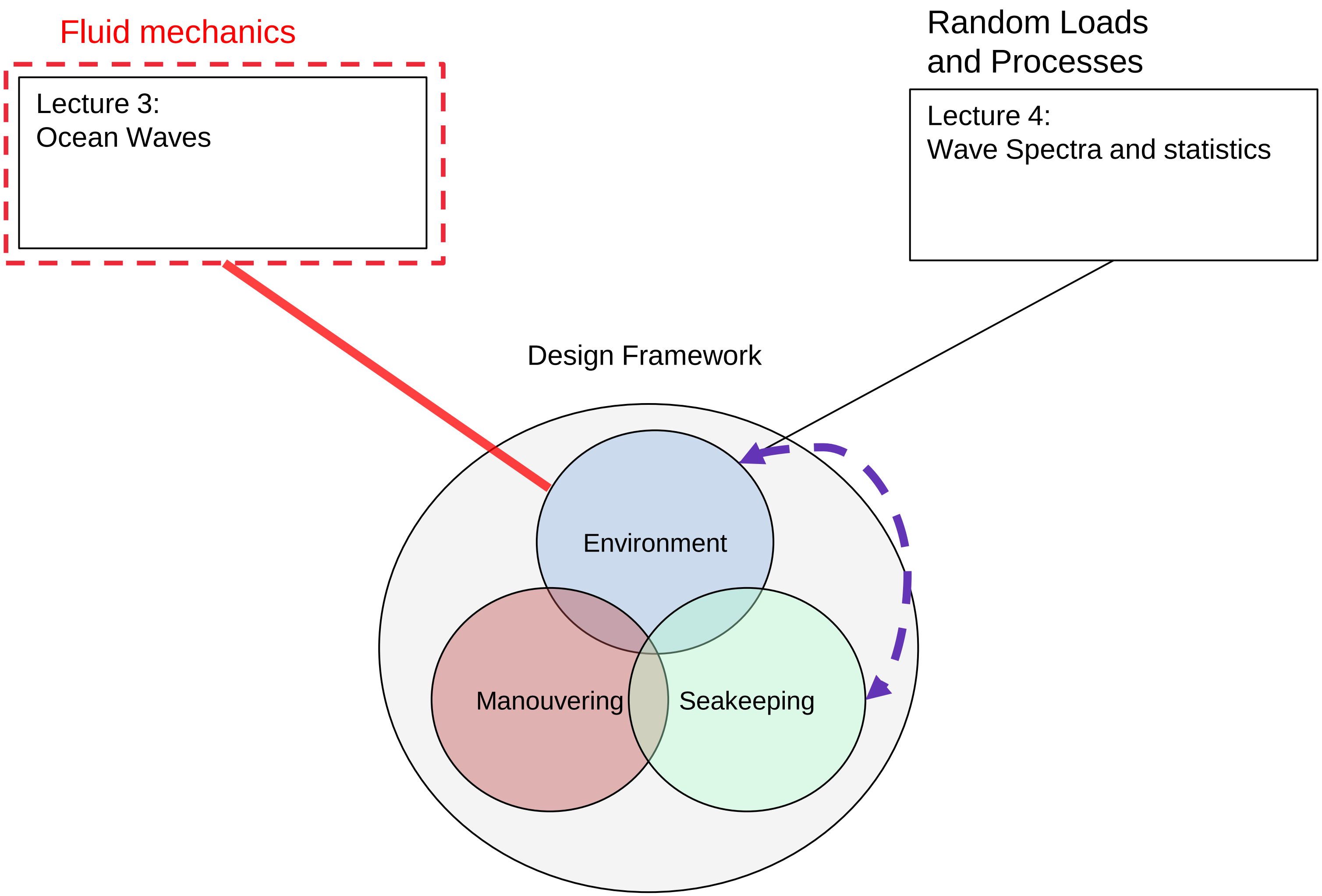
Aalto University
School of Engineering

MEC-E2004 Ship Dynamics (L)

Lecture 3 – Ocean waves

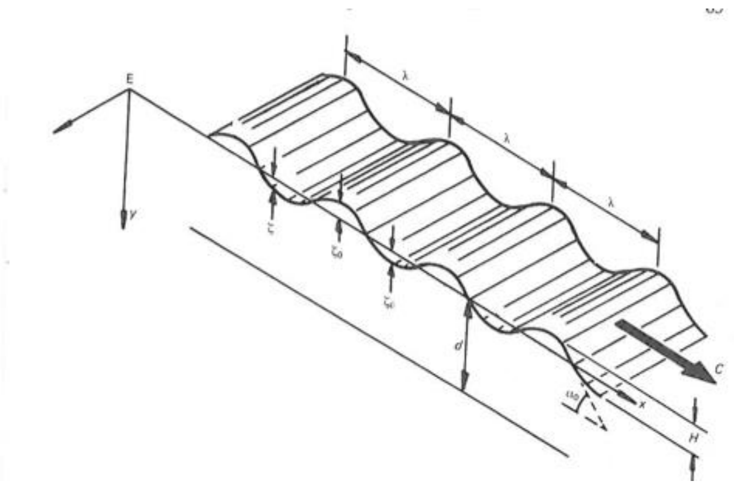
Where is this lecture on the course?



Contents

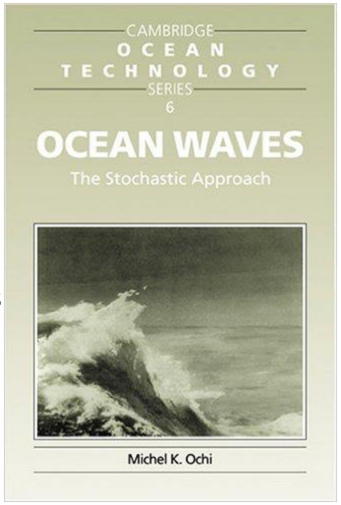
Aims

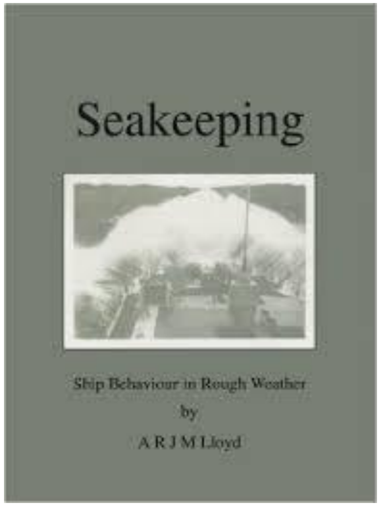
- To explain how sea surface waves and ocean waves form and develop
- To introduce the background to potential flow wave formation theories. Specific topics include :
  - ✓ Regular and irregular waves
  - ✓ Influence of water depth
  - ✓ Energy contents of waves
- To highlight the importance of non-linear wave model idealisations



Selected literature

- ➢ Lloyd, A.J.M.R., “Seakeeping, Ship Behavior in Rough Weather”, Chapters 3 & 4
- ➢ Ochi, M., “Ocean Waves - The Stochastic Approach”, Cambridge Series, Ocean Technology, 6, Chapter 1
- ➢ Lewis, E. V. “Principles of Naval Architecture - Motions in waves and controllability”, Vol. 3, Society of Naval Architects and Marine Engineers, Chapter 8
- ➢ DNV, “Environmental Conditions and Environmental Loads”, Recommended Pratice DNV-RP-C205



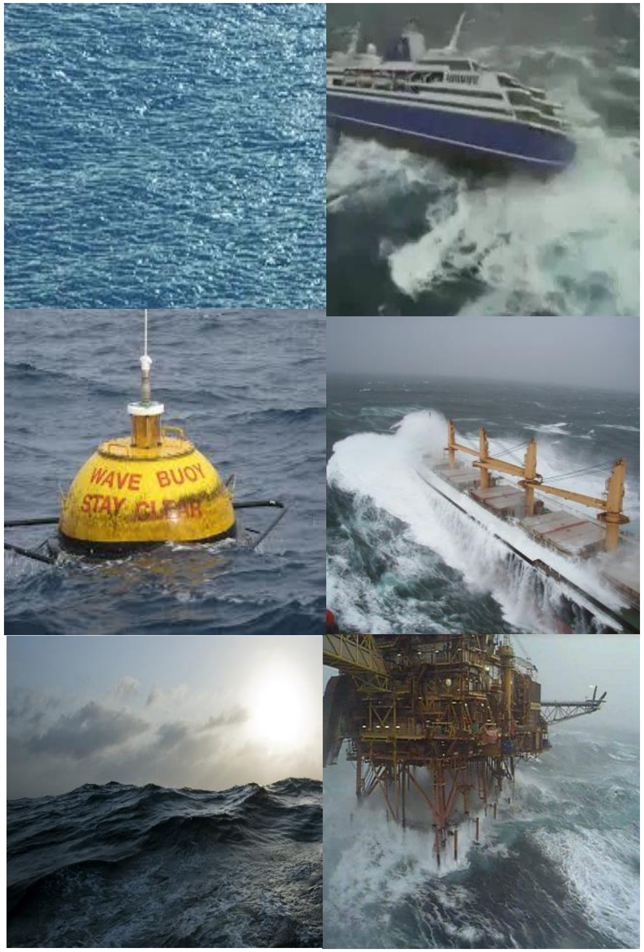


Motivation

- For ships and offshore structures…
  - Environmental effects lead to loads and motions
  - Dynamic response may be amplified
  - Manoeuvring capability is also affected

- The sea surface is highly nonlinear/irregular even for short time windows. Over longer periods of time this irregularity becomes even more obvious.

- Wave record measurements (e.g. measurements from wave buoys or satellite images) can help us support this visual finding
- Practically we need to assess ship motion responses and loads for these varying conditions and for this reason we develop our understanding on ocean waves and model wave kinematics
- Understanding of both linear and nonlinear methods may be key for design and operational assessment



Assignment 2

Grades 1-3:

- ✓ Select a book-chapter related to ocean waves
- ✓ Define the water depths for your ship's route and seasonal variations of wave conditions
- ✓ Based on potential flow theory, sketch what kind of waves you can encounter during typical journey (deep water, shallow water)
- ✓ Identify and select the most suitable wave spectra for your ship - Justify the selection.
- ✓ Discuss the aspects (e.g. likelihood) to consider in case of extreme events from viewpoint of operational area

- Grades 4-5:
  - ✓ Read 1-2 scientific journal articles related to ship dynamics
  - ✓ Reflect these in relation to knowledge from books and lecture slides

- Report and discuss the work.

Example

Mediterranean or Baltic Sea

- 9 months in open water
- 3 months in ice

Route: …

Water depth: …

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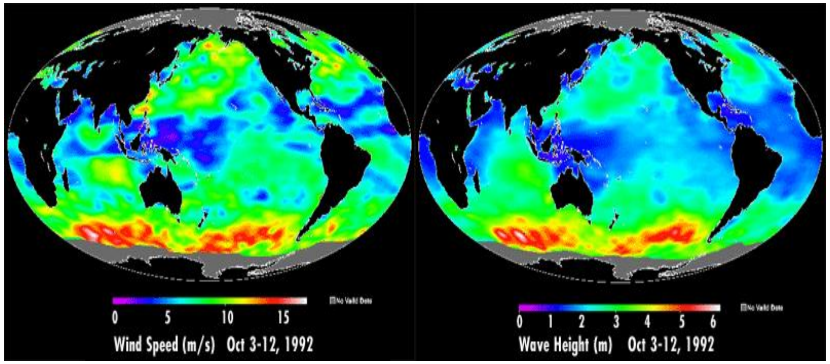
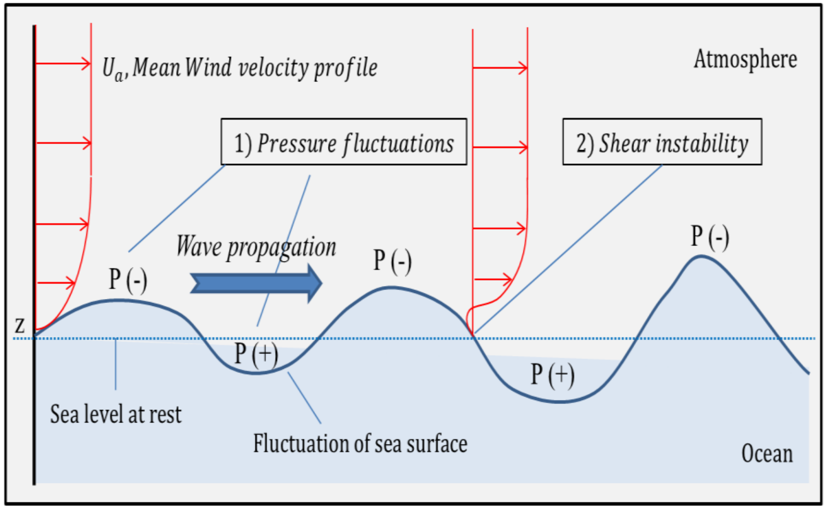
Wave Formation

