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Decision Support by Interval SMART/ SWING—Incorporating Imprecision in the SMART and SWING Methods

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ABSTRACT

Interval judgments are a way of handling preferential and informational imprecision in multicriteria decision analysis (MCDA). In this article, we study the use of intervals in the simple multiattribute rating technique (SMART) and SWING weighting methods. We generalize the methods by allowing the reference attribute to be any attribute, not just the most or the least important one, and by allowing the decision maker to reply with intervals to the weight ratio questions to account for his/her judgmental imprecision. We also study the practical and procedural implications of using imprecision intervals in these methods. These include, for example, how to select the reference attribute to identify as many dominated alternatives as possible. Based on the results of a simulation study, we suggest guidelines for how to carry out the weighting process in practice. Computer support can be used to make the process visual and interactive. We describe the WINPRE software for interval SMART/SWING, preference assessment by imprecise ratio statements (PAIRS), and preference programming. The use of interval SMART/SWING is illustrated by a job selection example.

Subject Areas: Decision Support Systems, Imprecision, Multicriteria Decision Making, and Uncertainty Modeling.

INTRODUCTION

Multicriteria decision analysis (MCDA) is an approach to systematically evaluate a set of alternatives with multiple criteria. Interval judgments provide a convenient way to account for preferential uncertainty, or imprecision, and incomplete information (Weber, 1987). In MCDA models, intervals can be used, for example, to

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describe the range of allowed variation in the weight ratio and value estimates due to imprecision.

Interval modeling has been applied in various MCDA methods. ARIADNE (Alternative Ranking Interactive Aid based on DomiNance structural information Elicitation; Sage & White, 1984; White, Sage, & Dozono, 1984) was the first decision support system to use interval judgments through direct constraints on values and weights. HOPIE (Holistic Orthogonal Parameter Incomplete Estimation; Weber, 1985) was based on holistic interval judgments on a set of hypothetical alternatives allowing also constraints for pairwise comparisons of the alternatives. Preference programming (Arbel, 1989; Salo & Hämäläinen, 1995, 2003) generalizes the pairwise comparisons of the AHP (Analytic Hierarchy Process; Saaty, 1980, 1994; Salo & Hämäläinen, 1997) to intervals. In PAIRS (Preference Assessment by Imprecise Ratio Statements; Salo & Hämäläinen, 1992), the attributes are also compared in pairs, but the alternatives are evaluated within a value tree framework. In PRIME (Preference Ratios In Multiattribute Evaluation; Salo & Hämäläinen, 2001), the attribute weights are elicited through interval tradeoff comparisons of value differences. Lee, Park, Eum, and Park (2001) and Eum, Park, and Kim (2001) have developed extended interval methods for identifying dominance and potential optimality.

In this article, we discuss the use of interval judgments in the SMART (Simple Multi-Attribute Rating Technique; Edwards, 1977; von Winterfeldt & Edwards, 1986) and SWING (von Winterfeldt & Edwards, 1986) methods. They are simple multiattribute weighting methods based on ratio estimation. While the idea of modeling imprecise information with intervals is not new, the use of intervals explicitly in SMART and SWING has not been previously presented in the literature. In our discussion, we deal with SMART and SWING simultaneously as one method, and refer to this generalized method as interval SMART/SWING.

In practice, the true usefulness of the methods is determined by procedural aspects. Easy-to-use approaches such as SMART and SWING are nowadays the common basis of many applied MCDA studies (Belton & Stewart, 2001). Thus, we believe that the related generalized interval SMART/SWING approach would be of interest to the practitioners as it preserves the cognitive simplicity of the original methods.

Computationally the interval SMART/SWING weighting process leads to a similar optimization problem as in the PAIRS method, and the tools presented with PAIRS can be directly used in the calculations. However, from the procedural and practical elicitation viewpoints, the interval SMART/SWING method has characteristics that should be addressed in the determination of the weight intervals and in the analysis of the results. These originate mainly from the fact that in interval SMART/SWING the preference comparisons are done with respect to a certain reference attribute only. The main objective of this article is to discuss these procedural and practical aspects of the method. We shall, for example, discuss the implications of having a certain reference attribute, and study the effects of using different attributes as a reference. Based on the results of a simulation study, we also suggest guidelines for how to select the reference attribute.

Computer support is needed to solve the overall value intervals, and it can facilitate the process by making it interactive and visual. To help the reader get an idea of the practical possibilities, we shall also describe the WINPRE (Workbench for INteractive PREference Programming; Hämäläinen & Helenius, 1997) software, which supports interval SMART/SWING, PAIRS, and preference programming approaches.

This article is organized as follows. First, we describe the relevant ratio estimation methods. Then, we discuss the use of intervals in preference judgments, and practical and procedural issues related to the selection of the reference attribute. The use of the method with the WINPRE software is demonstrated next by an illustrative example, and finally, we give the concluding remarks.

RATIO ESTIMATION METHODS

Multiattribute value theory (MAVT) is an MCDA approach, in which the overall values of the alternatives are composed of the ratings of the alternatives with respect to each attribute, and of the weights of the attributes. If the attributes are mutually preferentially independent (Keeney & Raiffa, 1976), an additive value function can be used to calculate the overall values. The overall value for alternative x is

$$v(x) = \sum_{i=1}^{n} w_i v_i(x_i), \tag{1}$$

where n is the number of attributes, $w_i \geq 0$ is the weight of attribute i, x_i is the consequence or the measurement value of alternative x with respect to attribute i, and $v_i(x_i)$ is its rating. One should note that other terms, such as a component value, an attribute value, and a score, are also used in literature to characterize $v_i(x_i)$. The sum of the weights is normalized to one, and the ratings are scaled onto the range [0, 1], for example, by using value functions. Weights w_i can be given directly by point allocation (Schoemaker & Waid, 1982), or by different weighting procedures such as SMART or SWING.

In SWING, the decision maker (DM) is first asked to consider a hypothetical alternative in which all the attributes are on their worst consequence levels. Then, the DM is asked to identify the most important attribute, that is, an attribute whose consequence he/she most preferably would change from its worst level to its best level. This is given a hundred points. Next, the DM is asked to identify an attribute, whose consequence he/she next preferably would change to its best level. To this, the DM is asked to assign fewer points to denote the relative importance of the change in this compared to the change in the most important attribute. The procedure continues similarly on the other attributes. The actual attribute weights are elicited by normalizing the sum of the given points to one.

