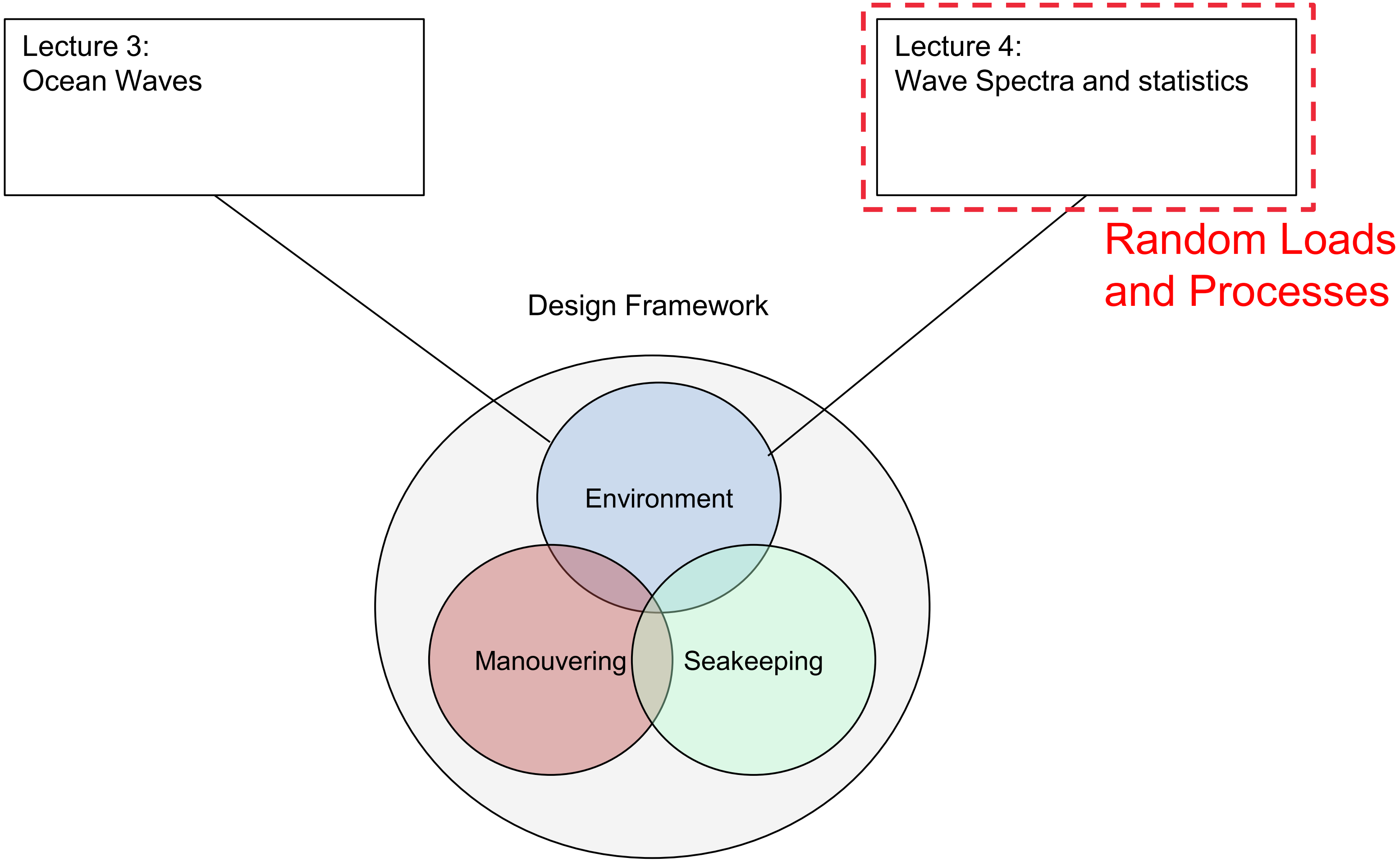
Aalto University
School of Engineering

MEC-E2004 Ship Dynamics (L)

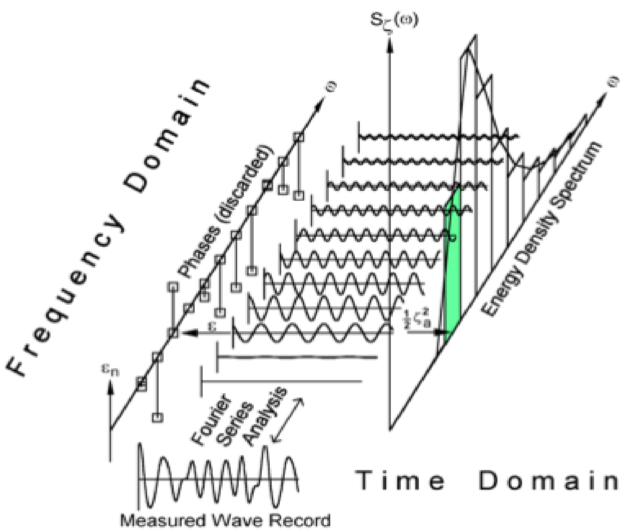
Lecture 4 –Irregular Seas

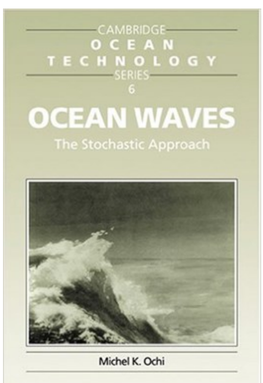
Where is this lecture on the course?

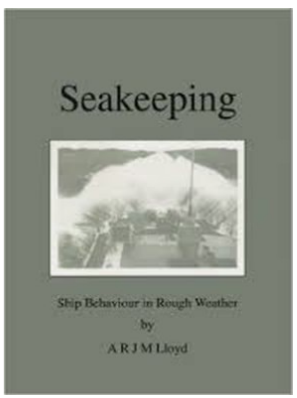


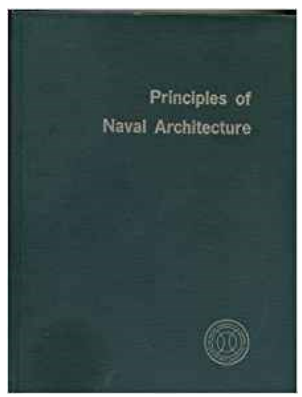
Contents

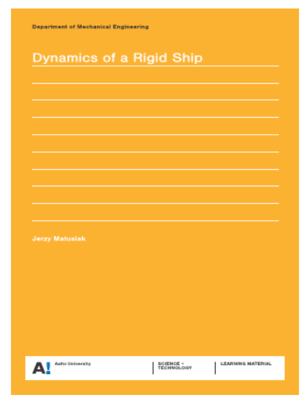
- **Aim :** To understand the wave spectrum and how it may be used to calculate short term ship responses in irregular seas ; Some brief introduction to Long term responses.
- <u>Literature</u>
  - Ochi, M., “Ocean Waves - The Stochastic Approach”, Cambridge Series, Ocean Technology, 6, Chapter 1
  - Lloyd, A.R.J.M, ”Seakeeping – Ship Behaviour in Rough Weather”, John Wiley & Sons, Chapters 3-4
  - Lewis, ”Principles of Naval Architecture – Vol. III”, SNAME, 1989
  - Matusiak, J., ”Dynamics of Rigid Ship”, Aalto University
  - Simon Haykin and Barry van Veen (2007), Signals and Systems, 2nd Edition, Wiley.