- Waves are typically generated by
  - ✓ Surface effects: wind
  - ✓ Submarine effects: earthquakes, landslides
- Two mechanisms for wind-generated waves
  - ✓ Pressure fluctuations in the sea surface [Phillips, Phillips, O. M.: 1957, ‘‘On the Generation of Waves by Turbulent Wind’’, J. Fluid Mech. 2, 417–445] journal link
  - ✓ Shear force in the interface of water and air [Miles, J. W.: 1957, ‘‘On the Generation of Surface Waves by Shear Flows’’, J. Fluid Mech. 3, 185–204] journal link
- Today we accept that usually the formation of waves starts from pressure fluctuations
  - ✓ waves are enlarged by shear forces
  - ✓ waves interact and combine to form longer waves



https://en.wikipedia.org/wiki/Wind_wave



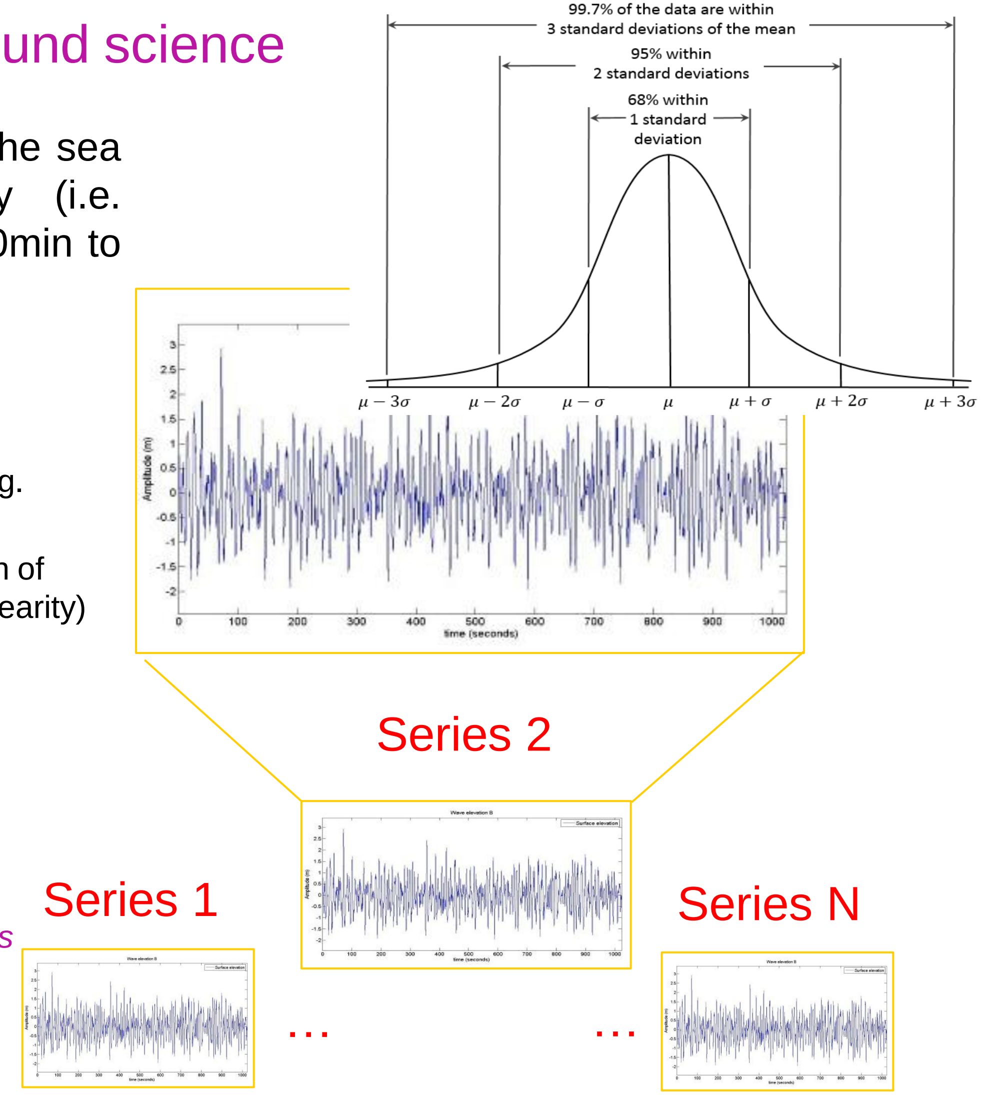
Ocean Waves – background science

- In terms of statistics of wave elevation the sea state may be considered stationary (i.e. statistically steady) for short period of 30min to 3 hrs. This in theory means that :

  ✓ Ocean waves can be described by Normal or Rayleigh probability distributions

  ✓ The sea has constant statistical properties (e.g. mean, standard deviation)

  ✓ Lifetime predictions are sequence/aggregation of short term responses (within the context of linearity)



- The tools that idealise waves combine principles of

  ✓ *Fluid mechanics*

  ✓ *Principles of stochastic processes / mechanics*

  ✓ *Probability theory*

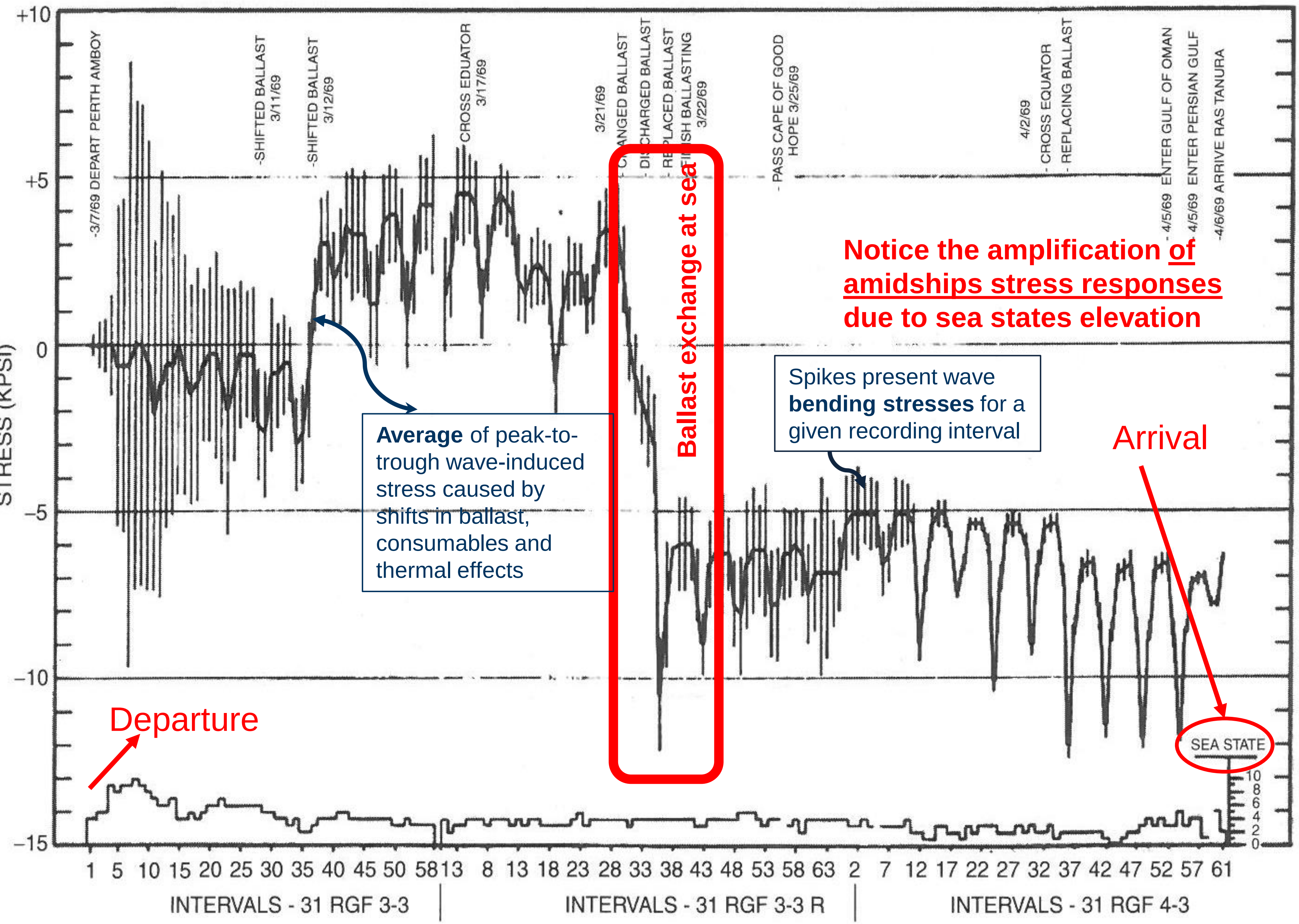


Fig. 4 Typical voyage variation in stresses, *R.G. Follis*, in ballast.

Further observations

To theoretically simulate the observations from the real signal presented we consider:

- Contribution of several components by summing all contributions

$$z(x,y,t) = \sum_i z_i(x,y,t)$$

- When we look at the result we can identify
  - ✓ Narrow band process, frequencies focused (useful for the assessment of loading and dynamic response)
  - ✓ White noise, frequency range is large

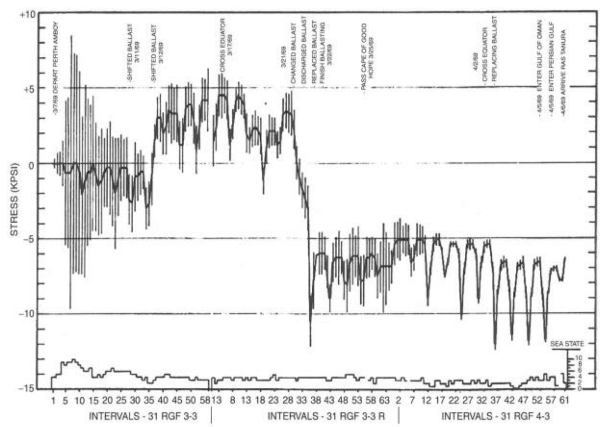
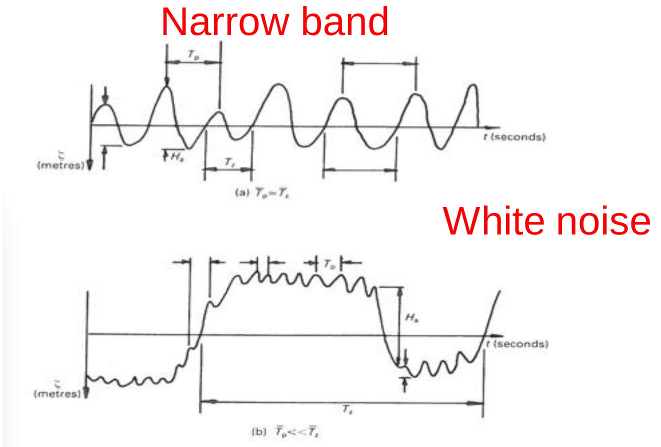


Fig. 4 Typical voyage variation in stresses, R.G. Follis, in ballast.

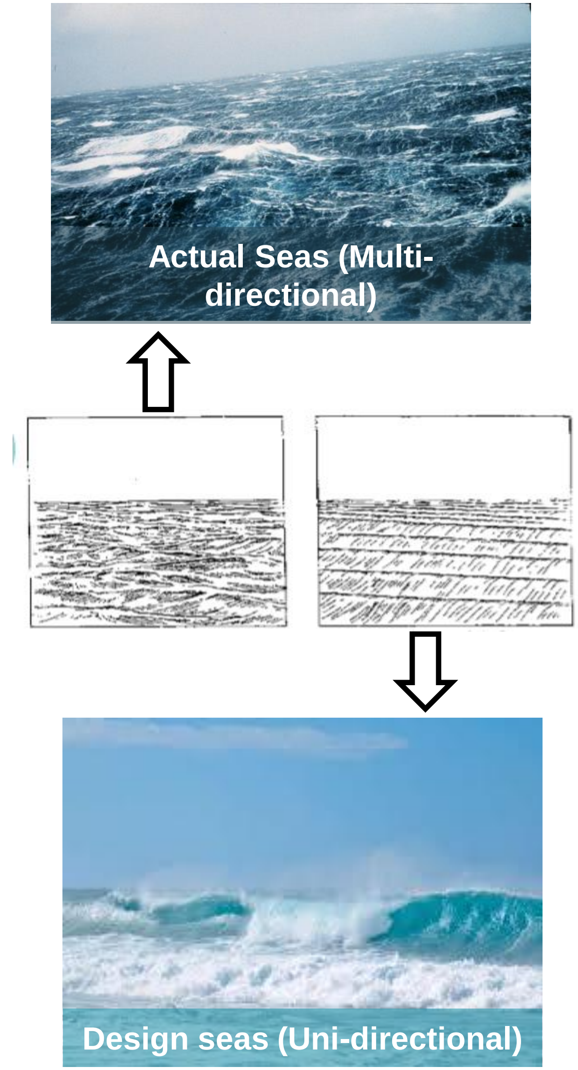


Wave patterns

- In open seas, observed wave patterns are complex and considered irregular, or multi-directional

- Near the coastal line, observed wave patterns are often considered regular, or uni-directional

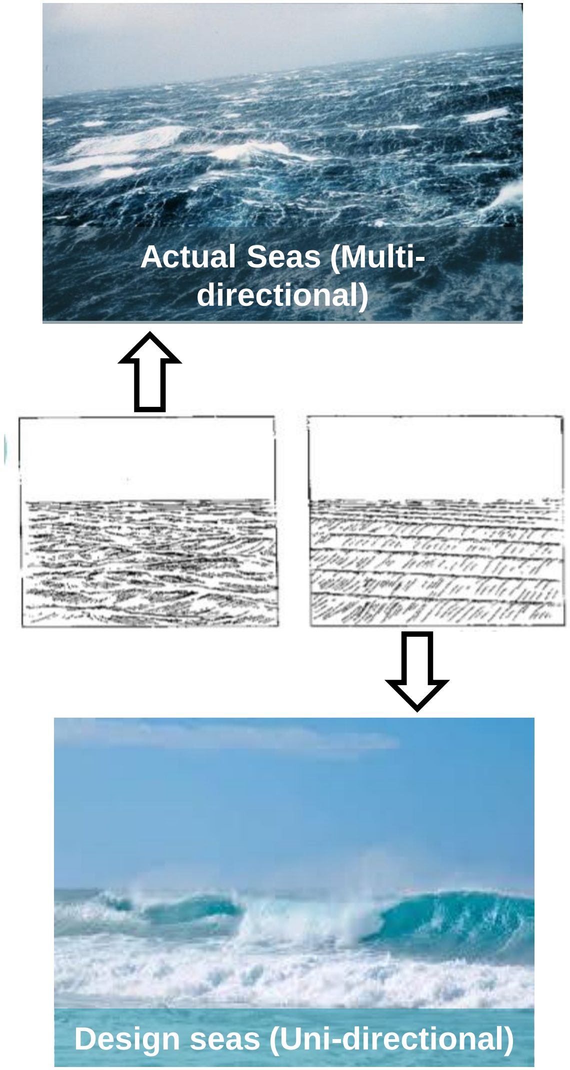
- **What the above mean ?**



Wave patterns

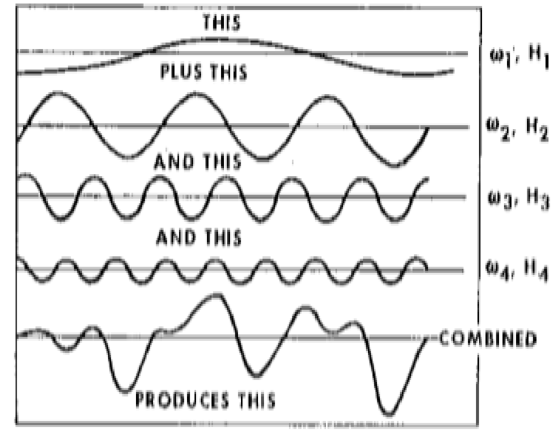
- In open seas, observed wave patterns are complex and considered irregular, or multi-directional
  - – This means that we observe numerous waves of different length and height and waves from all directions