In SMART, the DM assigns 10 points to the least important attribute. Then, he/she assigns more points to the other attributes to address their relative importance. The weights are elicited by normalizing the sum of the points to one. However, it has been stressed that the comparison of the importances of the attributes is meaningless, if it does not reflect the consequence ranges of the attributes as well (von Winterfeldt & Edwards, 1986; Edwards & Barron, 1994). These can be taken into account by applying SWING weighting to SMART. That is, in the comparison of the importances of the attributes, the DM should explicitly focus on

Table 1: A set of ratio methods classified by the type of judgments used.

	Exact Point Estimates	Interval Estimates
Minimum number of pairwise judgments	SMART, SWING	Interval SMART/SWING
More than minimum number of judgments allowed	AHP, Regression analysis	PAIRS, Preference programming

the attribute changes from their worst consequence level to the best level. Edwards and Barron (1994) named this variant as SMARTS (SMART using Swings), but the term SMART is also commonly used for this variant.

SMART and SWING are algebraic methods, that is, the weights are derived from $n - 1$ linearly independent judgments on preference relations (Weber & Borcherding, 1993), which is the minimum number of judgments required to elicit n weights. Another way is to derive the weights from a larger set of judgments with some estimation method (see Table 1). In an extreme case, the set of all the possible $n \times (n - 1)/2$ pairwise judgments is used. For example, in the AHP the weights are elicited from this set with the eigenvalue procedure (Saaty, 1980). Interval methods can be classified in the same way. When using interval estimates the minimum number of judgments is $2 \times (n - 1)$, as both the lower and upper bounds are given for the preference relations. Interval SMART/SWING uses the minimum number of judgments, but there are also interval methods which allow the use of more judgments, such as PAIRS and preference programming.

INTERVAL SMART/SWING

In interval SMART/SWING, we generalize the SMART and SWING methods (i) by allowing the reference attribute to be any attribute, not just the most or the least important one, and (ii) by allowing the DM to use interval judgments on the weight ratio questions and on the evaluation of the alternatives to represent related imprecision.

Reference Attribute

By allowing the reference attribute to be any attribute, interval SMART/SWING makes it possible to use, for example, some easily measurable attribute, such as money, as a reference. This can often make the weight elicitation process easier and consequently decrease imprecision related to the weights. This kind of an approach has also been recommended in other methods, for example, in the tradeoff and Even Swap methods, it has been suggested to make the easiest tradeoffs first (Keeney, 1992; Hammond, Keeney, & Raiffa, 1998, 1999).

In this generalization the reference attribute is given a fixed number of points, while the other attributes receive points that reflect their relative importance. In practice, any number of points can be assigned to the reference attribute, as far as the points assigned to the other attributes are relative to these points. For example, if the DM is familiar with the SMART method, it is natural to assign 10 points to the reference attribute in interval SMART/SWING, too.

The actual weights are elicited by normalizing the sum of the points to one, as in SMART and SWING. Thus, the distinction between these methods is based on procedural differences only. If the DM is consistent in his/her weighting, the weights elicited with different methods should be the same. However, behavioral research has shown that different weighting methods may give diverging results (Weber & Borcherding, 1993; Pöyhönen & Hämäläinen, 2001). As possible explanations for this, it has been suggested, for example, the DMs' tendency to give points in multiples of 10, or tendency to consider the ranking of the attributes rather than the strength of the preferences (Pöyhönen, Vrolijk, & Hämäläinen, 2001). Thus, here as well as on any other MCDA method, the DM should be well informed about the proper use of the method to avoid such procedural biases. This is especially important when using any attribute as a reference, as the points given can be both higher and lower than those for the reference attribute.

Interval Judgments

The other generalization of interval SMART/SWING is to allow the DM to reply with intervals to the weight ratio questions to describe possible imprecision in these. Then, the reference attribute is given a fixed number of points, but the points for the other attributes are given as intervals representing the imprecision in the judgments. From these intervals we can derive constraints for the attributes' weight ratios in a straightforward manner by taking the extreme ratios of the points given to the reference attribute and the other attributes, that is,

$$\frac{ref}{\max_i} \leq \frac{w_{ref}}{w_i} \leq \frac{ref}{\min_i}, \quad \forall i = 1, \ldots, n, \quad i \neq ref, \tag{2}$$

where *ref* stands for the points given to the reference attribute and $\max_i$ ($\min_i$) for the maximum (minimum) number of points given to a nonreference attribute i. For example, if the reference attribute was given 1.0 point and attribute i an interval from 0.5 to 3 points, the constraints for the weight ratio between these would be $1/3 \leq w_{ref}/w_i \leq 1/0.5 = 2$, or with another notation $w_{ref}/w_i \in [1/3, 2]$. The constraints in (2), in addition to the weight normalization constraint $\sum_{i=1}^{n} w_i = 1$, determine the feasible region of the weights, S.

Similar intervals can be given to the ratings of the alternatives to describe imprecision in these. In practice, these intervals can be assigned directly (e.g., $0.2 \leq v_i(x_i) \leq 0.5$) or, for example, by setting bounds for the value functions from which the intervals can be derived (Salo and Hämäläinen, 1992).

As a result, one gets overall value intervals for the alternatives describing the possible variation in the overall values due to allowed variation in the weights and the ratings. The lower bound for the overall value of alternative x ($\underline{v}(x)$) is elicited as its minimum, when allowing the weights and the attribute values to vary within the given constraints. That is,

$$\underline{v}(x) = \min_{w \in S} \sum_{i=1}^{n} w_i \, \underline{v}_i(x_i), \tag{3}$$

where $\underline{v}_i(x_i)$ is the lower bound for $v_i(x_i)$, and $w = (w_1, \ldots, w_n) \in S$. The upper bound is obtained analogously. The minimization problem (3) can be solved by linear programming. Technically this problem is similar to the one of the PAIRS method, and for computational details, see Salo and Hämäläinen (1992).

An analysis of the alternatives' value intervals can be employed to determine the dominance relations between the alternatives (Weber, 1987; Salo & Hämäläinen, 1992). Alternative x dominates alternative y if the value of x is greater than the value of y for every feasible combination of the weights, that is, if

$$\min_{w \in S} \sum_{i=1}^{n} w_i(\underline{v}_i(x_i) - \bar{v}_i(y_i)) > 0. \tag{4}$$

More specifically, one can say that this is a definition of pairwise dominance.