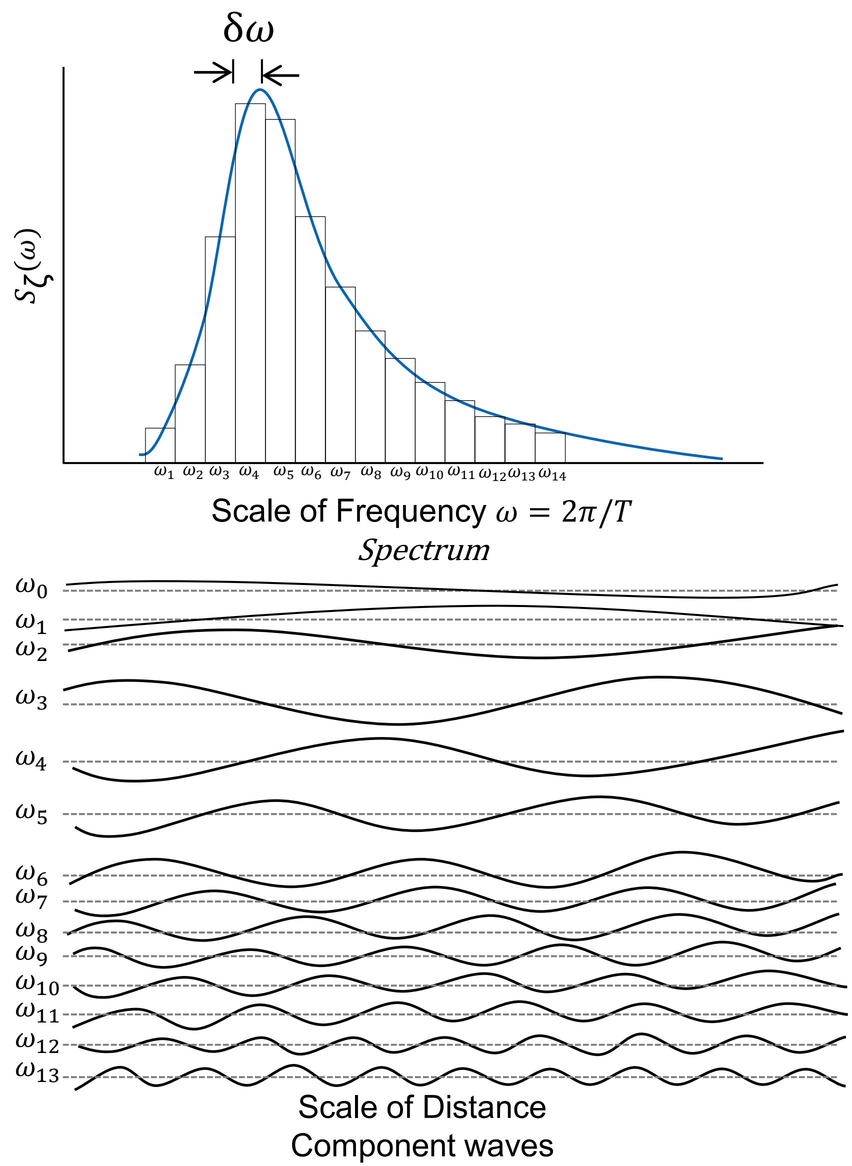


Motivation

- Ships operate in varying wave conditions. We should be able to evaluate Loads and motions under different wave heights and lengths

- Even if the ship is rigid it experiences varying pressures around hull . We have to know the influence of this varying pressure on motions, hydrodynamics pressures, shear forces and bending moments

- Hydrodynamic idealisations are possible in both frequency and time domains.
  - Frequency domain is useful for screening the worst conditions for our ship
  - Time domain to perform non-linear simulations at a given sea state



Assignment 2

Grades 1-3:

- ✓ Select a book-chapter related to ocean waves
- ✓ Define the water depths for your ship's route and seasonal variations of wave conditions
- ✓ Based on potential flow theory, sketch what kind of waves you can encounter during typical journey (deep water, shallow water)
- ✓ Identify and select the most suitable wave spectra for your ship - Justify the selection.
- ✓ Discuss the aspects (e.g. likelihood) to consider in case of extreme events from viewpoint of operational area

Example
Mediterranean or Baltic Sea
- 9 months in open water
- 3 months in ice

Route: …
Water depth: …

- Grades 4-5:
  - ✓ Read 1-2 scientific journal articles related to ship dynamics
  - ✓ Reflect these in relation to knowledge from books and lecture slides

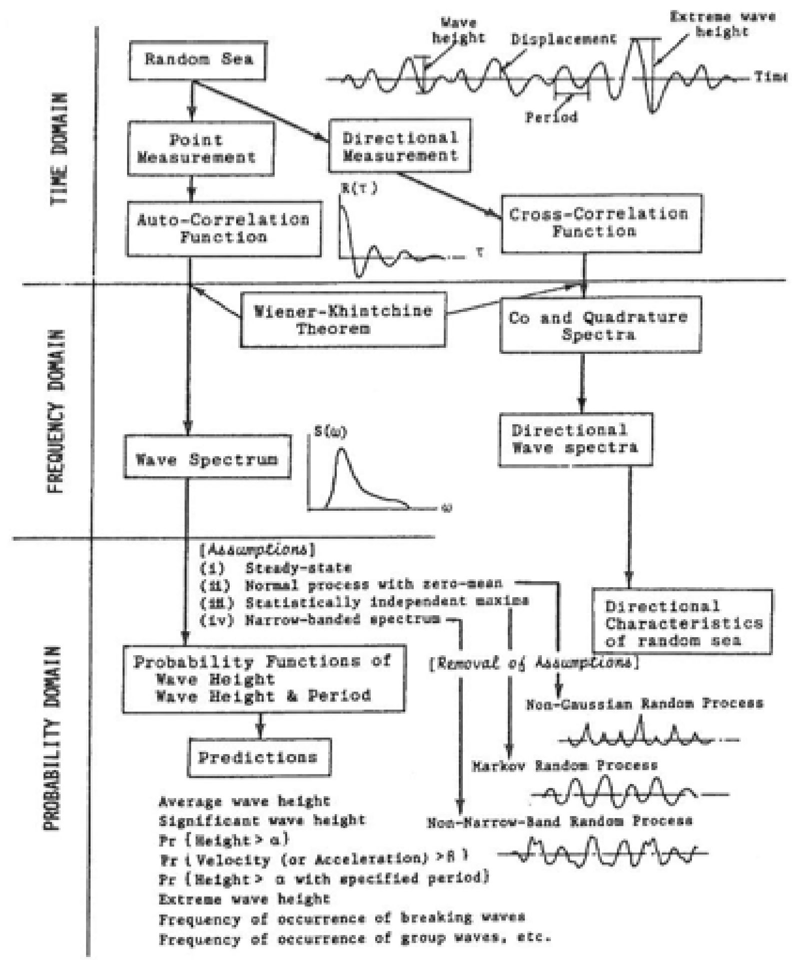
- Report and discuss the work.

