)$ denote a possible split of the dollar, where player 1 receives $x_1$ and player 2 receives $x_2$. Player 1's utility from a division $(x_1, x_2)$ is $x_1$. But now, when considering whether to accept or reject an offer, player 2 also cares about how much better they are doing than player 1. Specifically, player 2's utility from accepting an offer is now $(1-K)x_2+K(x_2-x_1)$, $K \in (0,1)$. When they are proposing, their utility is $x_2$. These preferences are common knowledge, and players have common discount factor $\delta \in (0,1)$.

   The bargaining procedure proceeds as in the alternating offer bargaining model from class. In each period $t \in \{1, 2, \ldots T\}$ a player makes an offer $(x, 1-x)$, where $x \in [0,1]$, the other player chooses to accept or reject. Player 1 makes offers in odd periods, player 2 makes offers in even periods. The game ends either after period $T$ or whenever an offer is accepted. If no one accepts an offer, both players get 0. Figure 1 illustrates this game in the $T = 2$ case.

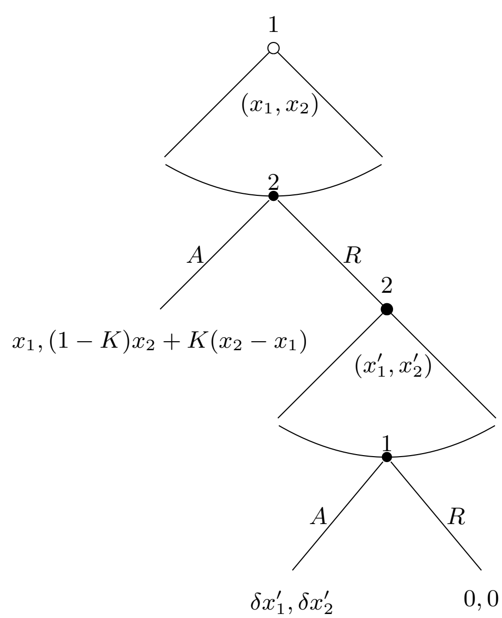


Figure 1: Game for 1b. Payoffs are defined analogously in (a) and (c)

(a) *(5 Points)* Suppose $T = 1$. As a function of $K$, what is the unique subgame perfect nash equilibrium.

(b) *(10 Points)* Suppose $T = 2$. What is the unique subgame perfect nash equilibrium of this game, as a function of $K$ and $\delta$?

(c) *(20 Points)* Now suppose $T = \infty$, the game is repeated until a player accepts. Describe a stationary, subgame perfect equilibrium of this game.

(d) *(5 Points)* Let $V_1(K, \delta)$ be player 1's payoff in the equilibrium you found in (c) for a given $K, \delta$. How do $\lim_{K\to 0} \lim_{\delta\to 1} V_1(K, \delta)$ and $\lim_{\delta\to 1} \lim_{K\to 0} V_1(K, \delta)$ compare? What does this tell you about the role of $K$ in the bargaining process.

2. The following question concern the two extensive forms at the end of the exam.

(a) *(5 Points)* How many strategies does player1 have in each of the extensive forms? How many subgames are there?

(b) *(10 Points)* What is the *reduced* normal form for extensive form 1? (In terms of showing work, a clear reduced normal form is sufficient for full credit on this question.)

(Note that this is also the reduced normal form for extensive form 2.)

(c) *(15 Points)* What are the pure strategy Nash equilibria in the normal form you found in (b)? Do any seem more plausible than the others?

(d) *(15 Points)* Are there any subgame perfect equilibrium in extensive form 1 where player 2 plays $B_R$ as part of their equilibrium strategy and player 1 gets a payoff of 10? If yes, provide an example, if no, prove it. What about in extensive form 2?

(e) *(15 Points)* How would your answer to (d) change if we replaced subgame perfection with almost Perfect Bayesian equilibrium?

(f) *(20 Points)* Construct a sequential equilibrium in each extensive form where $C$ is played.

(If you have extra time, does anything seem odd about the equilibrium you've constructed in the first extensive form? The second?)

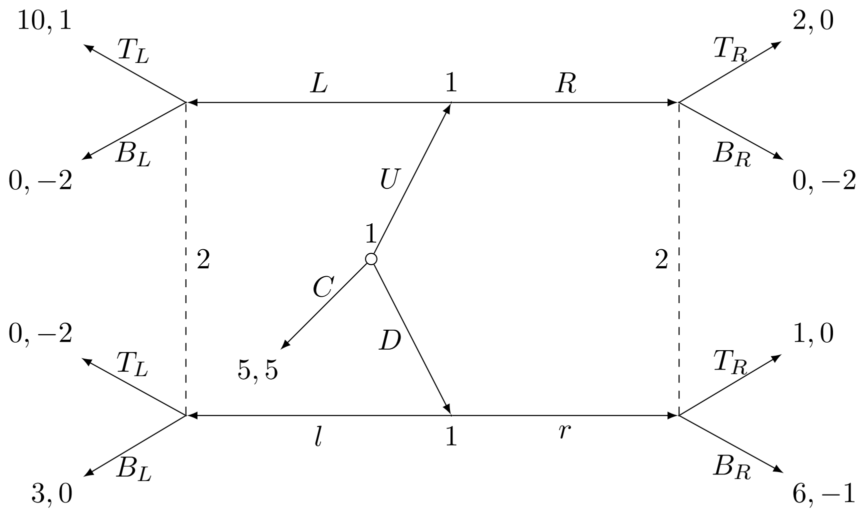


Figure 2: Extensive Form 1

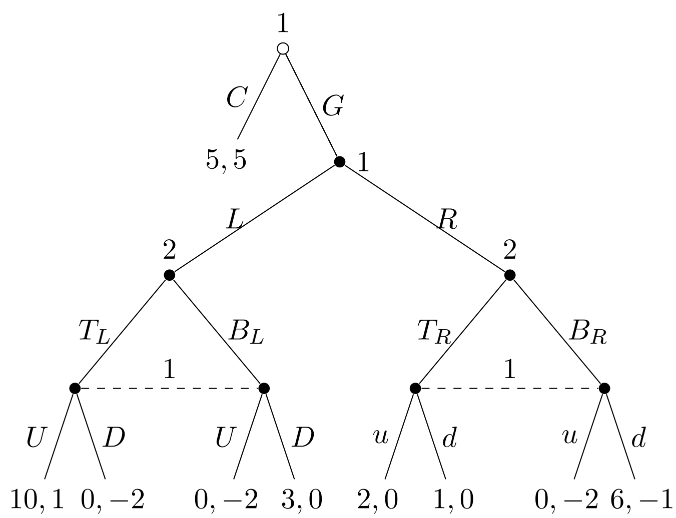


Figure 3: Extensive Form 2