Example

To illustrate the interval SMART/SWING analysis, consider a case with two alternatives (A and B) and three attributes (1, 2, and 3). *Attribute 1* is chosen as the reference attribute and given 1.0 point (see Figure 1). *Attribute 2* is given an interval from 0.5 to 2.0 points and *attribute 3* an interval from 1.0 to 3.0 points to reflect judgmental imprecision in the importances of these. The weight ratio constraints are derived from the ratios of these according to (2), and they are $w_1/w_2 \in [1.0/2.0, 1.0/0.5] = [1/2, 2]$ and $w_1/w_3 \in [1.0/3.0, 1.0/1.0] = [1/3, 1]$. These define the feasible region of the weights S. Figure 2 illustrates this region on the weight space $\{w = (w_1, w_2, w_3)\colon w_i \geq 0, \sum_{i=1}^{3} w_i = 1\}$. For simplicity, we assume that there is no imprecision in the ratings (i.e., the lower and upper bounds of each rating interval are the same) and set these as $\underline{v}_1(A) = \bar{v}_1(A) = 0.0$, $\underline{v}_1(B) = \bar{v}_1(B) = 1.0$, $\underline{v}_2(A) = \bar{v}_2(A) = 1.0$, $\underline{v}_2(B) = \bar{v}_2(B) = 0.8$, $\underline{v}_3(A) = \bar{v}_3(A) = 1.0$, and $\underline{v}_3(B) = \bar{v}_3(B) = 0.0$.

As a result we get the overall values intervals of [0.60, 0.83] for *alternative A* and [0.31, 0.65] for *alternative B* (see Figure 1). Now, *alternative A* dominates *alternative B*, as the value of A is greater than the value of B for any single weight combination within the feasible region S. Thus, although the overall value intervals overlap, on the basis of dominance, *alternative A* can be considered to be the best alternative.

Remarks

In interval SMART/SWING, the preference relations between the nonreference attributes are not explicitly stated. However, upper bounds for the weight ratios between these can be implicitly derived from the constraints (2). For example, in a case of three attributes, an upper bound for the weight ratio between two nonreference attributes 2 and 3 can be derived from constraints $ref/\max_2 \leq w_{ref}/w_2$ and $w_{ref}/w_3 \leq ref/\min_3$, from which we get $w_2 \leq \max_2/ref \times w_{ref} \leq \max_2/ref \times (ref/\min_3 \times w_3) \Rightarrow w_2/w_3 \leq \max_2/\min_3$. In our example, $\max_2 = 2$ and $\min_3 = 1$, and by calculating the lower bound similarly we get the weight ratio interval $w_2/w_3 \in [1/6, 2]$ (the dotted lines in Figure 2). However, these constraints are clearly redundant as they do not restrict the feasible region more than the other constraints do.

The feasible region of the weights obtained with interval SMART/SWING will never become empty, as may happen in the methods using more than minimum

Figure 1: Interval SMART/SWING analysis with three attributes (1, 2, and 3) and two alternatives (*A* and *B*). A screen capture from the WINPRE software.

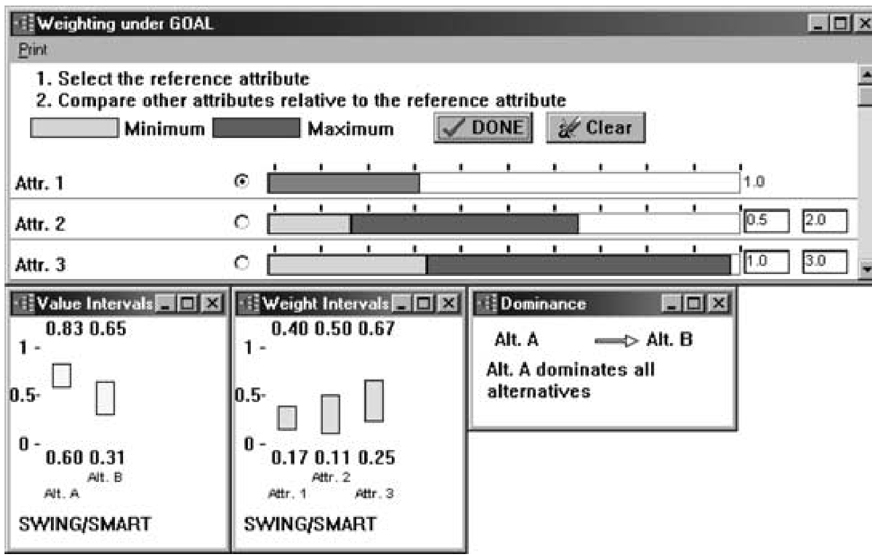
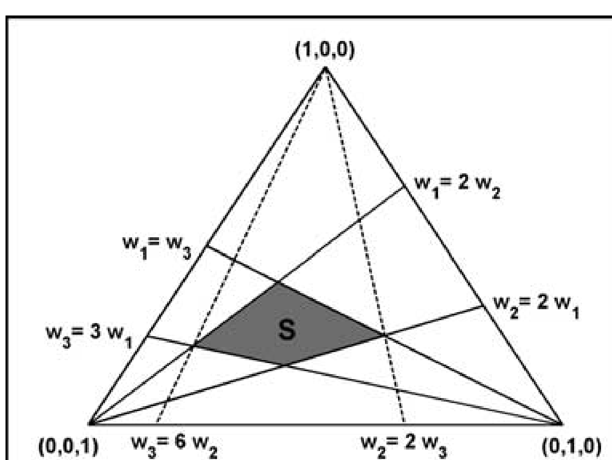


Figure 2: Feasible region *S* on the simplex representing the weight space.



number of judgments (e.g., in PAIRS). On the other hand, in these methods an empty feasible region would indicate inconsistency in the DM's preference assessment. In such a case the DM is requested to evaluate his/her preferences. Although in interval SMART/SWING the DM cannot give inconsistent judgments, it would often be useful to separately check the correctness of the statements. This can be carried out by assessing a few weight ratio constraints also between the nonreference attributes, even if this is not explicitly required by the method (Weber & Borcherding, 1993).