Waves and Probability – the basics

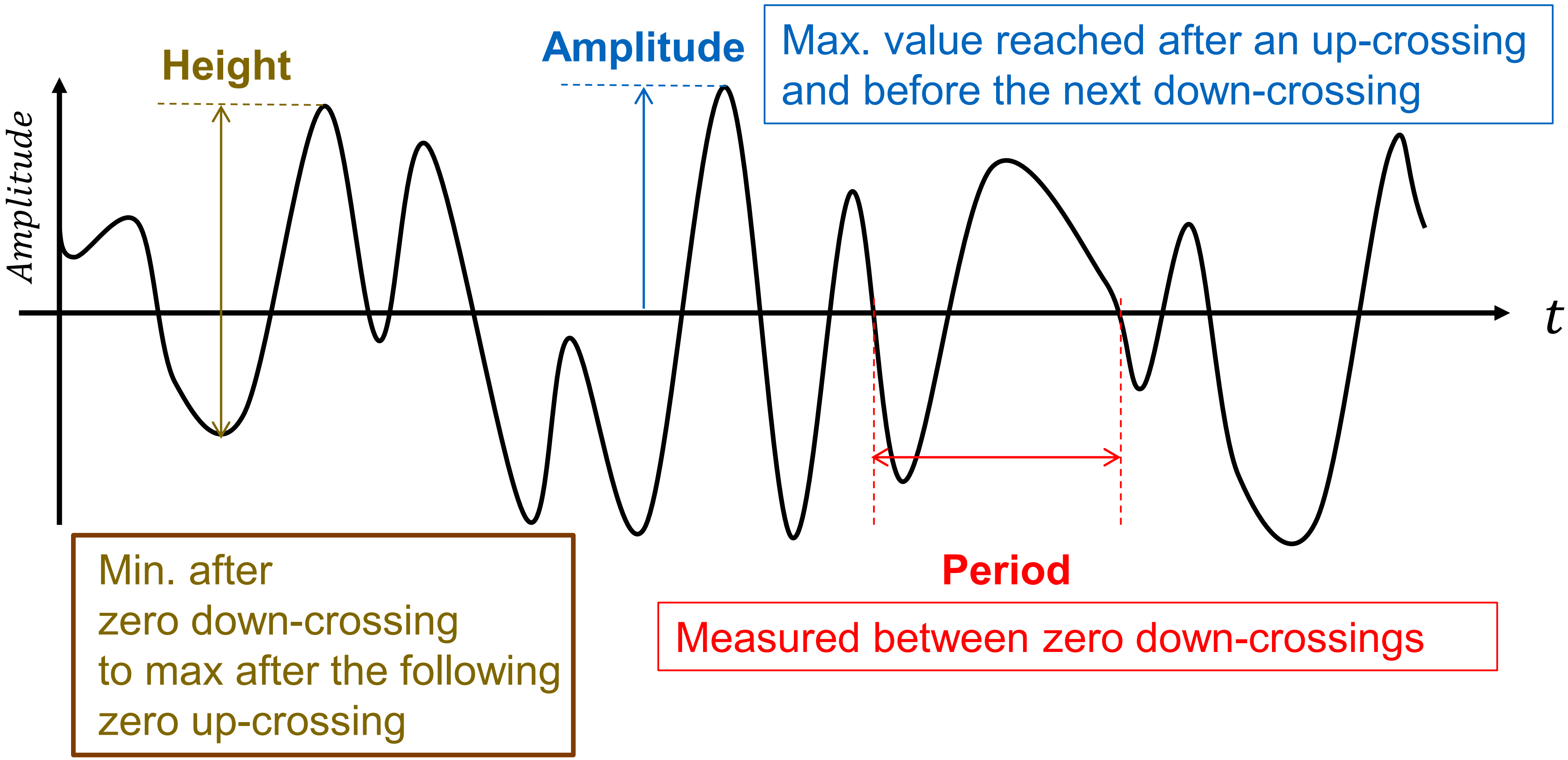
- When we measure the wave elevation of the random sea at a specific point we should move from time domain to probability domain through
  - ➢ Auto-correlation function, Fourier transformation
  - ➢ Wave spectrum

- Assumptions that are necessary to obtain the probabilities are:
  - ➢ Steady state process (probabilities will not be changing much from one transition to the next – stable probabilities)
  - ➢ Normal process (Gaussian / Bell type / Standard distribution) with zero mean
  - ➢ Statistically independent maxima (occurrence of one event does not affect the other)
  - ➢ Narrow-banded statistical process (spectrum energy with focus on single frequency or small number of frequencies)

- If we remove these assumptions also the way to assess probabilities, e.g. for extreme loads change



The Irregular Wave



Statistical representation of irregular wave

- Mean water elevation:

$$\bar{\zeta} = \sum_{n=1}^{N} \frac{\zeta_n}{N}$$

- Mean peak amplitude $\bar{\zeta}_a$
- Mean wave height $H_a = 2 \times \bar{\zeta}_a$
- Variance of the water elevation

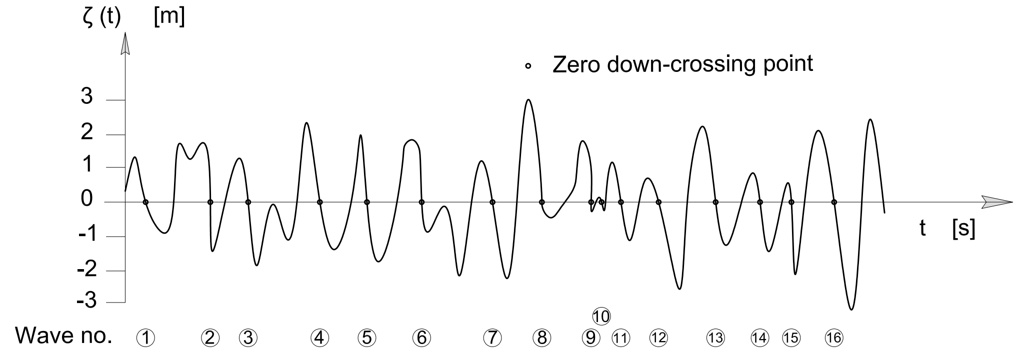
$$m_0 = \sum_{i=1}^{N} \frac{(\zeta_n - \bar{\zeta})^2}{N}$$