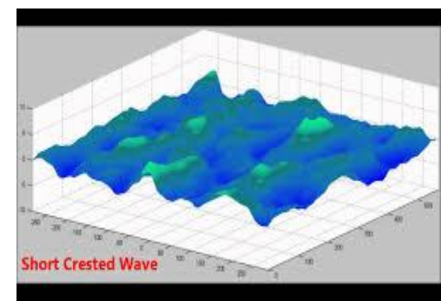
- Near the coastal line, observed wave patterns are often considered regular, or uni-directional
  - – This means that we observe the prevailing direction of waves progression and the length of the waves

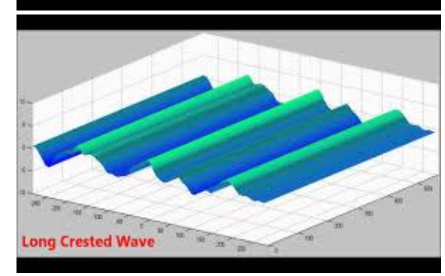


Key definitions

- A **regular wave** (also known as single wave component) has a single frequency, wavelength and amplitude (height)
- **Irregular waves** can be viewed as the superposition of a number of regular waves with different frequencies and amplitudes
- **Long-crested waves** are waves formed toward the same direction; **Short-crested waves** are waves formed toward different directions
- **Short term (ST) wave loads** generally relate with ocean or coastal wave formations over 0.5h-3h.
- **Long term (LT) wave loads** are assessed over lifetime (e.g. 20 years for ships) and comprise of a sequence of short term events. LT predictions consider multiple sea areas, routes, weather (see IACS URS11, Rec. 14, BSRA Stats etc.)







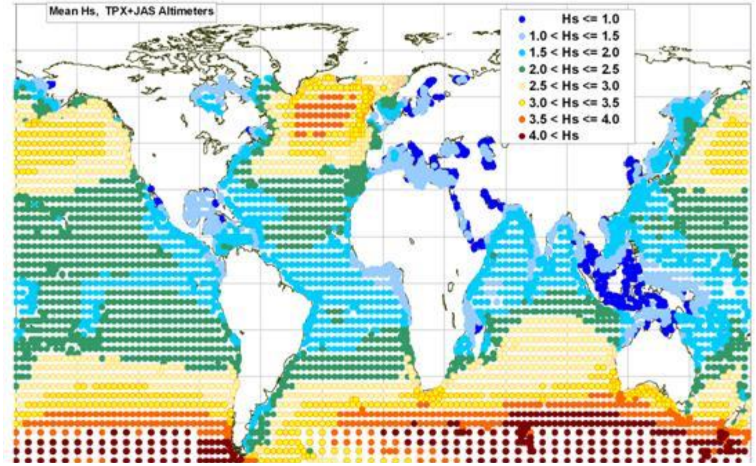
Modelling of random processes by probability theory

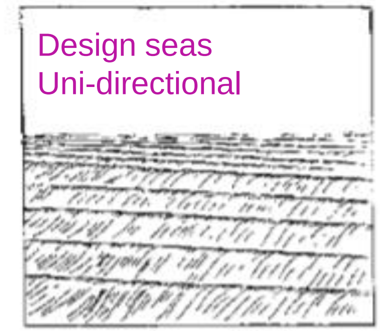
- A reasonably realistic yet simplified analysis assumes that the sea surface is a linear random process, i.e. a linear stochastic process. This means that
  - Probability distributions may be idealised as Gaussian distributions i.e. a normal distribution with zero mean and a variance sum of component variances (e.g. wave elevation)
  - Fourier analysis/transformations may be used to sum sinusoidal terms etc.
- From modelling perspective we tend to assume small spatial areas and time increments to ensure that variations in statistical properties are minimal

$$z(x,y,t) = \sum_i z_i(x,y,t)$$

$$\langle z(x,y,t) \rangle = 0, \langle \ \rangle = mean$$

$$\langle z^2 \rangle = \sum_i \langle z_i^2 \rangle, \langle z_i^2 \rangle = \text{variance}$$



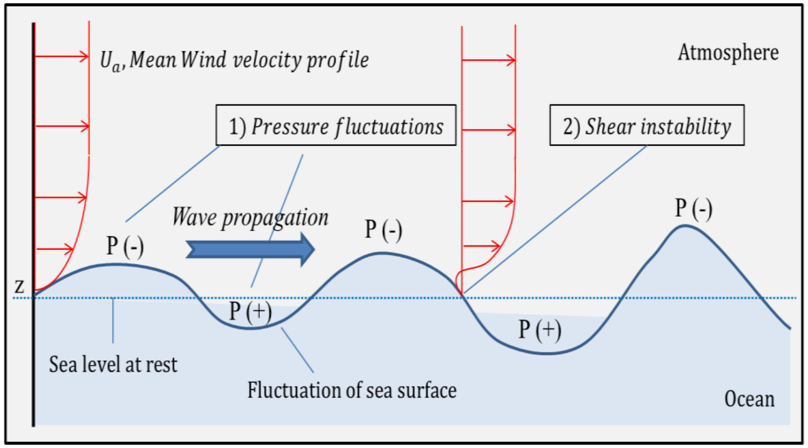


Wave elevation patterns in 3D are defined by a mean and a variance

Wind Wave Formation

Both regular (unidirectional) and irregular (multi-directional) waves have similar formations

- Mechanisms
  - Pressure fluctuations
  - Shear instability forces
- How are waves formed? (40 sec) [YouTube link]



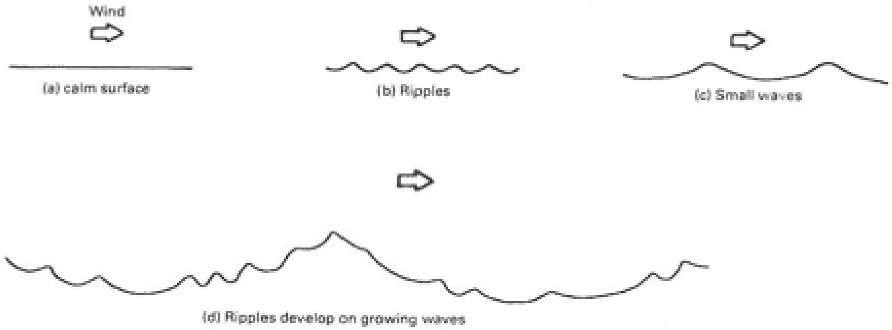


Fig. 4.1 — Wind-generated waves.

Wind Wave Formation

Growth of spectrum by component accumulation

- **Sea Spectra Simplified by Walter H. Michel (1967, SNAME)**

- How regular waves combine into an irregular pattern and how the consequent irregular behavior of a vessel at sea can be predicted on the basis of statistical formulations.

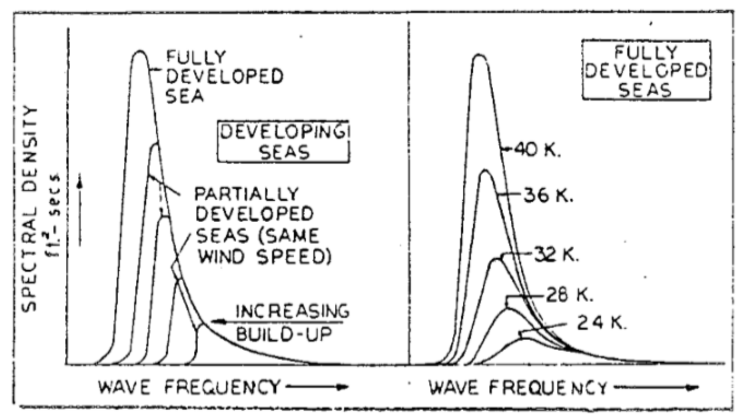


Fig. 4 Growth of spectrum

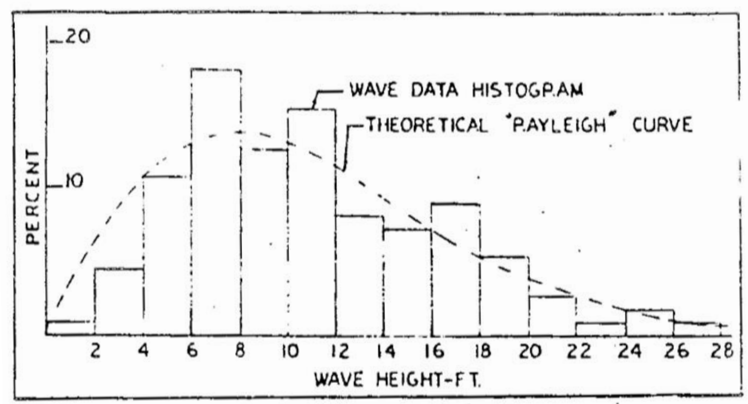
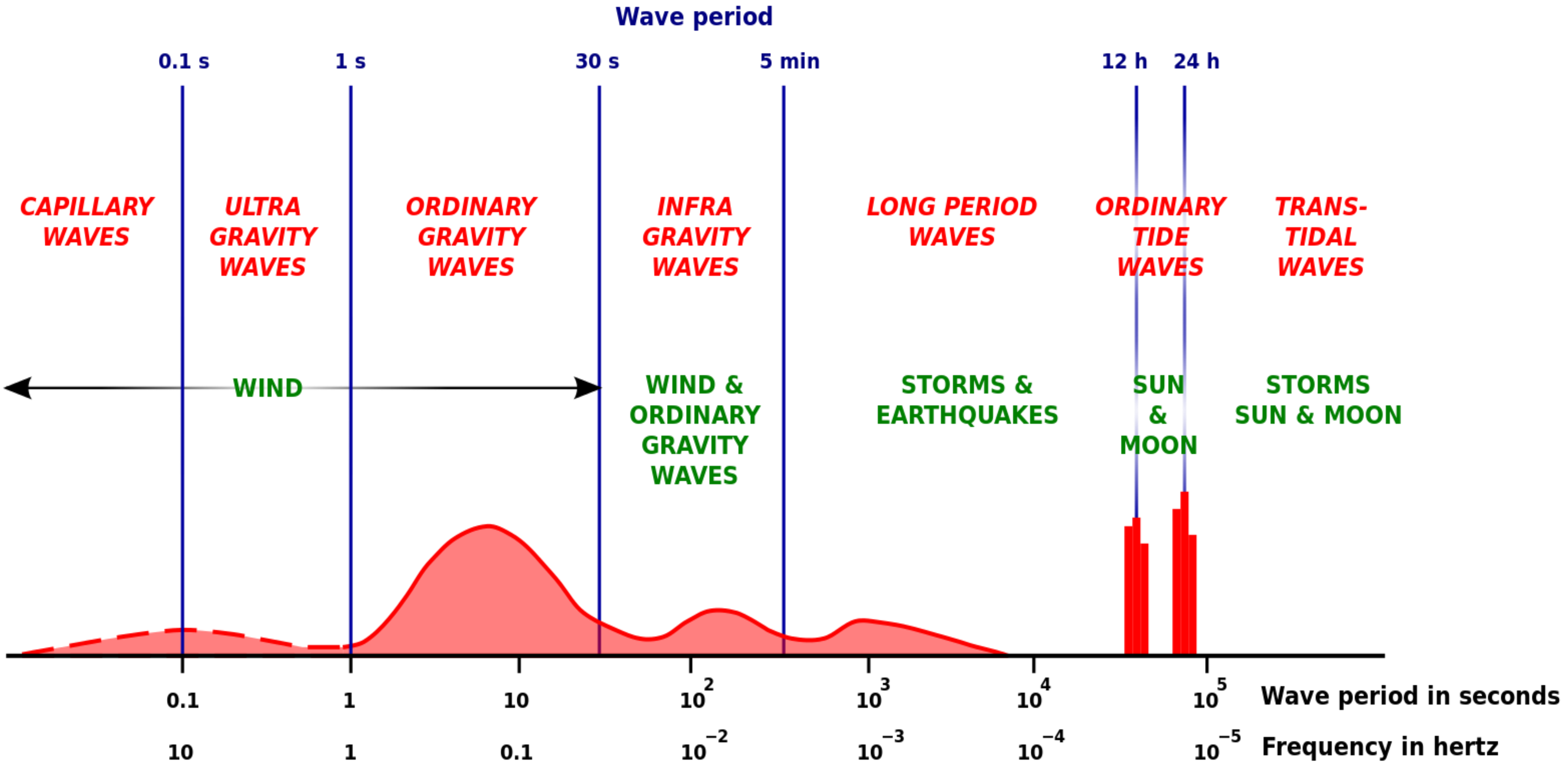