In this article, we only concentrate on nonhierarchical value trees having one attribute level. However, the method can also be applied in hierarchical trees with many attribute levels, similarly as PAIRS. Then, the interval weighting is carried out on each branch of the value tree separately. For computational details see Salo and Hämäläinen (1992).

HOW TO SELECT THE REFERENCE ATTRIBUTE

A common goal in MCDA is to identify dominated alternatives. In interval SMART/SWING, the choice of the reference attribute may affect the occurrence of the dominances. Next we shall discuss how the reference attribute can be efficiently selected, that is, so that as many dominated alternatives as possible are identified with as few procedural actions as possible.

In general, the smaller the feasible region is, the more dominated alternatives are likely to be identified. Therefore, a natural way is to select the reference attribute so that imprecision intervals become as tight as possible. However, procedurally the evaluation of these intervals is carried out only after selecting the reference attribute. Thus, in general this piece of information cannot be assumed to be available at this phase of the process. Yet there are also cases where the DM may be able to easily identify the attribute with least imprecision beforehand, for example, the above-mentioned money may be such an attribute to many DMs.

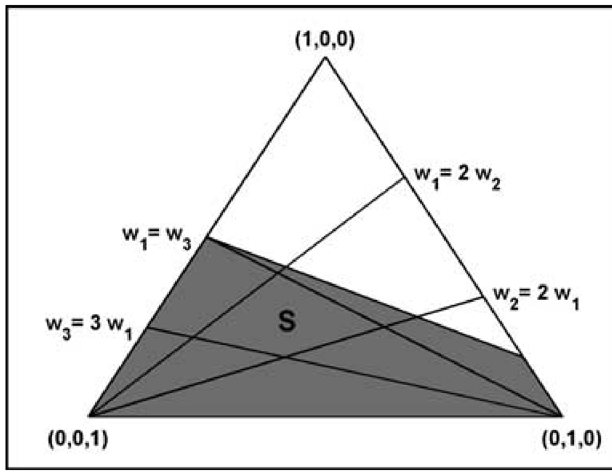
On the other hand, the shape of the feasible region and its position on the weight space may also have an effect on the occurrence of the dominances. To illustrate this, let us further consider our example in the previous section (Figure 2). From the whole weight space we can identify an area in which *alternative A* dominates *alternative B* (the shaded area in Figure 3). This area can be formed according to (4), that is, by including all the weight vectors $w = (w_1, \ldots, w_n)$, such that $\sum_{i=1}^{n} w_i(\underline{v}_i(A) - \bar{v}_i(B)) > 0$, in it. If the feasible region of the weights is now within this area as a whole, the corresponding dominance occurs, which is clearly the case in our example (Figure 3). Thus, if considering the shape of the feasible region, it would be desirable that the feasible region would be evenly stretched into all directions, so that it would be entirely included in as many of these areas of dominance as possible.

The size of the feasible region could be analytically measured, for example, by an area (or a content in a general case), or it could be approximated, for example, with the consistency measure of Salo and Hämäläinen (1997). However, there are not straightforward analytical ways to simultaneously take into account the shape and the position of the region. Thus, we carried out a simulation experiment to study the effects of selecting different attributes as a reference.

Simulation Study

The objective of the simulation study was to find out what would be the best choice for the reference attribute. The strategies compared were the ones where the ith most important attribute ($i = 1, \ldots, n$) was chosen as a reference attribute. Thus,

Figure 3: The dark coloring indicates an area, where *alternative A* dominates *alternative B*.



we assumed that the DM can specify the ranking for the relative importances of the attributes.

We generated a set of problems, and in each problem instance the efficiency of each strategy was measured. The goal was to determine whether there are statistical differences between the average efficiencies of the strategies. In addition, we studied the effects of the problem size, which was characterized by the number of the attributes (n) and the number of the alternatives (m). We conducted 1,000 simulation rounds on each combination of the values of $n = 3, 5, 8$ and $m = 3, 5, 8$. We did not study any larger problems, because the effects of parameter variation already emerged with these values. The simulations were carried out with the MATLAB software.

On each simulation round, the problem instance was generated as follows. We randomly generated pointwise (i.e., the lower and the upper bounds were the same) measurement values x_i from (0, 1) normal distribution for each alternative x on each attribute i. Thus, these values were independent of each other. The ratings $v_i(x_i)$ were then derived from these values by mapping the measurement value ranges linearly to interval [0, 1]. That is,

$$v_i(x_i) = (x_i - \underline{x}_i)/(\bar{x}_i - \underline{x}_i), \tag{5}$$

where $\bar{x}_i$ and $\underline{x}_i$ represent the maximum and minimum measurement values of attribute i, respectively. By assuming that unit increases in the measurement values are equally preferred on each attribute, the weight of an attribute $i(w_i^o)$ is relative to the range of the corresponding measurement values, that is,

$$w_i^o = (\bar{x}_i - \underline{x}_i) \Big/ \sum_{j=1}^{n} (\bar{x}_j - \underline{x}_j). \tag{6}$$

Thus, as a result of this process we got a problem instance having pointwise estimates for both the weights of the attributes and the ratings of the alternatives.

The imprecision on each strategy was modeled by assigning error ratio R (Salo & Hämäläinen, 2001) on all the ratios between the generated weights of the reference attribute (w^o_{ref}) and any other attribute $i(w^o_i)$. Thus, we assumed that relatively each weight ratio had equal imprecision assigned. In practice, each weight ratio was multiplied by factor R to get the upper bound for it, and divided by R to get the lower bound. That is,

$$\frac{1}{R}\frac{w^o_{ref}}{w^o_i} \leq \frac{w_{ref}}{w_i} \leq R\frac{w^o_{ref}}{w^o_i}, \quad \forall i = 1, \ldots, n, \quad i \neq ref, \tag{7}$$

where w^o_i is the initially generated pointwise weight of attribute i, and ref denotes the reference attribute. For example, if $w^o_{ref} = 0.5$, $w^o_2 = 0.2$ and $R = 1.5$, the weight ratio interval for w_{ref}/w_2 would be $[(1/1.5) \times (0.5/0.2),\ 1.5 \times (0.5/0.2)] = [1.67, 3.75]$. As a result, we got constraints for the same weight ratios that would have been assigned with the interval SMART/SWING process. The simulations were carried out with error ratio $R = 1.5$. In addition, to study the possible effects of error ratio R, simulations with $R = 1.2$, 1.4, 1.8, 2, and 3 were carried out for the case $n = m = 5$.