- Standard deviation

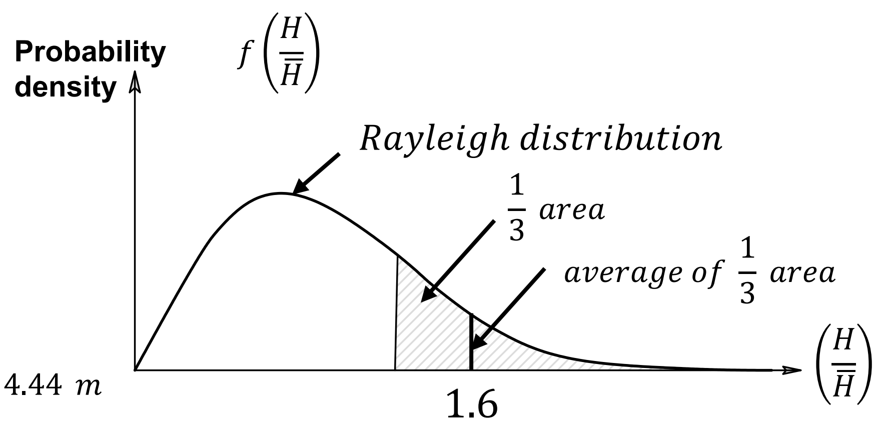
$$\sigma_0 = \sqrt{m_0}$$

- Significant wave heights is the average wave height of one-third of the highest waves

$$H_{1/3} = \frac{1}{5}\sum_{i=1}^{5} H_i = \frac{1}{5}(5.5 + 4.8 + 4.2 + 3.9 + 3.8) = 4.44\ m$$



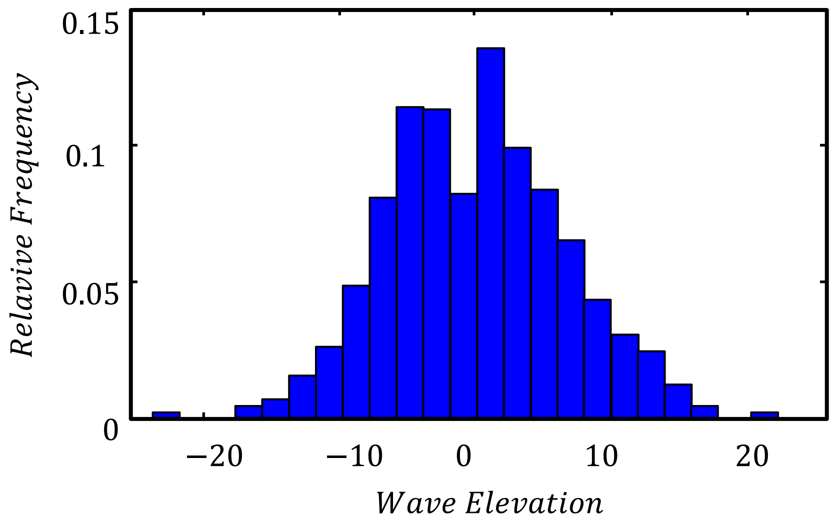
Rank i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H\ [m]$	5.5	4.8	4.2	3.9	3.8	3.4	2.9	2.8	2.7	2.3	2.2	1.9	1.8	1.1	0.23
$T[s]$	12.5	13	12	11.2	15.2	8.5	11.9	11	9.3	10.1	7.2	5.6	6.3	4	0.9
Wave no	7	12	15	3	5	4	2	11	6	1	10	8	13	14	9

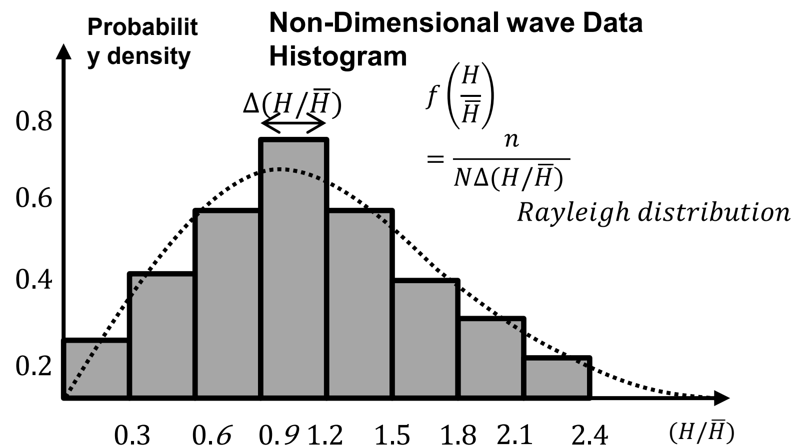


Water elevation histogram

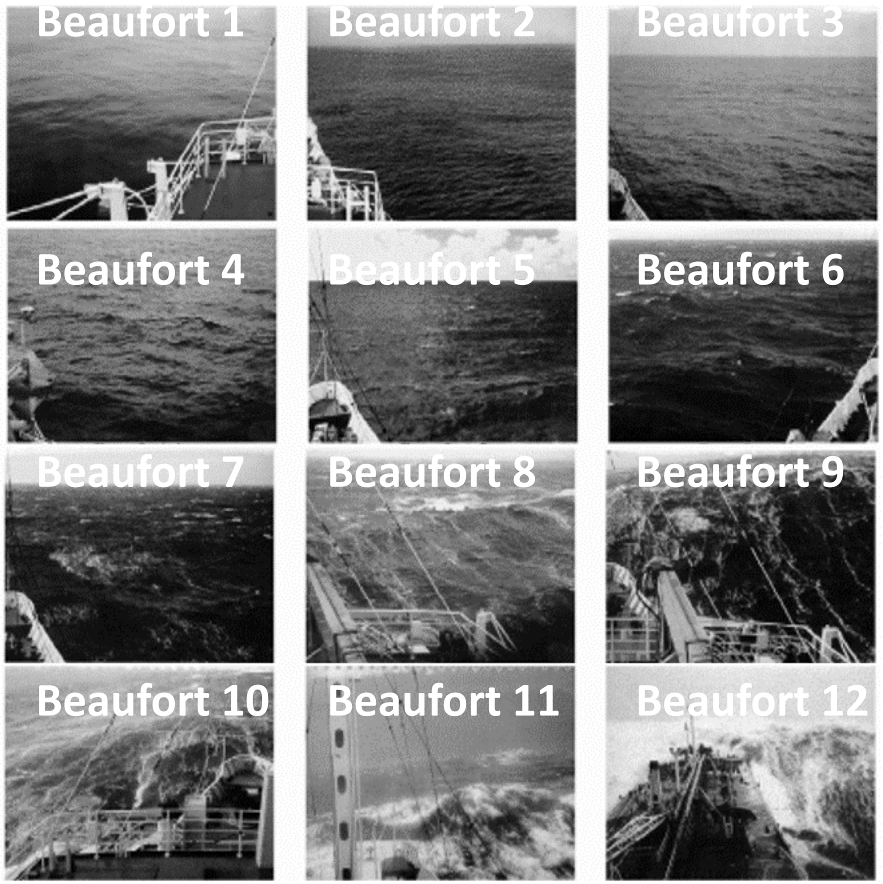
- The range of water elevation variation can be described using a histogram
  - Water surface measurement should be collected at a particular location and at regular intervals
  - The elevations are grouped into elevation ranges
  - The histograms are usually represented in a non-dimensional form; wave height/average wave height and plotted against the probability density.
  - The Rayleigh probability density function is usually used