Fig. 5 Histogram of wave height measurements

Classification of Waves – wave period basis



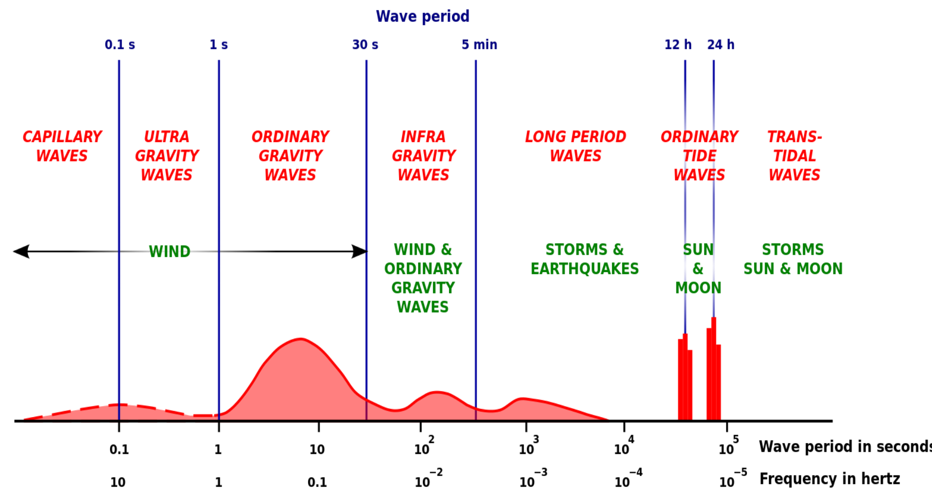
Classification of Waves – wave period basis

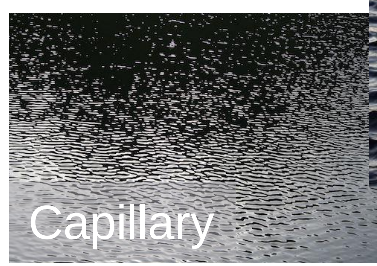
Wave period:

- time interval between the passage of successive crests at a fixed point

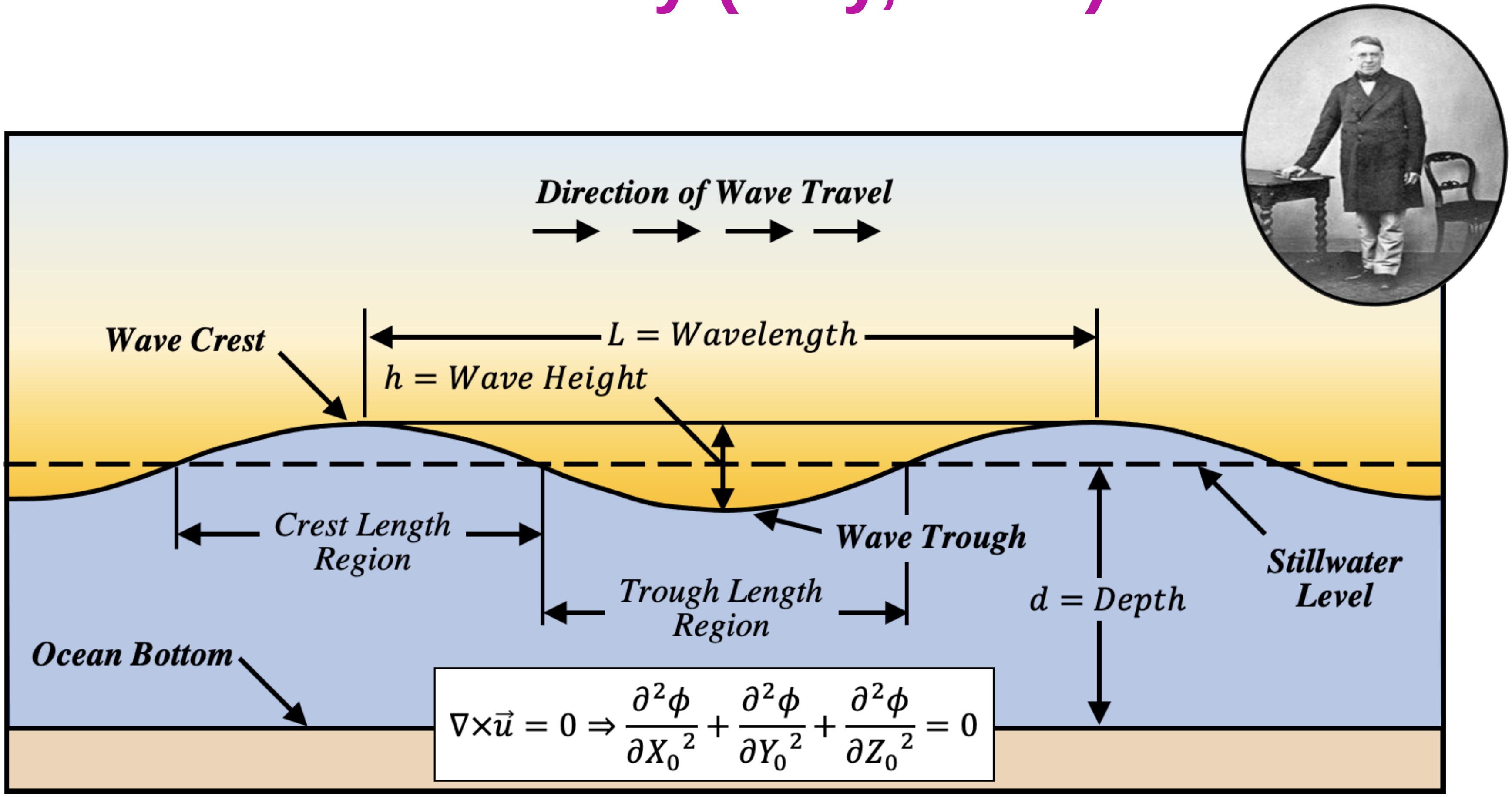
Wave classifications and Periods

- Capillary: less than 0.1 sec
- Ultra-gravity: from 0.1 sec to 1.0 sec
- Ordinary gravity: from 1.0 sec to 30 sec
- Infra-gravity: from 30 sec to 5 min
- Long-period: from 5 min to 12 hours
- Ordinary: from 12 hours to 24 hours
- Trans-tidal: 24 hours and up

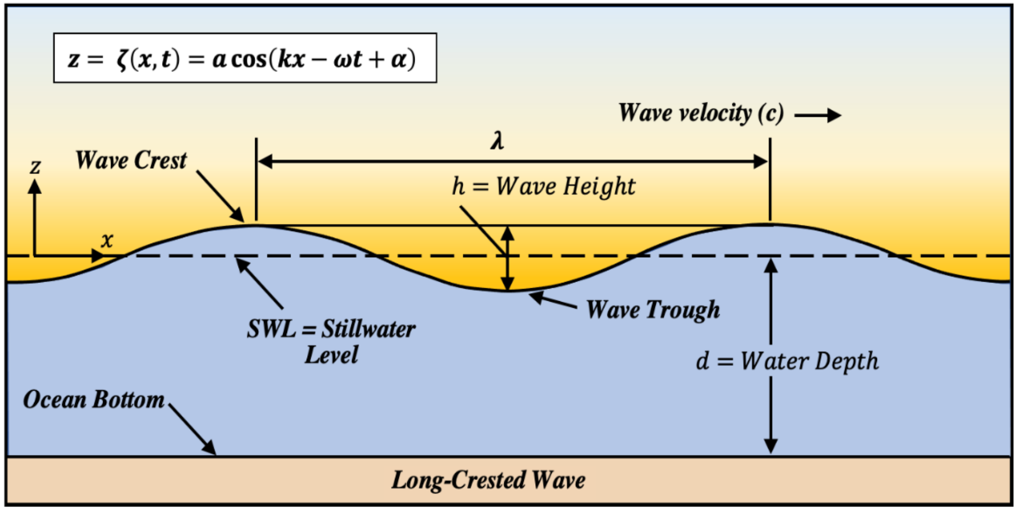




Linear Wave Theory (Airy, 1845)



Linear wave theory - Nomenclature

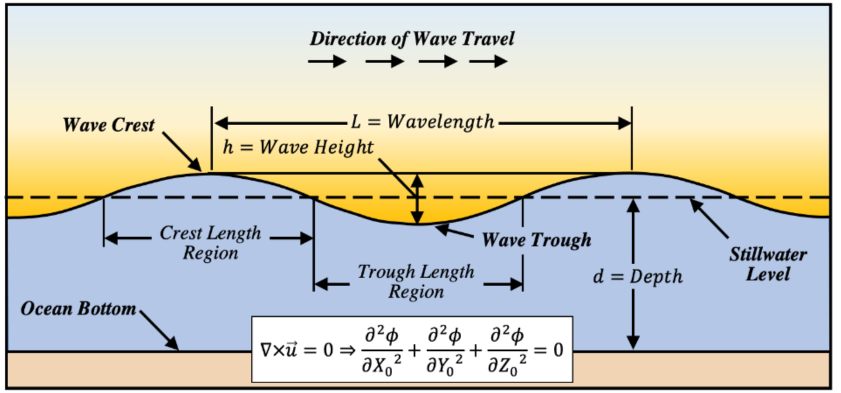


Variable	Definition	Units
a	wave amplitude	[m]
h	wave height	[m]
ω	wave frequency	[rad/s]
T	wave period	[s]
λ	wavelength	[m]
k	wave number	[1/m]
c	wave or phase velocity	[m/s]
d	water depth	[m]
α	phase angle (arbitrary)	[rad]

Equation	Relationship
$z = \zeta(x,t) = a\cos(kx - \omega t + \alpha)$	sinusoidal wave equation
$h = 2a$	wave height to amplitude
$\omega = 2\pi/T$	wave frequency to period
$k = 2\pi/\lambda$	wave number to wavelength
$c = \lambda/T = \omega/k$	wave speed to wavelength

Linear Wave Theory - Assumptions

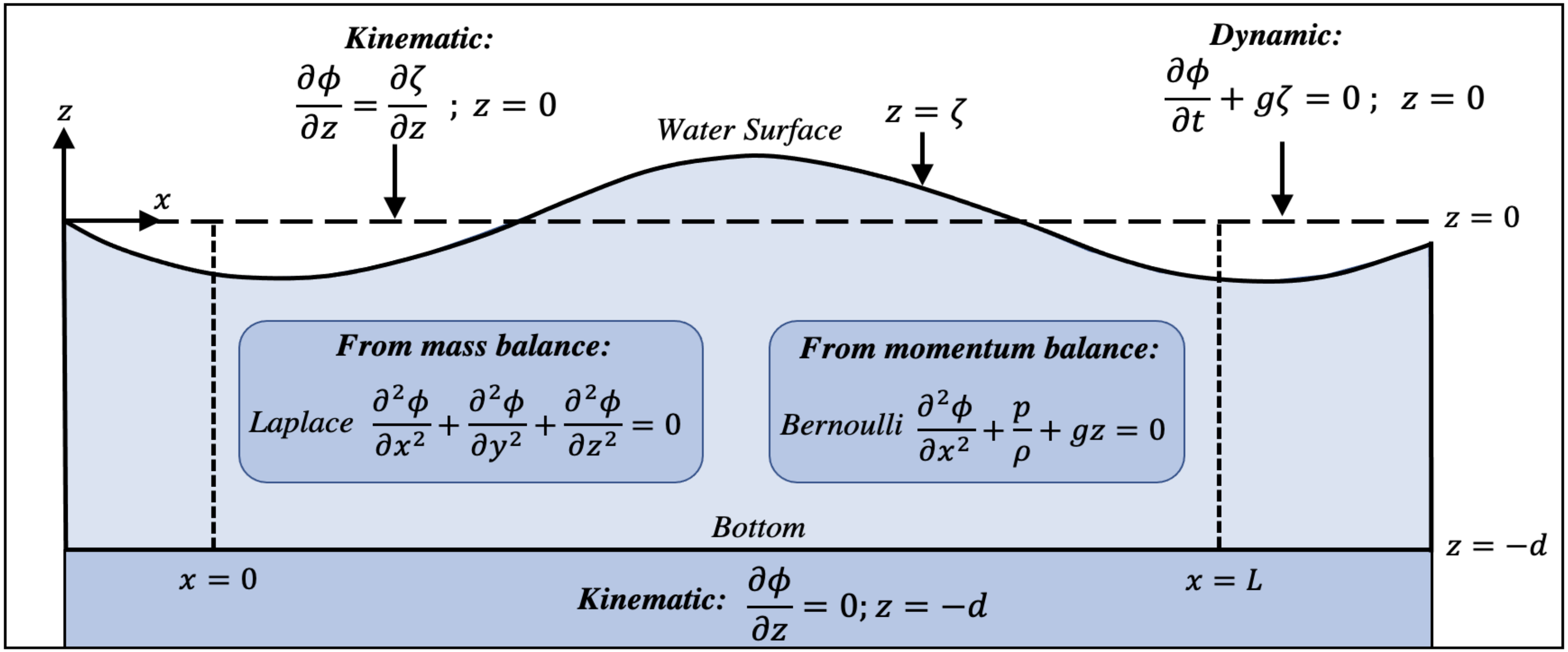
- Single component small amplitude wave
- 2D wave motions and do not change with time
- Ideal fluid- inviscid and irrotational flow
- Water is incompressible
- Viscosity, turbulence and surface tension are neglected



- Laplace velocity potential equation with boundary conditions is used to idealize (1) the seabed and (2) the deforming sea surface

$$\nabla \times \vec{u} = 0 \Rightarrow \frac{\partial^2 \phi}{\partial {X_0}^2} + \frac{\partial^2 \phi}{\partial {Y_0}^2} + \frac{\partial^2 \phi}{\partial {Z_0}^2} = 0$$