This setting appears realistic in many cases, as real events are often normally distributed. However, some real life cases would require essentially different distributions for the weights and the ratings. For example, a case having specified grading scales on each attribute would require a simulation setting with fixed ranges for the measurement values. Thus, the simulation was also tested by using other distributions, for example, the uniform distribution both on the weight space and on the ratings of the alternatives. However, the conclusions drawn from the simulations carried out with these other distributions were essentially the same as with this setting.

Two further strategies were also studied: a strategy where imprecision intervals were assigned for all the pairwise judgments (PAIRS), and a strategy where the constraints were sequentially assigned for adjacent pairs of attributes, that is, between the most and the second important ones, the second and the third important ones, and so on. The objective was to have a reference to techniques not having a certain reference attribute selected. However, we do not discuss how to elicit the constraints in these strategies in practice, but take these as directly given.

The efficiency of each strategy was measured by two different measures. The first one was the average number of dominated alternatives obtained with each strategy. The second one was the average of the maximum loss of value, that is, the maximum value difference between initially (at point w^o) the best alternative x^* and all the other alternatives,

$$\max(v(x) - v(x^*)), \quad \forall w \in S, x \in X\backslash\{x^*\}, \tag{8}$$

where S is the feasible region of the weights, and X the set of all the alternatives. If the maximum loss of value is negative, the value of alternative x^* is greater than any other alternative at every point of the feasible region, that is, it dominates all the other alternatives.

The simulation results are presented in Tables 2 and 3. The strategies are named after the rank of the reference attribute in the order of importance. For

Table 2: The average numbers of dominated alternatives with each strategy.

			Strategy									
R	*m*	*n*	1	2	3	4	5	6	7	8	All	Seq
1.5	3	3	1.683 (84.2%)	1.657 (82.9%)	1.571 (78.6%)						1.729 (86.5%)	1.657 (82.9%)
		5	1.423 (71.2%)	1.386 (69.3%)	1.339 (67.0%)	1.299 (65.0%)	1.263 (63.2%)				1.592 (79.6%)	1.340 (67.0%)
		8	1.129 (56.5%)	1.109 (54.5%)	1.069 (53.5%)	1.037 (51.9%)	1.029 (51.5%)	0.997 (49.9%)	0.977 (48.9%)	0.950 (47.5%)	1.437 (71.9%)	0.958 (47.9%)
	5	3	3.506 (87.7%)	3.484 (87.1%)	3.423 (85.6%)						3.599 (90.0%)	3.484 (87.1%)
		5	3.117 (77.9%)	3.037 (75.9%)	3.023 (75.6%)	2.989 (74.7%)	2.945 (73.6%)				3.400 (85.0%)	2.972 (74.3%)
		8	2.611 (65.3%)	2.578 (64.5%)	2.557 (63.9%)	2.527 (63.2%)	2.495 (62.4%)	2.501 (62.5%)	2.460 (61.5%)	2.430 (60.8%)	3.167 (79.2%)	2.294 (57.4%)
	8	3	6.362 (90.9%)	6.323 (90.3%)	6.295 (89.9%)						6.489 (92.7%)	6.323 (90.3%)
		5	5.815 (83.1%)	5.778 (82.5%)	5.739 (82.0%)	5.702 (81.5%)	5.659 (80.8%)				6.255 (89.4%)	5.661 (80.9%)
		8	5.113 (73.0%)	5.066 (72.4%)	5.018 (71.7%)	5.016 (71.7%)	4.958 (70.8%)	4.957 (70.8%)	4.937 (70.5%)	4.888 (69.8%)	5.968 (85.3%)	4.581 (65.4%)
1.2	5	5	3.615 (90.4%)	3.606 (90.2%)	3.585 (89.6%)	3.568 (89.2%)	3.558 (89.0%)				3.745 (93.6%)	3.572 (89.3%)
1.4	5	5	3.250 (81.3%)	3.205 (80.1%)	3.176 (79.4%)	3.158 (79.0%)	3.095 (77.4%)				3.512 (87.8%)	3.153 (78.8%)
1.8	5	5	2.681 (67.0%)	2.596 (64.9%)	2.595 (64.9%)	2.524 (63.1%)	2.451 (61.3%)				3.092 (77.3%)	2.520 (63.0%)
2	5	5	2.455 (61.4%)	2.359 (59.0%)	2.352 (58.8%)	2.291 (57.3%)	2.180 (54.5%)				2.928 (73.2%)	2.246 (56.2%)
3	5	5	1.679 (42.0%)	1.622 (40.6%)	1.575 (39.4%)	1.594 (39.9%)	1.516 (37.9%)				2.277 (56.9%)	1.525 (38.1%)

Table 3: The average of the maximum loss of value with each strategy.

			Strategy									
R	*m*	*n*	1	2	3	4	5	6	7	8	All	Seq
1.5	3	3	−0.167	−0.157	−0.129						−0.187	−0.157
		5	−0.049	−0.038	−0.024	−0.014	0.001				−0.100	−0.020
		8	0.032	0.041	0.049	0.053	0.059	0.065	0.069	0.076	−0.042	0.088
	5	3	−0.067	−0.061	−0.045						−0.088	−0.061
		5	0.008	0.014	0.020	0.027	0.034				−0.039	0.033
		8	0.053	0.058	0.061	0.064	0.067	0.068	0.072	0.076	−0.010	0.108
	8	3	−0.033	−0.027	−0.019						−0.052	−0.027
		5	0.023	0.028	0.032	0.036	0.040				−0.018	0.045
		8	0.058	0.060	0.062	0.064	0.065	0.066	0.068	0.071	0.002	0.111
1.2	5	5	−0.070	−0.068	−0.066	−0.063	−0.060				−0.091	−0.060
1.4	5	5	−0.017	−0.012	−0.007	−0.002	0.004				−0.055	0.004
1.8	5	5	0.073	0.083	0.091	0.102	0.113				0.005	0.110
2	5	5	0.111	0.123	0.132	0.146	0.158				0.030	0.154
3	5	5	0.255	0.272	0.284	0.307	0.329				0.131	0.306

example, *strategy 1* represents the strategy where the most important attribute, that is, an attribute having the highest initial weight w_i^o, was chosen as a reference. "Seq" and "All" stand for the strategies using sequential and all the possible judgments, respectively. The percentages in Table 2 represent the share of the maximum number of alternatives that can be made dominated with each strategy ($m - 1$).