$$f(H/\bar{H}) = \frac{\pi}{2}(H/\bar{H}) \cdot e^{-\pi/4\left(H/\bar{H}\right)^2}$$





Irregular Waves - classification

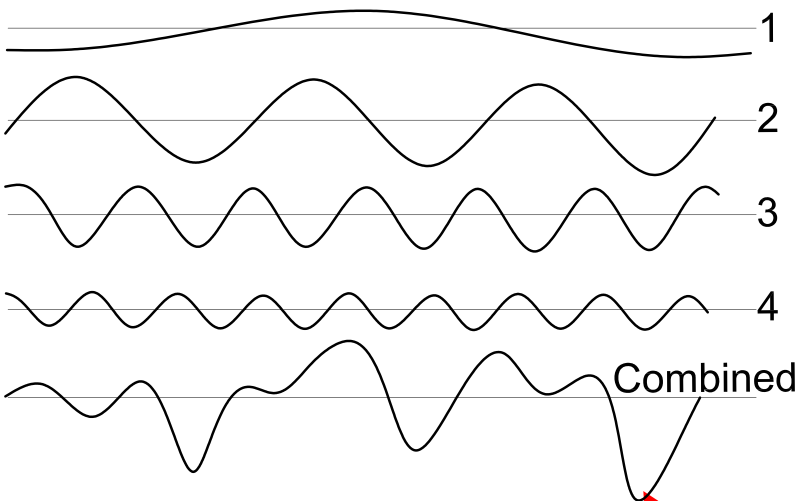


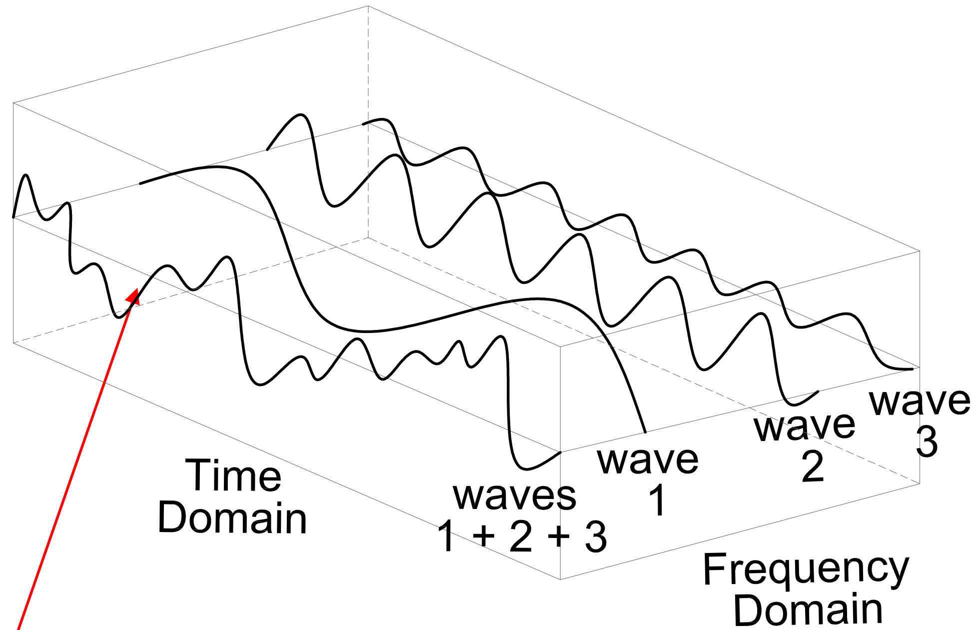
Determination of the wave spectra is carried out by observations of seaway and prevailing wind.

Description of wind	Beaufort number	Wind speed (knots)	$SWH\_(1/3)(m)$
Light air	1	2-3	1.00
Light breeze	2	4-7	1.40
Gentle breeze	3	8-11	1.65
Moderate breeze	4	12-16	2.25
Fresh breeze	5	17-21	3.10
Strong breeze	6	22-27	4.15
Moderate gale	7	28-33	5.40
Fresh gale	8	34-40	7.10
Strong gale	9	41-48	10.10
Whole gale	10	49-56	12.45
Storm	11	57-65	15.90
Hurricane	12	more than 65	

This table may be useful in obtaining wave spectra when wind speed is known.

The irregular wave formation





Combination of waves leads to irregular wave

Sea Spectra Simplified

By Walter H. Michel[1]

A dissertation on the simple wave elements that make up the complex sea, this paper is intended to give the practicing naval architect a clearer view of how regular waves combine into an irregular pattern and how the consequent irregular behavior of a vessel at sea can be predicted on the basis of recent statistical formulations.

Prologue

More than 13 years have elapsed since St. Denis and Pierson introduced to this Society the exciting new theory of sea-wave behavior and its effect on ships ("On the Motions of Ships in Confused Seas," *Trans. SNAME*, vol. 61, 1953). Since that time, much effort has been expended in proving, refining, and applying this theory in research activities until today we are on the threshold simple, regular wave. Although the theory is still in the throes of development and change, as more study and actual sea data are gathered, and, although there are still limitations to it (it does not as yet take good account of shallow water, or very steep waves, for example), it presents the most logical assessment of what the sea actually is and how it does what it does.