Linear Wave Theory – Representation of wave system



Linear wave theory- Potential Flow

- Bernoulli's equation for 2D system (X,Z)
  - Velocity potential equation for ideal fluid

$$\frac{\partial \Phi}{\partial t} - \frac{1}{2}(u^2 + w^2) - \frac{p}{\rho} - gZ = 0$$

$\frac{\partial \Phi}{\partial t}$:	*velocity potential with respect to time [m^2/s^2]*
p:	*pressure (relative to atmosphere) [Pa] [kg/(ms^2)]*
ρ:	*fluid density [kg/m^3]*
g:	*acceleration due to gravity [m/s^2]*
u, w:	*fluid particle velocity components [m/s]*
Z:	*wave elevation function [m]*

$$Z = \zeta(X,t) \qquad u = -\frac{\partial \Phi}{\partial X} \qquad w = -\frac{\partial \Phi}{\partial Z}$$

Linear wave theory- Potential Flow

- For regular progressive waves in deep water, velocity potential is defined as:

$$\Phi(X,Z,t) = -a\frac{g}{\omega}\frac{\cosh[k(Z+d)]}{\cosh(kd)}\sin(kX-\omega t+\alpha)$$

- For ideal flow condition Laplace's equation should be satisfied

$$\nabla^2\Phi = 0$$

- The linearised dynamic free surface condition (from Bernoulli's equation after neglecting higher order terms) becomes:

$$\frac{\partial\Phi}{\partial t} - g\zeta = 0 \qquad \zeta = \frac{1}{g}\frac{\partial\Phi}{\partial t} \qquad \text{on } Z = 0$$

Linear wave theory- Potential Flow

- The kinematic dynamic free surface condition becomes:

$$\frac{\partial \zeta}{\partial t} - w = 0 \qquad Z = \zeta = 0$$

- This combined with Eq.(4) leads to :

$$\frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial \Phi}{\partial Z} = 0 \qquad Z = 0$$

- The bottom no flow condition implies

$$\frac{\partial \Phi}{\partial Z} = 0 \quad \text{on } Z = -d$$

Linear wave theory- Potential Flow

- The dynamic free surface boundary condition becomes :

$$\frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = -a(-\omega)\, sin(kX - \omega t + \alpha)$$

$$\frac{\partial\Phi}{\partial Z} = -a\frac{gk}{\omega}\frac{\sinh[k(Z+d)]}{\cosh(kd)}\sin(kX - \omega t + \alpha)$$

$$\frac{\partial\Phi}{\partial Z} = -a\frac{gk}{\omega}\tanh(kd)\sin(kX - \omega t + \alpha) \quad \text{for } Z = 0$$

From $\frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\partial\Phi}{\partial Z}$ ⇨ $a\omega = \frac{agk}{\omega}\tanh(kd)$ or $\omega^2 = kg\tanh(kd)$

Linear wave theory- Dispersion relationship

- Dispersion relationship gives wave travelling trajectory

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kd)} = \sqrt{\frac{g}{k}\tanh\left(2\pi\frac{d}{\lambda}\right)}$$

- For shallow waters

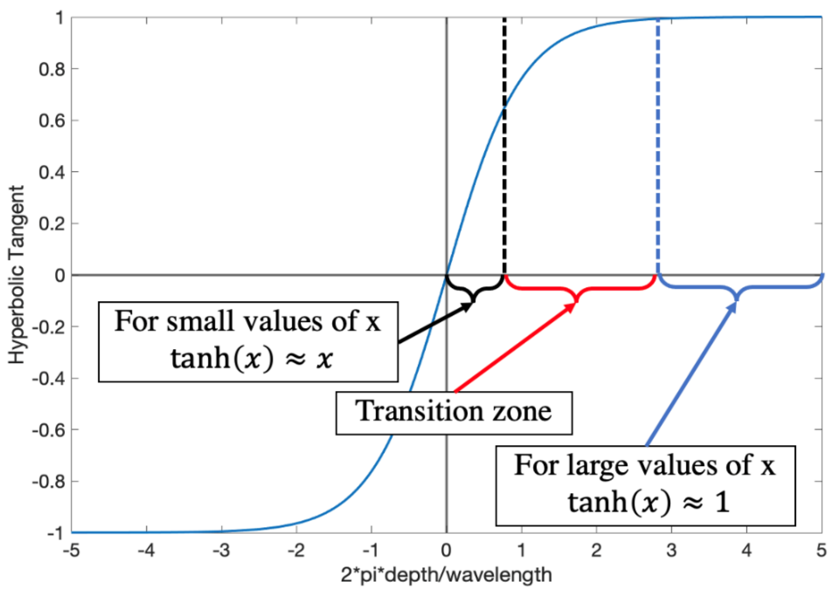
$$d/\lambda < 0.5\,, \tanh(kd) \to kd \;\therefore\; c = \sqrt{gd}$$

- For intermediate waters

$$0.5 < d/\lambda < 3.0\,, c = \sqrt{\frac{g}{k}\tanh\left(2\pi\frac{d}{\lambda}\right)}$$

- For deep waters

$$3.0 \ll d/\lambda \;or\; 3.0 \ll kd \;\therefore\; \tanh(kd) \to 1 \;\therefore c = \sqrt{g/k}$$



Linear wave theory- The influence of water depth

- Dependence on the distance from free surface in deep water

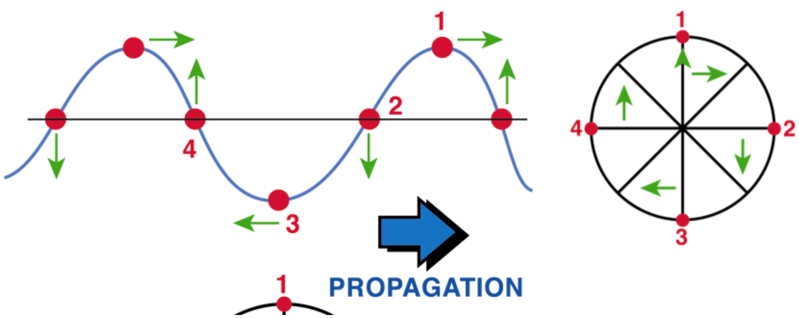
$$\frac{\cosh[k(z+d)]}{\cosh(kd)} = \frac{\cosh(kz)\cosh(kd) + \sinh(kz)\sinh(kd)}{\cosh(kd)}$$

$$= \cosh(kz) + \sinh(kz)\tanh(kd)$$

$$= \cosh(kz) + \sinh(kz)$$ in deep water $$= e^{kz}$$

- Using this exponential variation and Bernoulli's Eq. (ignoring higher order terms) the pressure relative to atmospheric (anywhere (i.e. X,Z) in the deep water fluid is :

$$\Delta p = \rho\left[\frac{\partial \Phi}{\partial t} - gz\right] = \rho g[e^{kz}\zeta(x,t) - z] \quad (7)$$



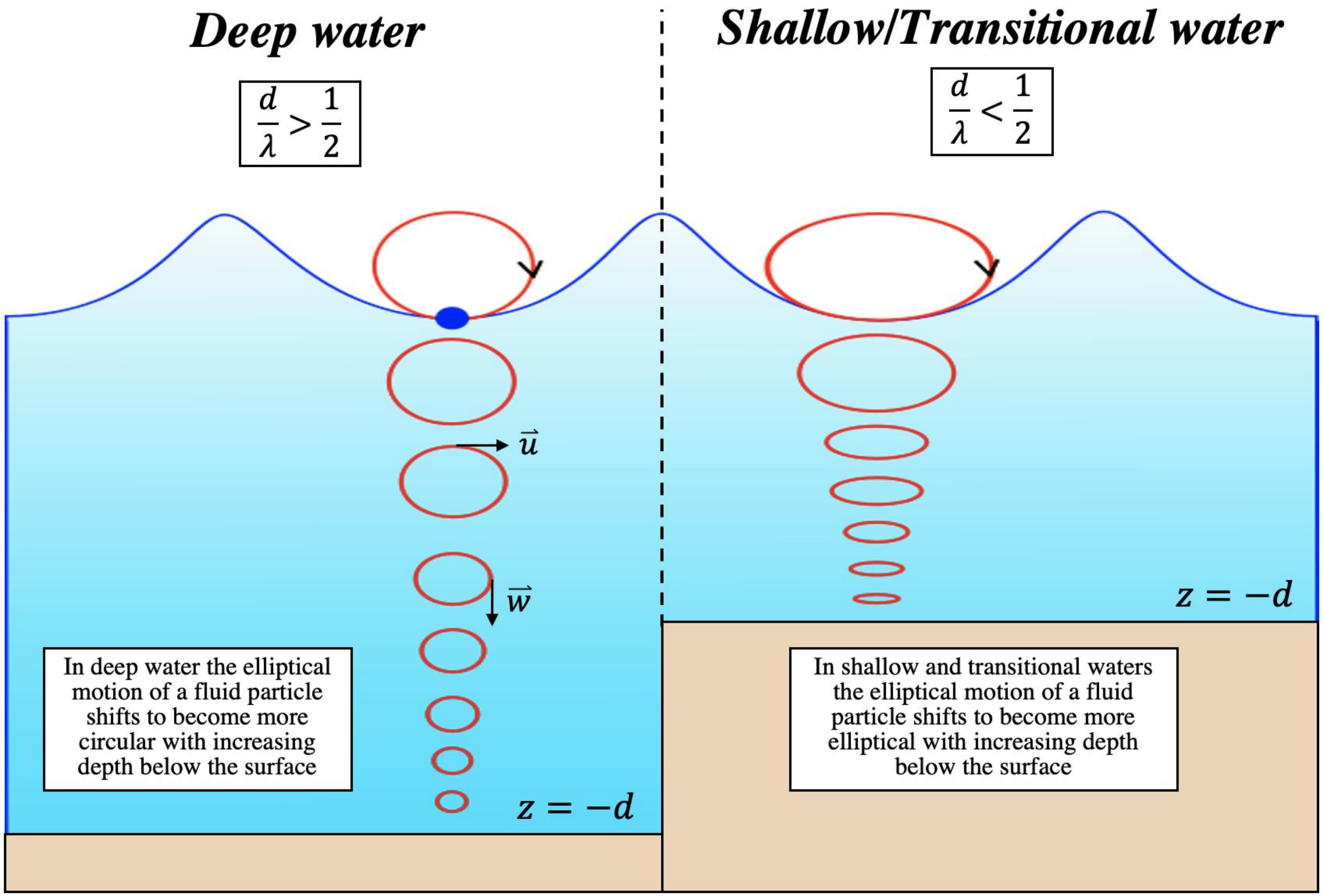
(HORIZONTAL)

$$U = \frac{H}{2}\,\frac{gT}{L}\,\frac{\cosh\,[2\pi(z+d)/L]}{\cosh\,(2\pi d/L)}\,\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

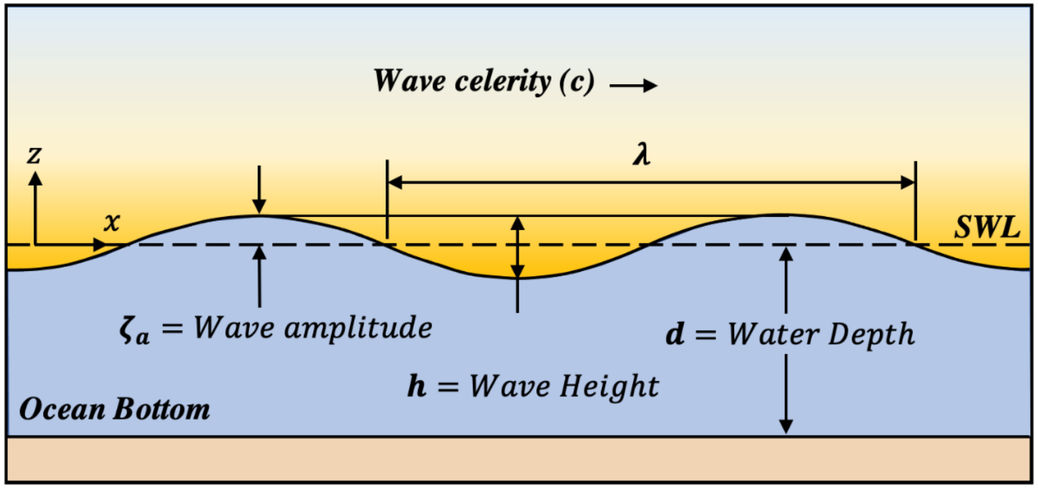
(VERTICAL)

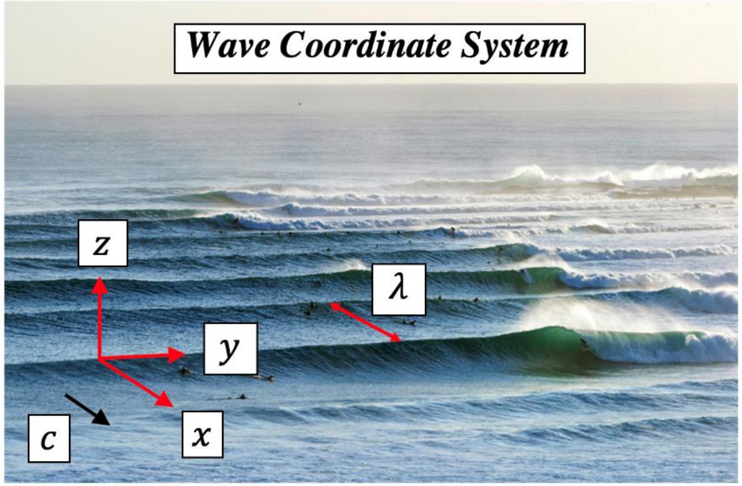
$$W = \frac{H}{2}\,\frac{gT}{L}\,\frac{\sinh\,[2\pi(z+d)/L]}{\cosh\,(2\pi d/L)}\,\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