Finally, the differences in the efficiencies between all the possible strategy pairs in all problem sizes were statistically studied. In practice, we calculated the differences both in the maximum loss of value and in the number of dominated alternatives for each strategy pair in each problem instance. Then we tested whether the averages of these differences significantly differed from zero. According to some normality tests (Lilliefors, Jarque-Bera) the data cannot be assumed to be normally distributed, and thus we used the nonparametric Wilcoxon sign rank test for this testing.

Discussion

The simulation results show that if the error ratios on all the preference judgments are the same, in general a strategy of having a more important attribute as a reference is significantly more efficient (with alpha level 0.05). In the statistical tests on the maximum loss of value, this applied on all problem sizes and strategy pairs except between strategies 4 and 5 in the case $n = m = 8$. In the tests on the number of dominated alternatives, there were a few strategy pairs, in which the strategy with a more important reference attribute was not significantly more efficient. These occurred mainly in the cases $n = 8$ between the strategies where an intermediate attribute was as a reference. However, for example, in all the tests between *strategy 1* and a strategy where the least important attribute was as a reference, *strategy 1* identified significantly more dominated alternatives.

In this respect the most efficient way is to choose the most important attribute as a reference attribute. On average, this strategy identified most dominances and had the smallest losses of value in all the problem sizes and with all the error ratios. However, the use of the strategy assumes that the most important attribute can be identified, but often this can be easily done. The use of the most important attribute as a reference also has other advantages. It is certainly meaningful to the DM, whereas comparisons to some less important attribute may become imprecise due to negligible importance of this. The DM has also presumably given more thought to the most important attribute than to the less important ones, and through this may have reduced the related imprecision.

On the other hand, the results also show that even a small reduction in the error ratio affects the efficiency more than choosing the most important attribute as a reference. For example, in the case $n = m = 5, R = 1.4$, all the strategies except *strategy 5* identified more dominances than *strategy 1* in the case $n = m = 5, R = 1.5$. Thus, if the DM can easily pick out an attribute containing least imprecision, this is likely to be worth choosing as a reference attribute instead of the most important one.

To sum up these observations, we suggest the following rules to select the reference attribute:

1. If the DM can easily identify an attribute containing least imprecision, this should be selected as a reference attribute.
2. If the imprecision related to the attributes cannot be differentiated, the most important attribute should be selected as a reference attribute.

If the DM can identify neither the attribute containing least imprecision nor the most important attribute, the attributes are likely to be such on equal terms that no specific recommendations can be given.

As a result of the initial weighting process, there may still be nondominated alternatives so that further adjustments to the parameters are required until the best alternative can be identified. The DM can try to give more precise preference judgments, for example, by tightening the already stated constraints. Another way is to try to reduce imprecision related to the values of the alternatives. Especially if the set of alternatives has been reduced by eliminating dominated alternatives, the workload needed to consider the imprecision related to the alternatives is also smaller.

Decision rules can also be applied to rank alternatives for which dominances do not hold (Salo & Hämäläinen, 2001). Rules based on centralization, such as the central values of the overall value intervals or the use of the central weights, can also be recommended here. However, some other rules such as maximin or maximax (i.e., maximizing the minimum or the maximum of the overall value interval) may cause bias when applied in interval SMART/SWING. This is because there are no explicit constraints between nonreference attributes, and thus these will generally have wider weight ratio intervals. Consequently, the alternatives strong in these attributes will also have wider intervals for the overall values.

Comparison to the Strategies Using Sequential and All the Possible Judgments

In the sequential strategy, the number of explicitly given judgments is $2 \times (n - 1)$, that is, the same as when using a reference attribute. However, all the explicitly given judgments are needed to elicit an upper bound for the weight ratio constraint between the most and the least important attribute. Consequently, this constraint would contain all the imprecision in these judgments. In comparison, by using some reference attribute, the bounds for the weight ratios between any nonreference attributes are elicited only from two explicitly given judgments, as demonstrated in the remarks of the previous section. Thus, by default the sequential strategy is expected to produce wider intervals than strategies using a certain reference attribute, and the more attributes we have the more inefficient the sequential strategy is expected to be. This is supported by the simulation results. In the cases $n = 3$, there is no difference as then the sequential strategy actually corresponds with *strategy 2*, but in the cases $n = 8$ the sequential strategy is generally the most inefficient one.

The DM can also carry out pairwise preference comparisons between all the attributes. Then the number of the given judgments increases from $2 \times (n - 1)$ to $n \times (n - 1)$. For example, in the case $n = 8$ this means an increase from 14 judgments to 56. However, as the first $2 \times (n - 1)$ judgments are given by using the same reference attribute, from these we can derive some constraints for all the attribute pairs, whereas the further judgments only tighten these constraints. Thus, expectedly the further judgments shall not be as efficient in identifying new dominances as the $2 \times (n-1)$ first judgments. The simulation results also clearly support this. For example, in the case $n = m = 8$, $R = 1.5$, by giving the first 25% (14 of 56) of all the possible pairwise judgments, 73.1% of the dominances were identified. If one further gave all the rest of the judgments, the percentage increased only to 85.3%. Thus, the result suggests that instead of assigning constraints on all the possible attribute pairs, the DM should consider other ways of trying to reduce the imprecision, for example, in the ratings of the alternatives.

From the behavioral viewpoint, it is plausible to assume that the imprecision related to some attribute decreases when more preference judgments on this are given, because then this attribute becomes more familiar. In the case of the sequential strategy, this would further reduce its efficiency, as the attributes under judgment change all the time. In the case of giving all the pairwise judgments there may be some influence in favor of this strategy. However, we see that this effect is so small compared to the extra workload needed to give all the judgments that we still cannot suggest using this strategy to further reduce the imprecision.

EXAMPLE WITH COMPUTER SUPPORT

As an example, we consider Vincent Sahid's job decision problem (Figure 4) adapted from Hammond et al. (1998). Vincent's task is to select the best job from five alternatives evaluated with respect to six attributes (Table 4). In the original example, the problem was approached with the Even Swaps method (Hammond et al., 1998, 1999). Now we describe how to apply interval SMART/SWING to model the possible imprecision in the example. One should note that the given intervals are based on our subjective interpretation of the case description, as the original example did not give these explicitly.