Even though this is now well recognized, much study

Wave superposition

- Consider two waves travelling past the same point ($x = 0$) with the same amplitude ζ and different frequencies

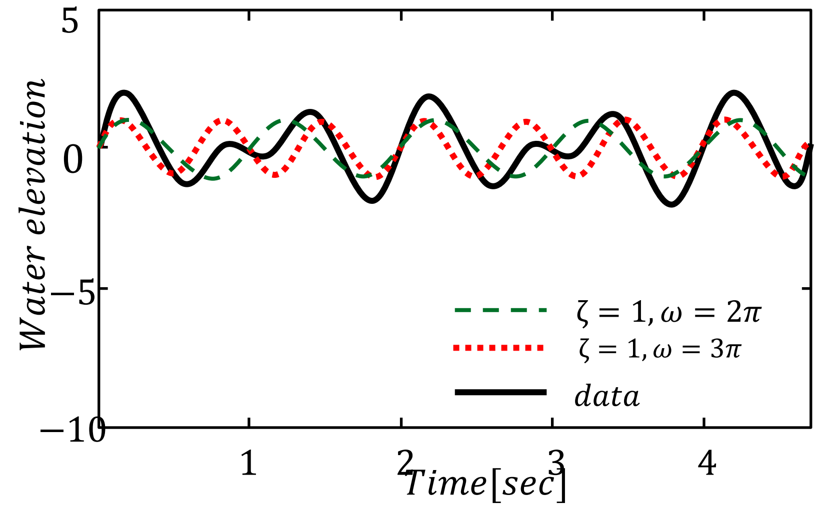
$$\zeta_1(t) = \zeta_0 sin(\omega_1 t)$$
$$\zeta_2(t) = \zeta_0 sin(\omega_2 t)$$

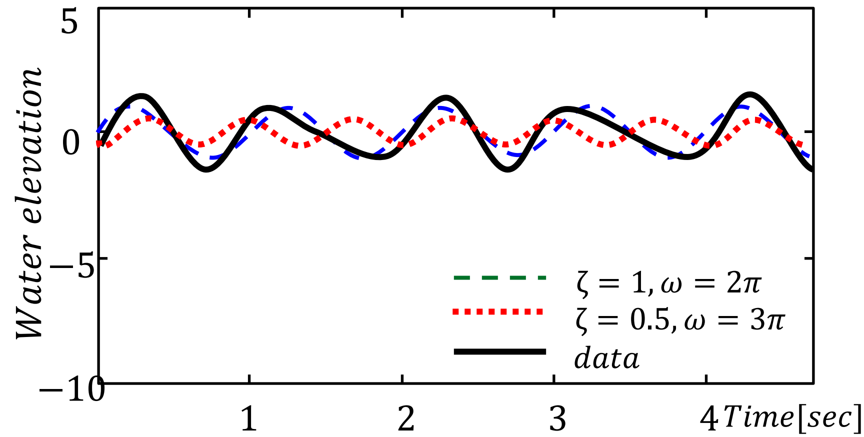
- The water elevation at any time

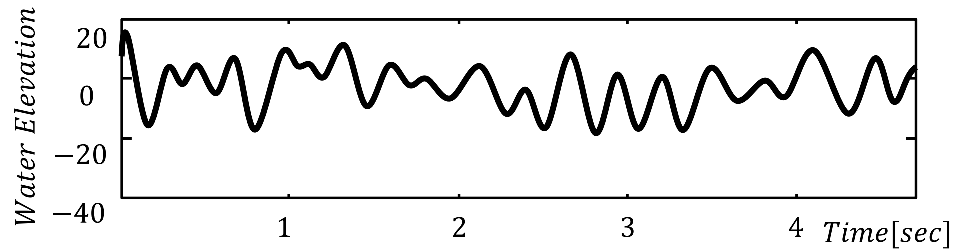
$$\zeta(t) = \zeta_1(t) + \zeta_2(t)$$

- For N waves, the amplitude should equal

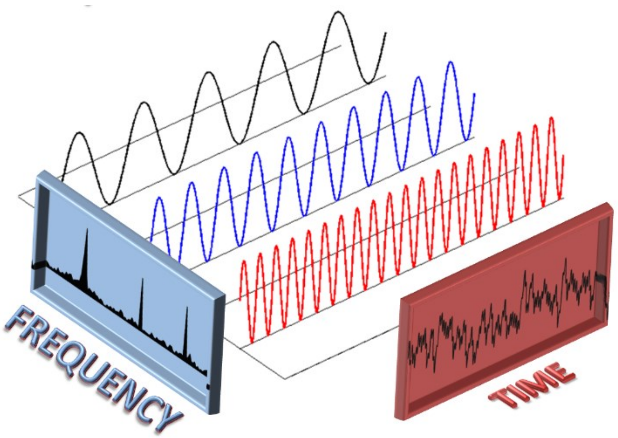
$$\zeta(t) = \bar{\zeta} + \sum_{n=1}^{N} \zeta_{n0} sin(\omega_n t + \varepsilon_n)$$





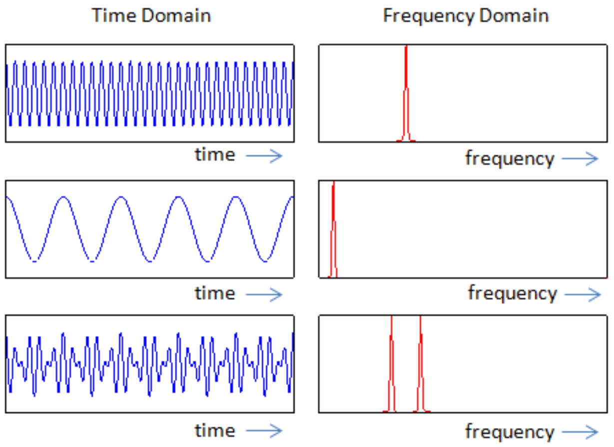


Fourier analysis



- Fourier Transform FT
  - Defines the different frequency components (amplitude and phase angle) in a given wave train.

- The FFT provides the frequency domain information $(X_0, X_1, \ldots, X_k)$ of a recorded signal in time domain $(x_0,\ t_0)$, $(x_1,\ t_1)$,$(x_n,\ t_n)\ldots$
  - x_n is the recorded wave magnitude at time t_n, with Sampling frequency f_s.
  - k refers to the frequency domain
  - X_k is a complex number (real and imaginary) gives information of wave magnitude and phase.
  - Number of points in frequency domain is N/2 (as x_n refers to magnitude and X_k to magnitude and phase)



Fourier analysis

- A Fast Fourier Transformation (FFT) gives a complex number
  - ✓ Not in a physically meaningful form
  - ✓ Should be scaled to find amplitudes and phase.