Linear wave theory- The influence of water depth



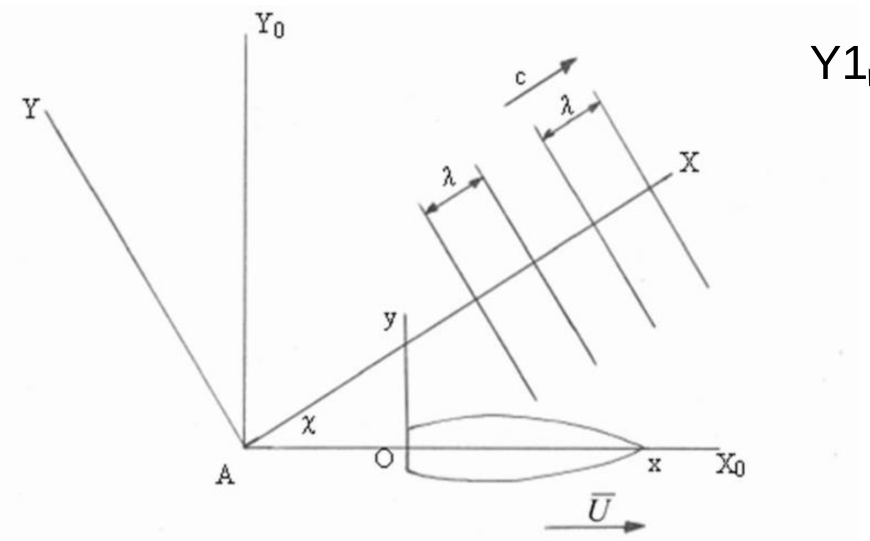
Linear wave theory- presense of moving ship

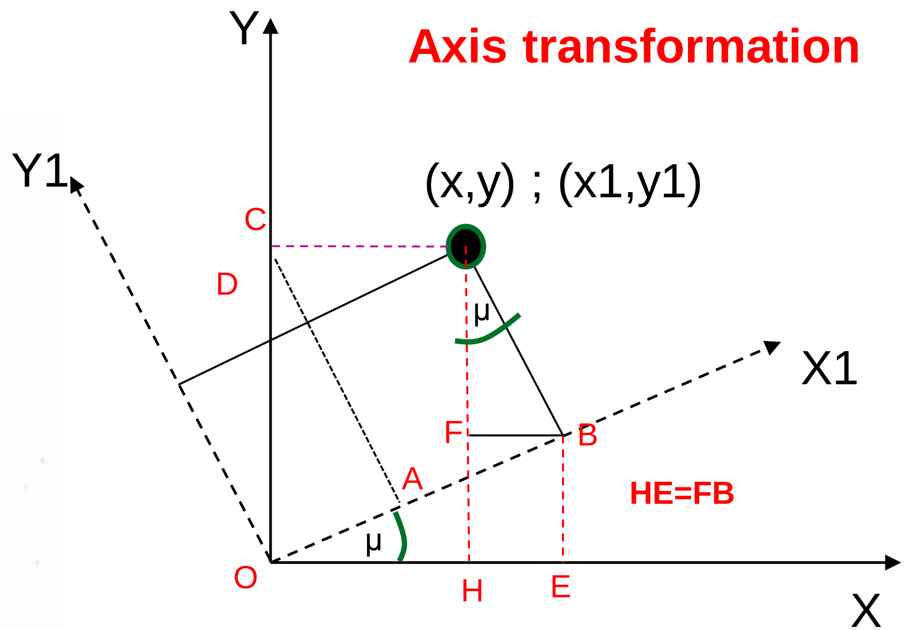




Variable	Definition	Units
ζ	instantaneous water surface level from SWL	[m]
ζ_a	wave amplitude from mean level to crest	[m]
h	wave height	[m]
λ	wavelength	[m]
c	wave celerity	[m/s]
T	wave period	[s]
α	instantaneous wave slope	[-]
α_0	maximum wave slope	[-]
h/λ	wave steepness	[-]

Linear wave theory- presense of moving ship





X1 = **OA+AB**= ysinμ+xcosμ
Y1 = **AC-DC**= ycosμ-xsinμ

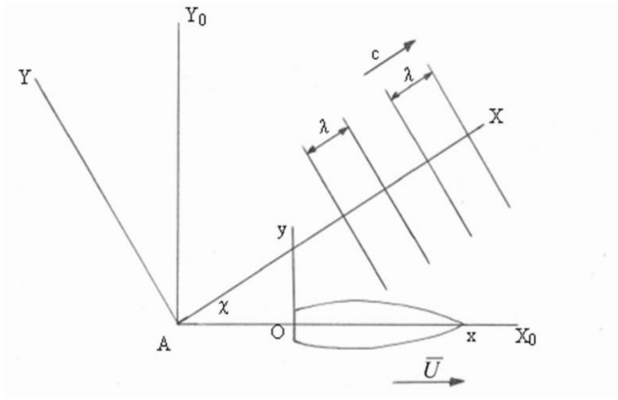
X = **OE-HE**= x1cosμ – y1sinμ
Y = **EB+FG**=x1sinμ+y1cosμ

Ref : Lloyd, A.J.M.R., Seakeeping, “Ship Behavior in Rough Weather”, Chapter 3

Linear wave theory- presense of moving ship

- Considering transformation of coordinates for a regular wave propagating at an angle χ (i.e. from AXYZ to $AX_0Y_0Z_0$ for $Z = Z_0$ axes systems) becomes

$$X = X_0 \cos\chi + Y_0 \sin\chi$$



- The transformation coordinate to OXZ

$$X_0 = x + \overline{U}\, t \quad and \quad y = Y_0$$

- Combining both leads to

$$\begin{aligned}\zeta(x, y, t) &= a\cos(kx\cos\chi + ky\sin\chi + k\overline{U}\cos\chi\, t - \omega t + \alpha) \\ &= a\cos(kx\cos\chi + ky\sin\chi - \omega_e\, t + \alpha)\end{aligned}$$

- Where ω_e is the encounter frequency, i.e. the frequency of the wave as seen by an observer of the ship moving in waves

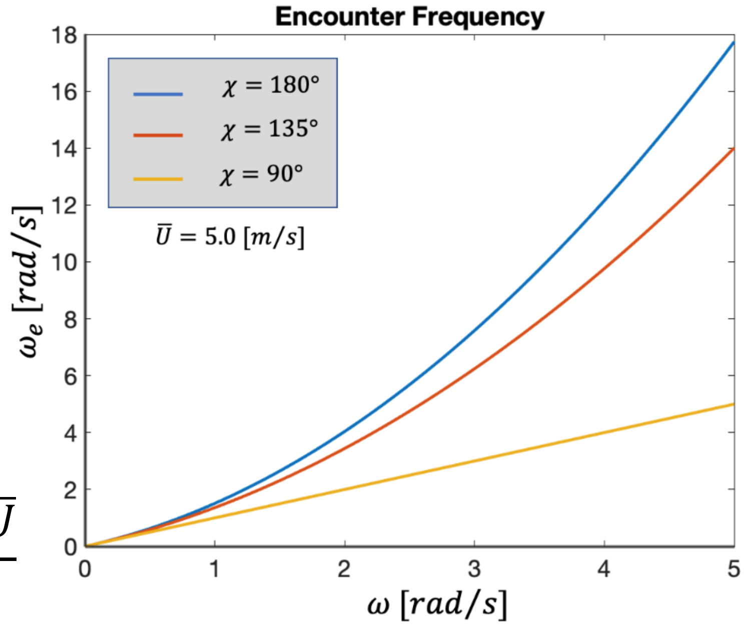
Ref : Lloyd, A.J.M.R., Seakeeping, “Ship Behavior in Rough Weather”, Chapter 3

Linear wave theory- encounter frequency

- For deep water the encounter frequency is:

$$\omega_e = \omega - k\bar{U}\cos\chi \rightarrow k(c - \bar{U}\cos\chi)$$

$$\omega_e = \omega - \frac{\omega^2\bar{U}}{g}\cos\chi$$



- For head waves

$$\chi = 180° \rightarrow \cos\chi = -1 \qquad \omega_e = \omega + \frac{\omega^2\bar{U}}{g}$$

- For quartering waves

$$\chi = 135° \rightarrow \cos\chi = -0.7071 \quad \omega_e = \omega + 0.7071\frac{\omega^2\bar{U}}{g}$$

- For beam waves

$$\chi = 90° \rightarrow \cos\chi = 0 \qquad \omega_e = \omega$$

Ref : Lloyd, A.J.M.R., Seakeeping, "Ship Behavior in Rough Weather", Chapter 3

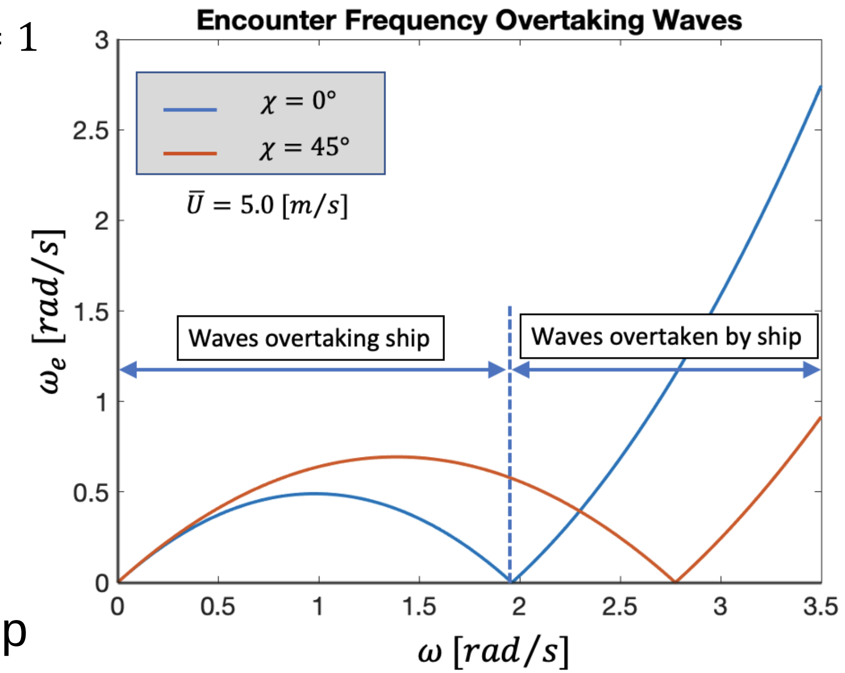
Linear wave theory- encounter frequency

- For following waves $\chi = 0° \rightarrow \cos\chi = 1$

$$\omega_e = \omega - \frac{\omega^2 \bar{U}}{g} = \omega\left(1 - \frac{\omega \bar{U}}{g}\right)$$

$$\omega_e = 0 \qquad \begin{cases} \omega = 0 \\ \dfrac{\omega \bar{U}}{g} = 1 \end{cases} \qquad \bar{U} = c$$

$$\omega_e < 0 \qquad \frac{\omega \bar{U}}{g} > 1 \qquad \bar{U} > c$$



- For $\dfrac{\omega \bar{U}}{g} < 1$ the waves overtake ship
- The maximum value of encounter frequency

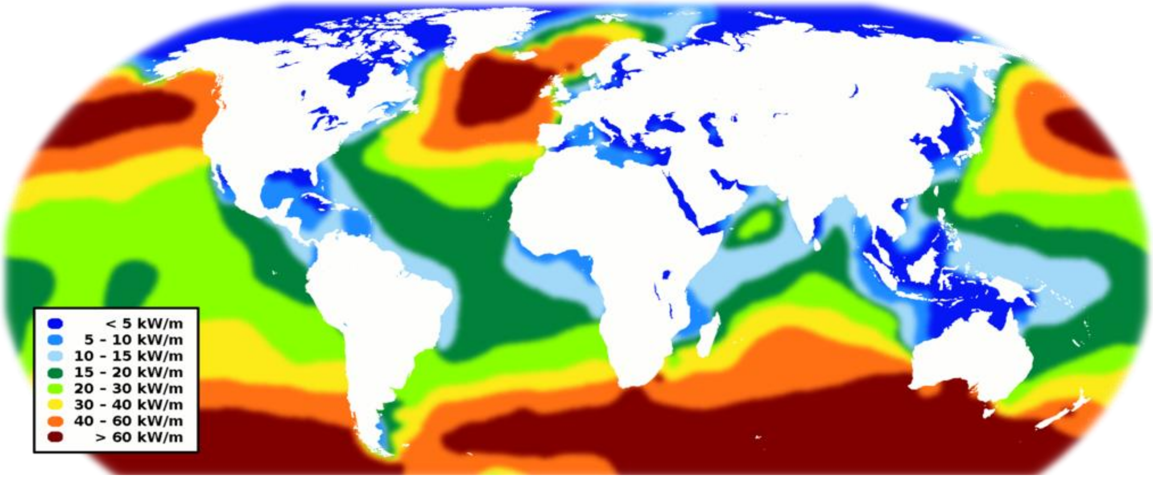
$$\frac{d\omega_e}{d\omega} = 0 \rightarrow 1 - 2\omega\frac{\bar{U}}{g} = 0 \qquad max(\omega_e) = \frac{g}{4\bar{U}}$$

- Relationship holds true for all $\chi < 90°$

Encounter frequencies treated only as positive values

Ref : Lloyd, A.J.M.R., Seakeeping, "Ship Behavior in Rough Weather", Chapter 3

Wave Energy and Power



World map showing wave energy flux in kW per meter wave front

- Total energy of wave system:
  - Kinetic – due to water particle velocity
  - Potential – due to portion of fluid mass being above wave trough