Figure 4: Value tree for Vincent Sahid's job decision.

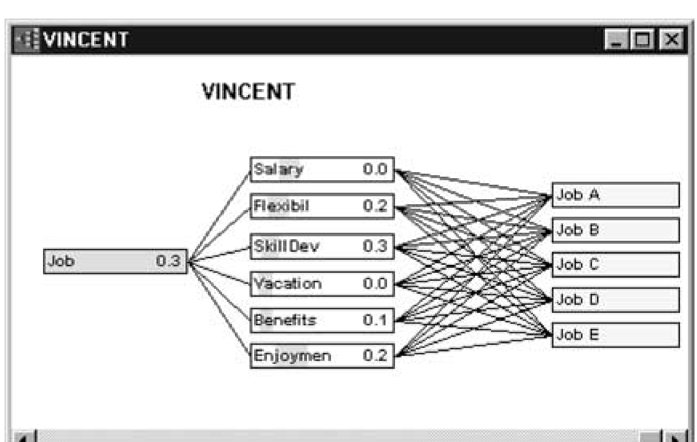


Table 4: Consequences table for Vincent Sahid's job decision (Hammond et al., 1998).

	Job A	Job B	Job C	Job D	Job E
Monthly salary	$2000	$2400	$1800	$1900	$2200
Flexibility of work schedule	Moderate	Low	High	Moderate	None
Business skills development	Computer	Manage people, computer	Operations, computer	Organization	Time management, multiple tasking
Vacation (annual days)	14	12	10	15	12
Benefits	Health, dental, retirement	Health, dental	Health	Health, retirement	Health, dental
Enjoyment	Great	Good	Good	Great	Boring

We illustrate the process by using the WINPRE software, available in the Decisionarium Web site (www.decisionarium.hut.fi; Hämäläinen, 2003). WINPRE provides a graphical user interface to support different phases of the analysis, for example, the creation of the value tree, the elicitation of the attribute weights and the analysis of results. The analysis of the results is truly interactive, as WINPRE gives instantaneous feedback on how the overall values and dominance relations change due to changes in the attribute weights and in the alternatives' ratings. Another software developed later by our research team to support interval ratio methods is PRIME Decisions (Salo, Gustafsson, & Gustafsson, 1999). It supports the PRIME method, and allows interval SMART/SWING to be used in the weight elicitation. For a detailed discussion of PRIME Decisions, see Gustafsson, Salo, and Gustafsson (2001).

Interval SMART/SWING is suitable for this problem especially for the following reasons. First, there are different types of imprecision (Wallsten, 1990 or French, 1995) related to the attributes, which can all be modeled with intervals. Second, there are relatively many attributes. Thus, with interval SMART/SWING the number of attribute comparisons does not become too high, as it depends only linearly on the number of attributes.

In attributes *business skills development* and *benefits*, there may be imprecision, for example, due to incomplete job descriptions. We model this by using intervals to cover the possible differences between the given job descriptions and the reality. Attributes *flexibility of work schedule* and *enjoyment* are evaluated by classifying the alternatives into a set of verbal explanations. However, often there is some imprecision around these explanations, for example, two alternatives may both be classified as *good* on some attribute, although in practice the other one may be somewhat better. This is modeled by associating a rating interval with each of the verbal explanations. For example, on attribute *enjoyment* we use intervals: *boring* = [0.0, 0.2], *good* = [0.5, 0.7], and *great* = [0.8, 1.0] (see Figure 5).

Exact point estimates can also be used by setting the upper and lower bounds of the intervals the same. In this example, the consequences in attributes *salary* and

Figure 5: Interval evaluation for the attribute *enjoyment*.

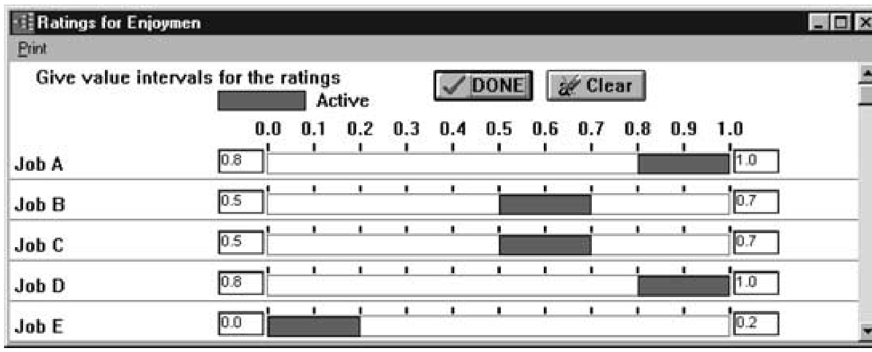


Table 5: Value intervals for the attributes.

Attribute and Its Range	Job A	Job B	Job C	Job D	Job E
Monthly salary [$1800, $2400]	[1/3, 1/3]	[1, 1]	[0, 0]	[1/6, 1/6]	[2/3, 2/3]
Flexibility of work schedule [0, 1]	[0.5, 0.7]	[0.2, 0.4]	[0.8, 1.0]	[0.5, 0.7]	[0.0, 0.0]
Business skills development [0, 1]	[0.3, 0.7]	[0.7, 1.0]	[0.5, 0.8]	[0.0, 0.3]	[0.6, 0.9]
Vacation [10, 15]	[0.8, 0.8]	[0.4, 0.4]	[0.0, 0.0]	[1.0, 1.0]	[0.4, 0.4]
Benefits [0, 1]	[0.8, 1.0]	[0.3, 0.4]	[0.0, 0.0]	[0.5, 0.6]	[0.3, 0.4]
Enjoyment [0, 1]	[0.8, 1.0]	[0.5, 0.7]	[0.5, 0.7]	[0.8, 1.0]	[0.0, 0.2]

vacation for each alternative are pointwise estimates, which are mapped linearly on the value scale. Table 5 presents each alternative's value intervals on the attributes.

In attribute weighting, there may be imprecision, for example, due to the DM's inability to assess his/her weights precisely. Figure 6 presents the interval SMART/SWING weighting in our example. *Monthly salary* is chosen as the reference attribute for two reasons. First, it is an easily measurable and understandable attribute. Thus, the attribute comparisons can be expected to be an easier process than, for example, in the case where *business skills development* is to be compared with the other attributes. Second, *salary* is the most important attribute (jointly with *enjoyment*), and thus all the comparisons are carried out to less important attributes.