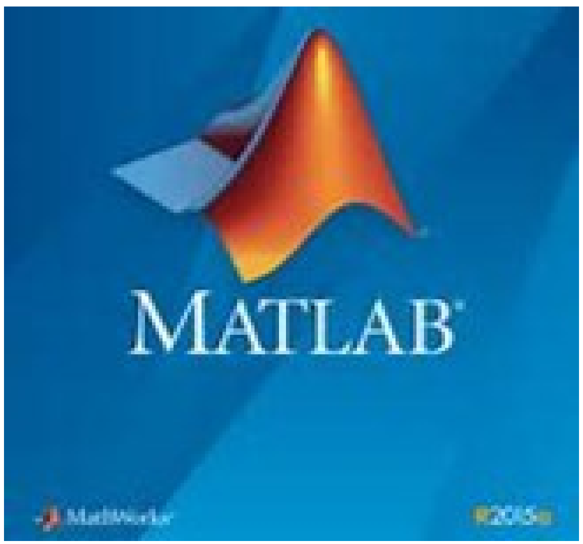
$$Magnitude = 2\frac{|X_k|}{N}$$

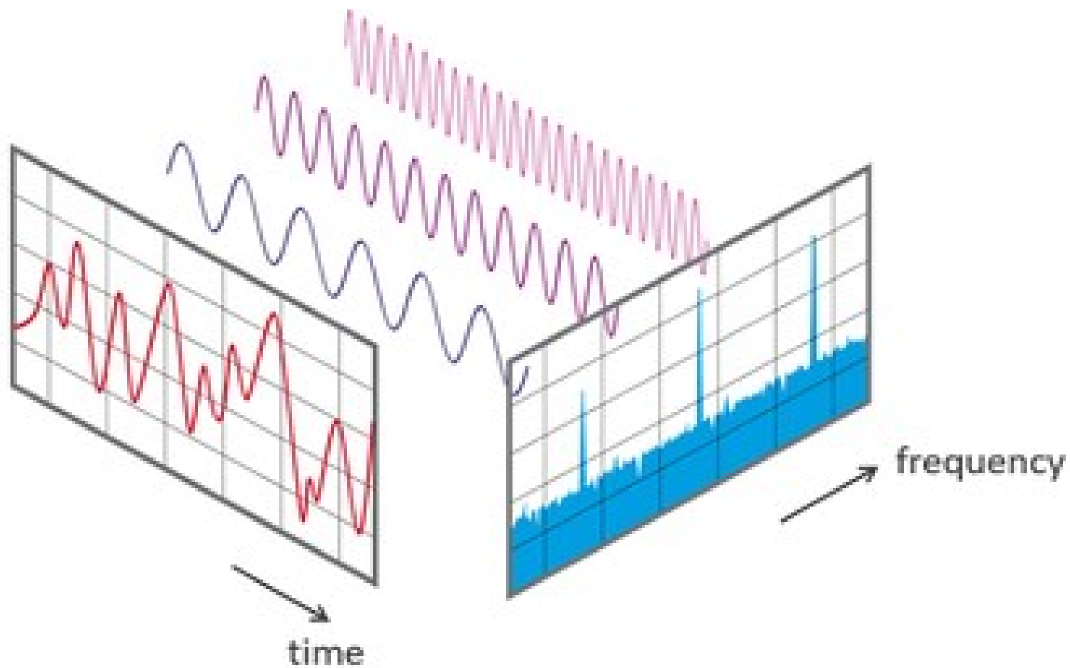
  - ✓ The phase angle is determined from taking the tangent of the real and complex parts of the FFT output.
  - ✓ MATLAB can solve the complex FFT to obtain X_k, while scaling should be solved manually
  - ✓ The highest frequency depends on how rapidly the data is sampled ($Df = \frac{f_s}{N}$). Therefore, it can be measured as:

$$f_{max} = \frac{N}{2}Df$$

Spectrum idealisation practical

- A fast Fourier transform (FFT) is an algorithm that samples a signal over a period of time (or space) and divides it into its frequency components. These components are single sinusoidal oscillations at distinct frequencies each with their own amplitude and phase.





Spectrum idealisation practical

From Time to Frequency domain

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fft

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Fast Fourier transform

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Syntax

```
Y = fft(X)
Y = fft(X,n)
Y = fft(X,n,dim)
```

Description

`Y = fft(X)` computes the discrete Fourier transform (DFT) of X using a fast Fourier transform (FFT) algorithm. example

- If X is a vector, then `fft(X)` returns the Fourier transform of the vector.
- If X is a matrix, then `fft(X)` treats the columns of X as vectors and returns the Fourier transform of each column.
- If X is a multidimensional array, then `fft(X)` treats the values along the first array dimension whose size does not equal 1 as vectors and returns the Fourier transform of each vector.

`Y = fft(X,n)` returns the n-point DFT. If no value is specified, Y is the same size as X. example

- If X is a vector and the length of X is less than n, then X is padded with trailing zeros to length n.
- If X is a vector and the length of X is greater than n, then X is truncated to length n.
- If X is a matrix, then each column is treated as in the vector case.
- If X is a multidimensional array, then the first array dimension whose size does not equal 1 is treated as in the vector case.

`Y = fft(X,n,dim)` returns the Fourier transform along the dimension `dim`. For example, if X is a matrix, then `fft(X,n,2)` returns the n-point Fourier transform of each row. example