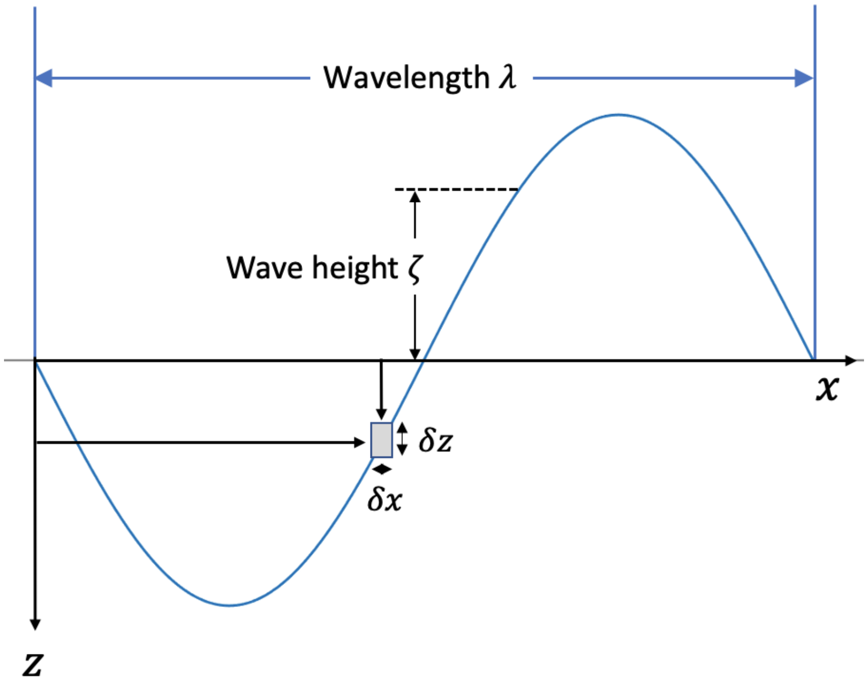
$$E_{TOT} = E_{PE} + E_{KE} = \frac{\rho g \zeta_0^2}{2}$$

Wave Energy – Potential Energy

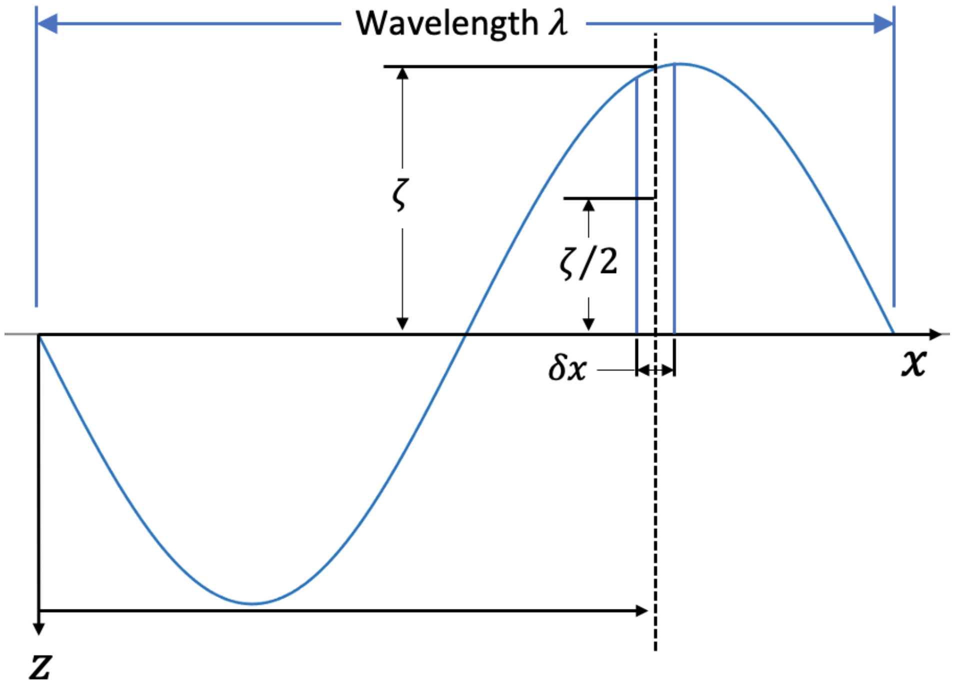
$$P.E. = mgh = (\rho g \delta x)\frac{\zeta}{2} = \frac{\rho g \zeta^2 \delta x}{2}$$

If we integrate this energy over the entire wavelength, we get the potential energy of the wave per unit width as:

$$\boldsymbol{E_{PE} = \frac{\rho g \lambda \zeta_0^2}{4}}$$



Wave Energy – Kinetic Energy



The wave has also kinetic energy (*K.E.*) and if the total velocity of the segment is q then the kinetic energy of the segment is:

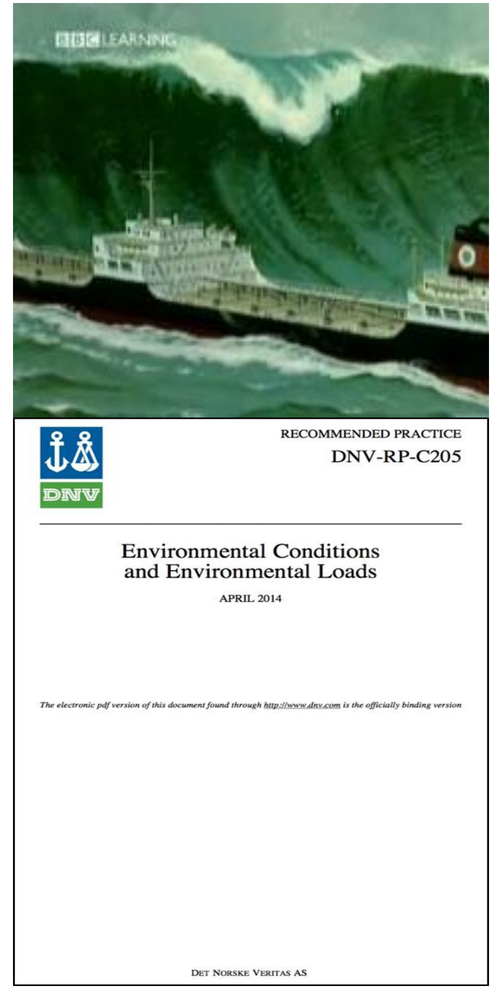
$$K.E. = \frac{1}{2} m q^2 = \frac{\rho q \delta x \delta z}{2}$$

Consequently, integrating this over the full wavelength and by utilizing the relationship between the speed and wavelength gives the total *K.E.* per unit width as:

$$E_{KE} = \frac{\rho g \lambda \zeta_0^2}{4}$$

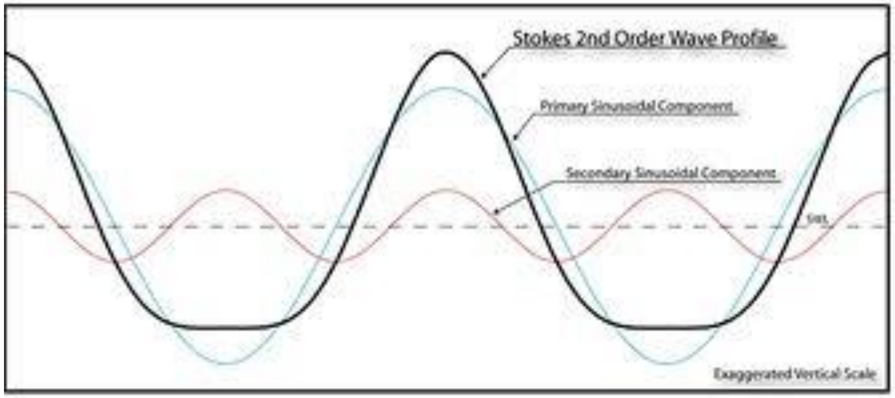
Deviations from Linear Wave Theory

- A linear wave model is very useful in practical engineering work. This is because :
  - ✓ It is easy to use
  - ✓ It complies well with the linear modelling of ship responses
  - ✓ It enables modelling of the sea by superimposing waves of different lengths and heights
- **In some cases, certain non-linear effects must be considered:**
  - ✓ The information provided by the linear wave model up to the still water level is not sufficient, e.g. we deal with local wave pressure loads on ship's side shell
  - ✓ An increase of wave steepness results in wave profiles that differ from the ideal cosine form
  - ✓ In some cases, certain non-linear effects have to be considered
- Suitably validated NL wave theories and hence NL wave idealisations can provide
  - ✓ Better agreement between theoretical and observed wave behavior.
  - ✓ Useful in calculating mass transport.

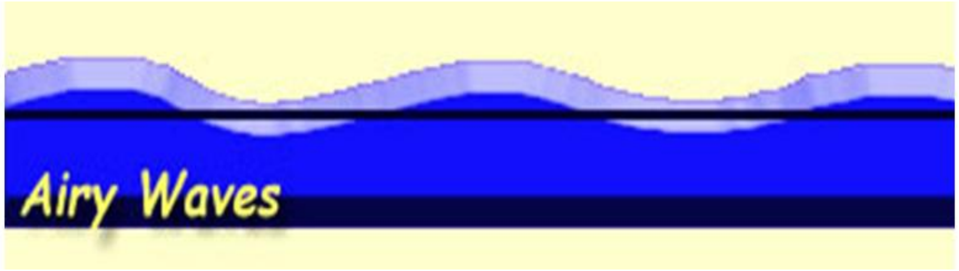


Stokes Waves - Introduction

- A method of dealing with steep regular (inviscid) waves in intermediate and deep water
  - (e.g. coastal and offshore structures)
  - Stokes series expansion is used mathematically (1st, 2nd, 3rd,… order)
  - Cannot handle shallow water waves
- What physically differs them from the linear model is the fact that they attempt to evaluate the pressures up to the actual water surface giving this method better accuracy for local pressures

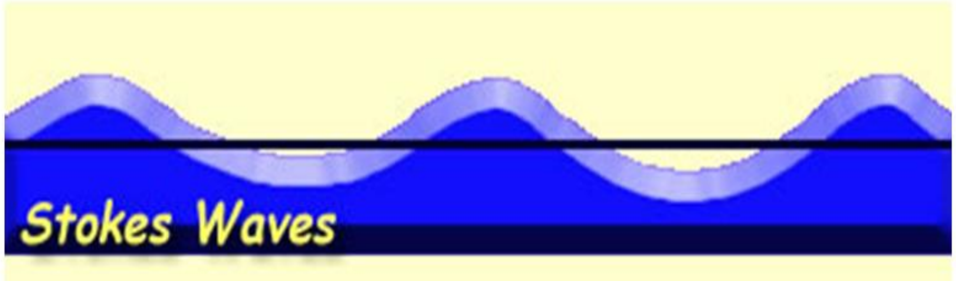


Deviations from Linear Theory – Airy vs Stokes waves



$$\phi = \frac{H}{2}\frac{g}{\omega}\frac{\cosh[k(d+z)]}{\cosh(kd)}\sin(kx-wt)$$

$$\eta(x,z,t) = a\sin(kx-wt)$$



$$\phi = \frac{H}{2}\frac{g}{\omega}\frac{\cosh[k(d+z)]}{\cosh(kd)}\sin(kx-wt) -$$

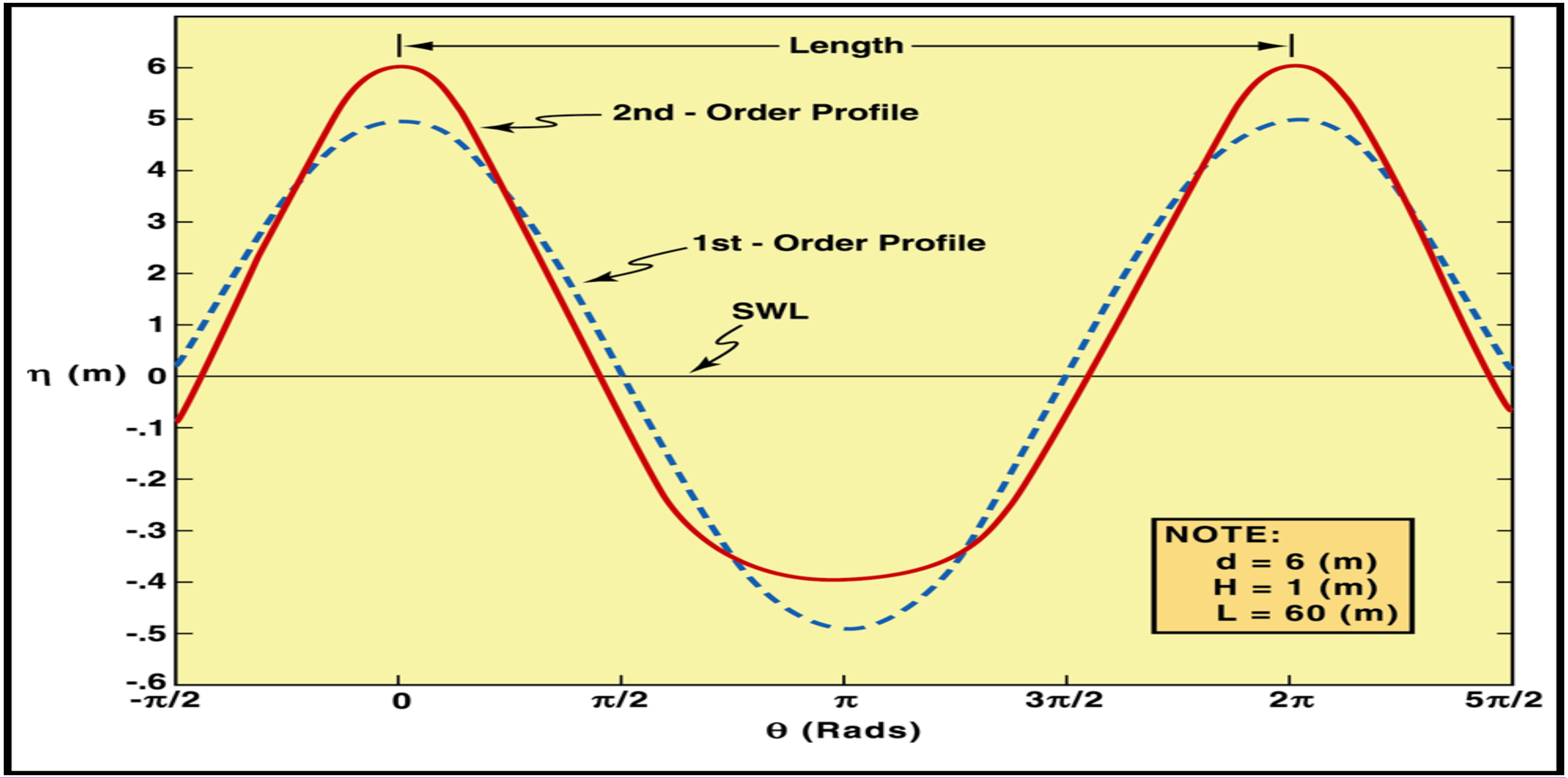
$$\frac{3}{32}\frac{H^2}{\omega}\frac{\cosh[2k(d+z)]}{\sinh^4(kd)}\sin 2(kx-wt)$$