As a result we get the overall value intervals for the alternatives and the possible dominance relations (Figure 7). Now alternatives *Job C* and *Job E* are dominated by *Job B* (and *Job E* also by *Job A*). Thus, any combination of the weights satisfying the given constraints cannot give *Job C* or *Job E* a better overall value than *Job B* has.

We can continue our analysis by specifying the given information to get more accurate results. We can, for example, define subclasses for the verbal descriptions. As alternatives *Job C* and *Job E* are dominated, new information is only needed for the classes concerning *Jobs A*, *B,* and *D*. For example, *Jobs A* and *D* both

Figure 6: Interval SMART/SWING weighting in Vincent Sahid's job selection example.

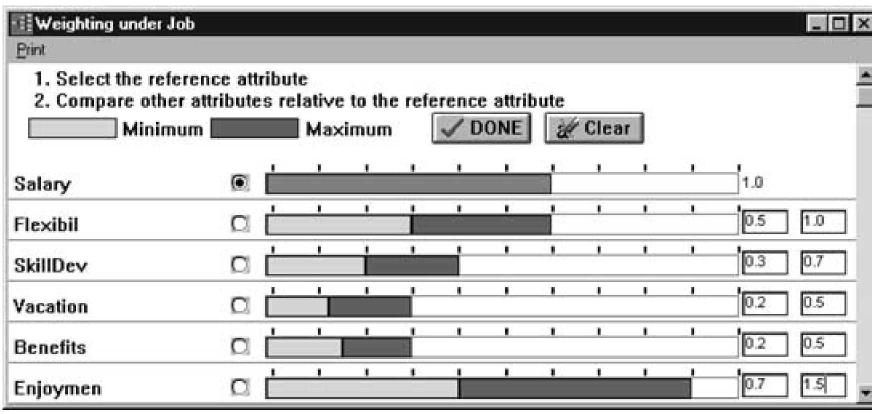
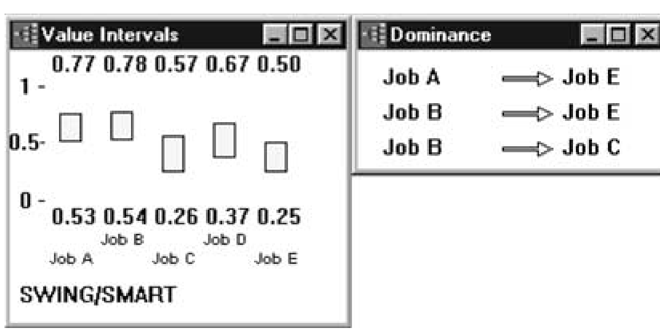


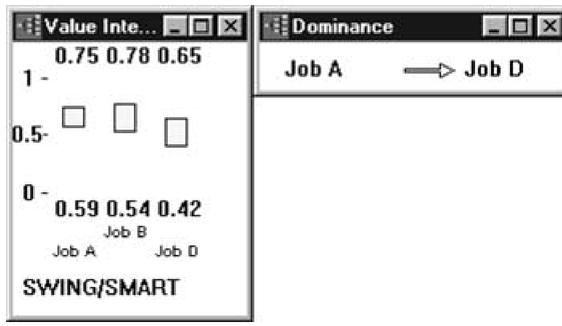
Figure 7: Overall value intervals and dominance relations.



have a moderate *flexibility* (rating interval [0.5, 0.7]) and great *enjoyment* (rating interval [0.8, 1.0]). By examining the situation more closely, the DM could end up concluding that these alternatives indeed are of equal *flexibility* (e.g., both having rating 0.6) and of equal *enjoyment* (e.g., both having rating 1.0). In light of this new piece of information, *Job A* begins to dominate *Job D* (Figure 8). Although the alternatives are still equally preferred on these two attributes, this new more precise piece of information has decreased the imprecision between these alternatives. Similarly we can continue by adjusting the other attributes or alternatives until the best alternative is found.

Another approach is to try to eliminate only the obviously inferior alternatives. We might not want to find out the best alternative, but instead an alternative that performs satisfactorily in all circumstances. For example, in this case we could arbitrarily select either *Job A* or *B* instead of trying to make the model more precise, as both of the alternatives perform reasonably well.

Figure 8: Overall value intervals and dominance relations in light of more precise information.



CONCLUSIONS

In view of the practical applicability of MCDA methods, the easiness of the method is often very important (Stewart, 1992). SMART and SWING are easy-to-use ratio estimation methods. In this article, we have generalized them to allow the selection of different reference attributes and the use of intervals to model imprecision. The aim is to provide the DM with a possibility to also model imprecision without making the methods too complex to use. Consequently, these methods can be adapted to cover a wider range of decision-making situations.

Technically, the operations are straightforward, as these can be carried out similarly to the PAIRS method. However, the DM should realize that the selection of the reference attribute can influence the amount of remaining imprecision. Based on the simulation results, we suggested practical rules to efficiently select the reference attribute. As a whole, the interval SMART/SWING method provides an easy way to model imprecision without a significant loss in the ease of weight elicitation process, assuming that the impacts of the behavioral aspects are recognized.

Computer support is available to facilitate the use of interval methods. We described the WINPRE software for visually supporting interval SMART/SWING, PAIRS, and preference programming. Our example demonstrated a particular way to carry out the interval SMART/SWING weighting process, that is, adding information step by step until the best alternative is found. With this kind of a process, the DM's tasks can be reduced, as dominated alternatives are eliminated during the analysis. WINPRE's dynamical way of showing the results online makes the identification of dominated alternatives fast and easy.

In this article, we have presented the basic way of applying intervals in the SMART and SWING methods. However, the proposed method could be extended in various ways. One may, for example, want to assign distributions on the given intervals to more accurately define the given imprecision. This would lead to a stochastic simulation approach, which has already been applied, for example, in the AHP and preference programming (Saaty & Vargas, 1987; Arbel & Vargas, 1993; Stam & Silva, 1997; Hahn, 2003). It would also be technically possible to use an interval as a reference, but then the interpretation of the intervals should be reconsidered. Nevertheless, any extensions to the proposed method as well as practical implications of these are beyond the scope of this article and will be a subject of future research. [Received: January 2003. Accepted: October 2004.]

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