$$\eta(x,z,t) = \frac{H}{L}\sin(kx-wt) + \text{NL terms}$$

$$\frac{H^2 k}{16}\frac{\cosh kd}{\sinh^3(kd)}(2+\cosh(2kd)\cos 2(kx-\omega t)$$

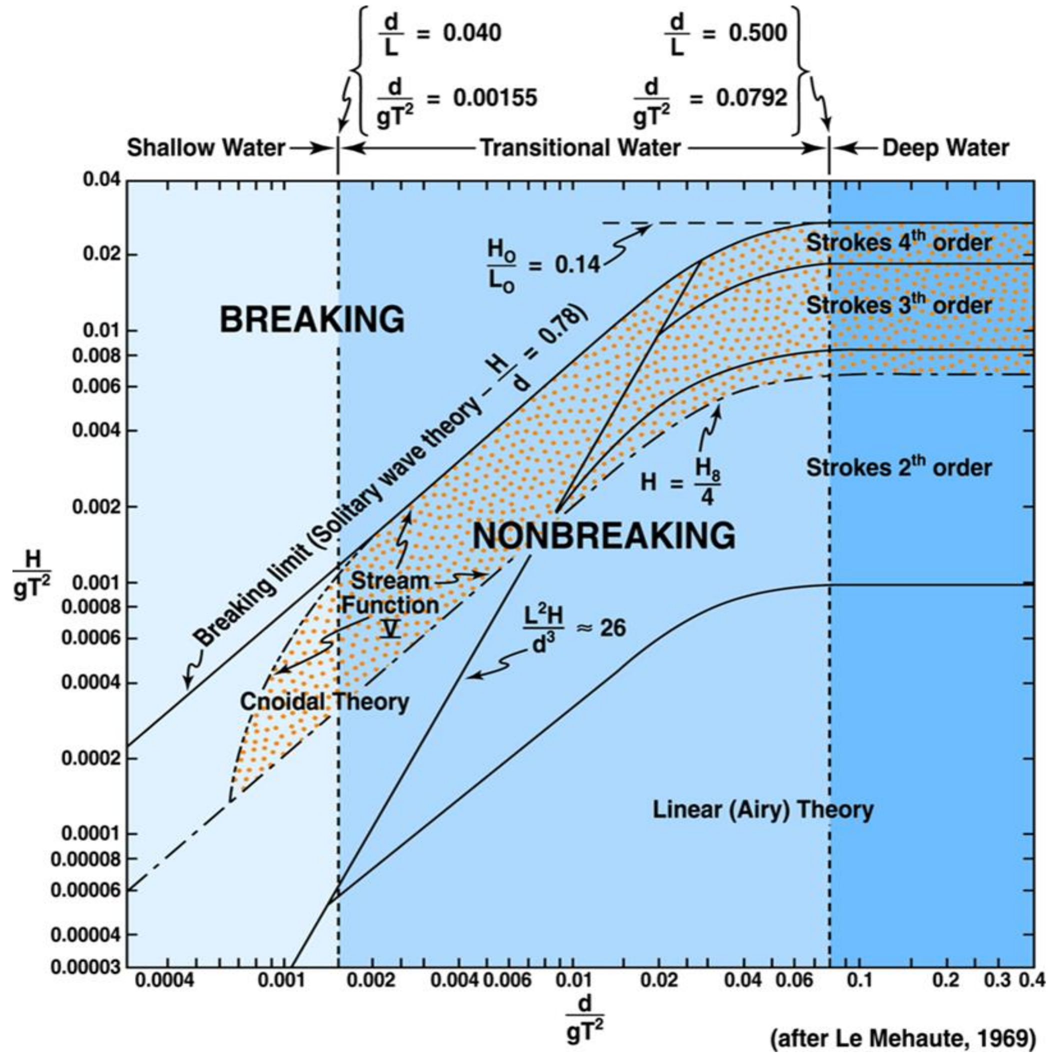
Deviations from Linear Theory – Airy vs Stokes waves

Higher order waves are more peaked at the crest, flatter at the trough and with distribution slightly skewed above SWL



Deviations from Linear Theory – Airy vs Stokes waves

- **Linear Wave Theory:** Simple, good approximation for 70-80 % engineering applications.
- **Nonlinear Wave Theory:** Complicated, necessary for about 20-30 % engineering applications.
- Both results are based on the assumption of non-viscous flow.



Aalto University
School of Engineering

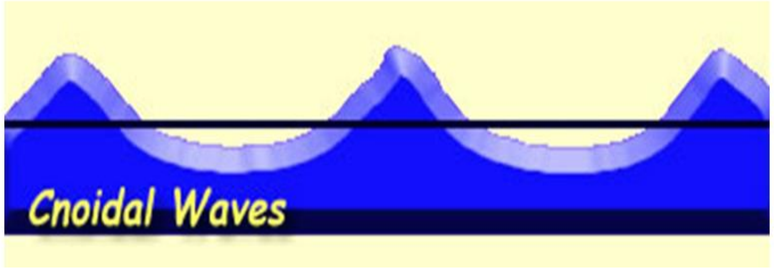
Ref : Le Méhauté, B. (1969). "An introduction to hydrodynamics and water waves." Springer.

Cnoidal Waves - trochoidal steepest waves

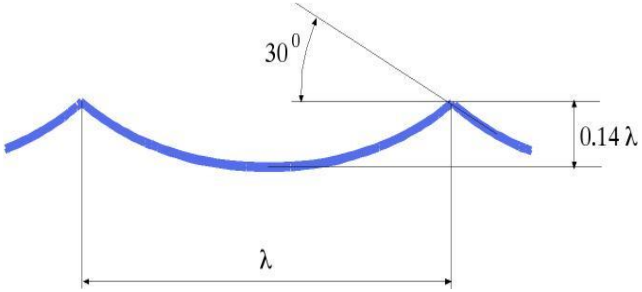
- The steepest possible waves that have sharper crests and flatter troughs
  - –Usually represented by a trochoid
  - –Not of a sine form like other waves
  - –Observed just before they break

$$\frac{h}{\lambda} \approx \frac{1}{7} \rightarrow h \approx 0.14\lambda$$

- An increase of wave steepness is associated with the sharpening of the crests while the **troughs get flatter**

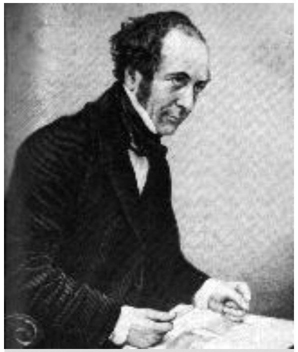


- We remove the irrotationality assumption from fluid mechanics
- Crossing swells, consisting of near-cnoidal wave trains.



Solitary Waves

- Solitary waves are described as waves of translation and differ from regular and irregular waves
  - Nonlinear wave propagation was studied in 1844 “Report on Waves” by Scott Russell link
  - “Solitary Wave Theory” by Walter Munk 1949 article link
- Wave pattern video (32 sec)
  - YouTube link



Scott Russell

THE SOLITARY WAVE THEORY AND ITS APPLICATION TO SURF PROBLEMS*

BY WALTER H. MUNK

Scripps Institution of Oceanography and Institute of Geophysics, University of California

Introduction

The purposes of this paper are: (a) to give a summary of useful relationships derived by means of the solitary wave theory, and to plot these relations using dimensionless parameters for the purpose of making the theory accessible to numerical examples;† (b) to review various studies at the Scripps Institution dealing with the application of this theory to surf problems; and (c) to discuss the problem of sand transport in or near the surf zone, in the light of the solitary wave theory.

This investigation represents part of a general project undertaken during the war for the purpose of providing useful wave forecasts for the amphibious forces. By 1943, methods for forecasting sea and swell had been developed[1,2] and a study of the transformation of waves in shallow water was initiated for the purpose of extending the wave forecasts right into the surf zone. It should be noted that the outer edge of the surf zone (the greatest depth where waves break) is usually the most critical from the point of view of bringing landing craft ashore.

The problem was attacked in three ways: (a) by field observations along the East Coast by the Woods Hole Oceanographic Institution and along the West Coast by the Scripps Institution of Oceanography; (b) by laboratory observations at the Beach Erosion Board wave tank, in Washington, D. C., and later at the Department of Engineering of the University of California in Berkeley, California; (c) by theoretical studies.

A theoretical investigation by Burnside,[3] based on the assumptions of constancy of wave periods, conservation of energy, and the linear shallow water (Airy) wave theory, reveals that the waves decrease somewhat in height after entering shallow water, reach a minimum height and then increase.[4,5] The *initial decrease* in wave height had been noticed by O'Brien in laboratory investigations. A comparison between the subsequent *increase in height* as derived from Burnside's equations with that obtained from field and laboratory observations mentioned above, showed the computed increase to be considerably smaller than the observed increase. This discrepancy became increasingly large the nearer one came to the breaking zone, the zone most important for practical forecasts.

One reason for this discrepancy is contained in an assumption underlying the linear Airy theory, namely that the wave height be small compared to

* Contribution from the Scripps Institution of Oceanography, New Series No. 406. This work represents results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department under contract with the University of California.
† PLATES 1-12 at end of the paper.

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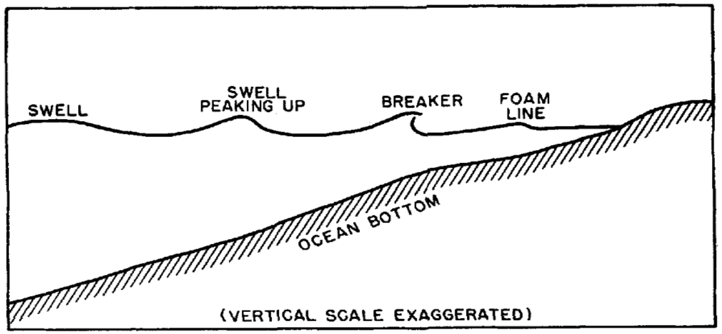
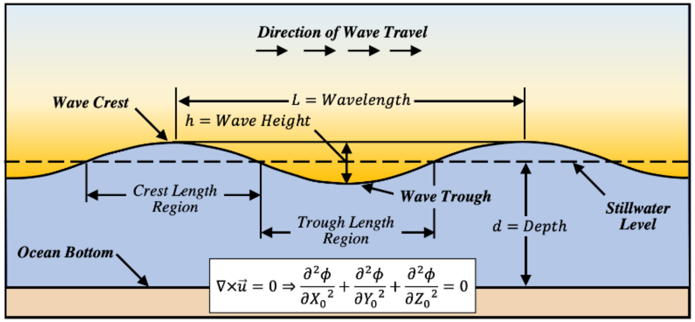
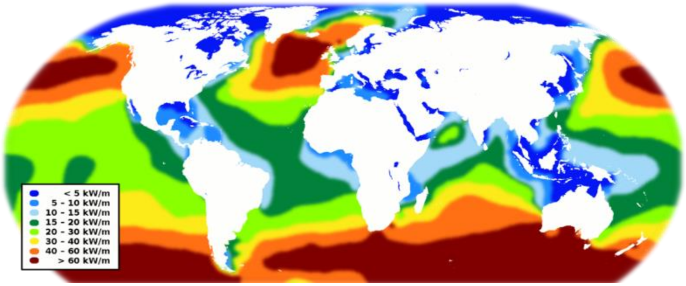


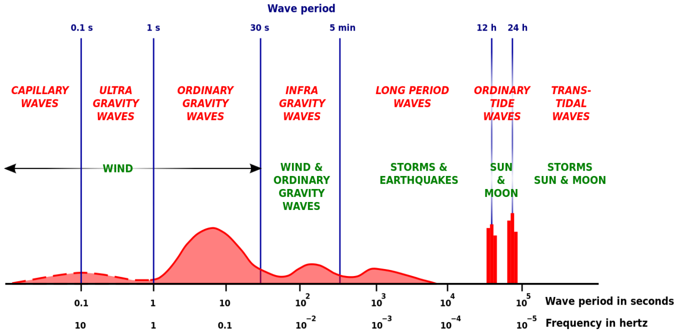
FIGURE 1. Schematic presentation of change in wave shape as wave advances into shallow water.

Summary

- Lecture content
  - The wave formation mechanisms
  - Linear theory of surface waves and associated terms
  - Deviations from the theory
- Sea surface can be described as…
  - Sum of sinusoidal wave components
  - Traveling in same direction, long-crested waves
  - Traveling in different directions, short-crested waves
- When partial wave properties are known, remainder can be estimated using potential flow theory (linear)
- Wave factors/properties relevant for design
  - Energy contents
  - Wave period/length and amplitude
  - The slope of waves
- Non-linearities exist in waves where the potential theory breaks down







Thank you !