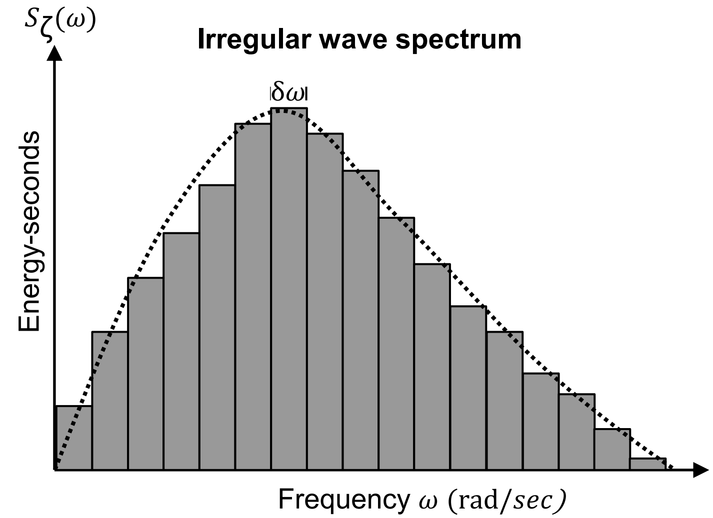
Wave energy spectrum

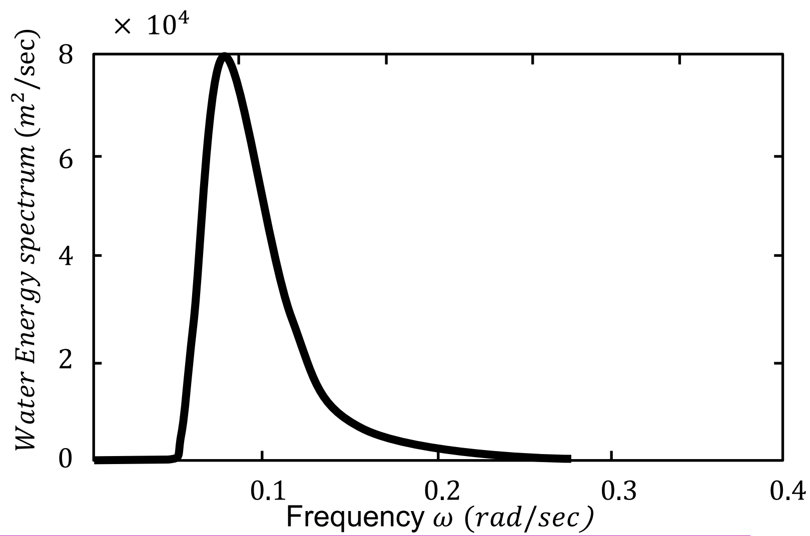
- Total energy of a sea state equals the area under the wave energy spectrum

$$E_{TOT} = \frac{1}{2}\rho g(\zeta_{a1}^2 + \zeta_{a2}^2 + \cdots + \zeta_{an}^2)$$

  - Where $\frac{1}{2}\rho g(\zeta_{an}^2)$ is the energy of each regular wave comprises the sea state.
  - To compare the energy of different sea states we express the energy without the constant ρg
  - Hence, the spectral ordinate for each frequency

$$S_\zeta(\omega_i) = \frac{\zeta_{i0}^2}{2\delta\omega}$$





Velocity and acceleration spectra

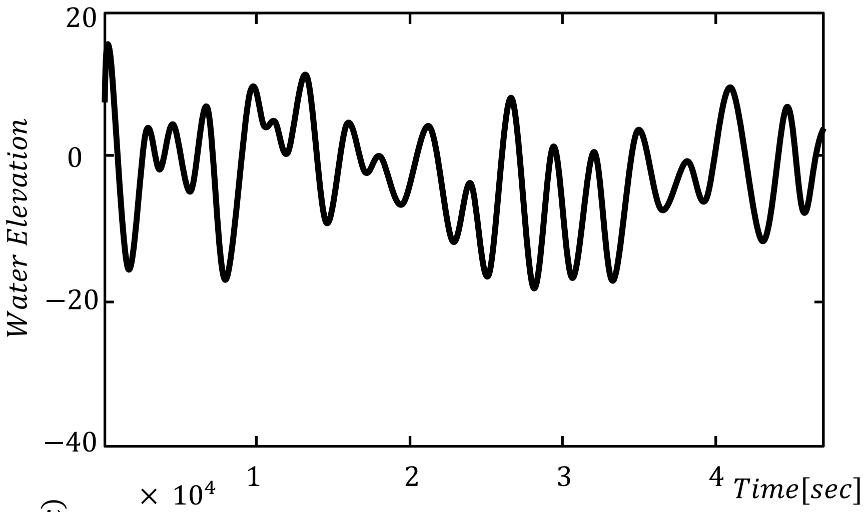
- The velocity and acceleration spectra ordinates, $S_{\dot{\zeta}}(\omega)$ and $S_{\ddot{\zeta}}(\omega)$, can be related to the wave energy ordinates:

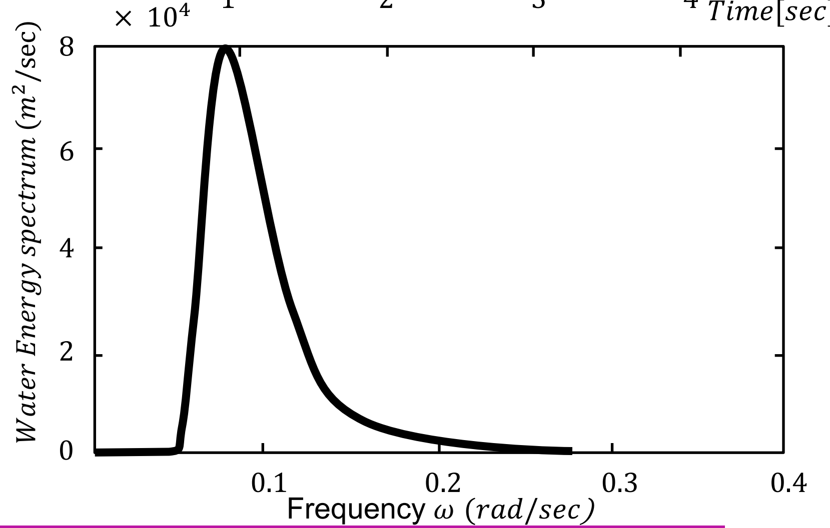
$$S_{\dot{\zeta}}(\omega) = \omega_i^2 S_{\zeta}(\omega_i)$$
$$S_{\ddot{\zeta}}(\omega) = \omega_i^4 S_{\zeta}(\omega_i)$$

- The areas under the velocity spectrum m_2 and acceleration spectrum m_4 (spectral moments):

$$m_2 = \int_0^{\infty} \omega^2 S_{\zeta}(\omega\,)$$
$$m_4 = \int_0^{\infty} \omega^4 S_{\zeta}(\omega\,)$$

  - *These are called the second m_2 and forth m_4 spectral moments*



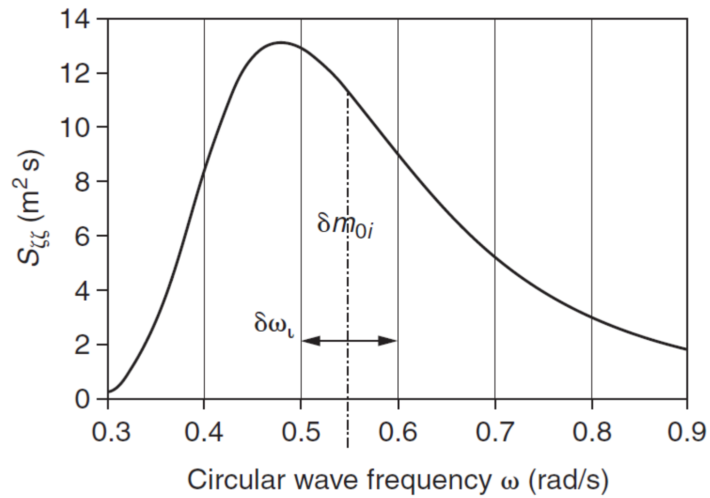


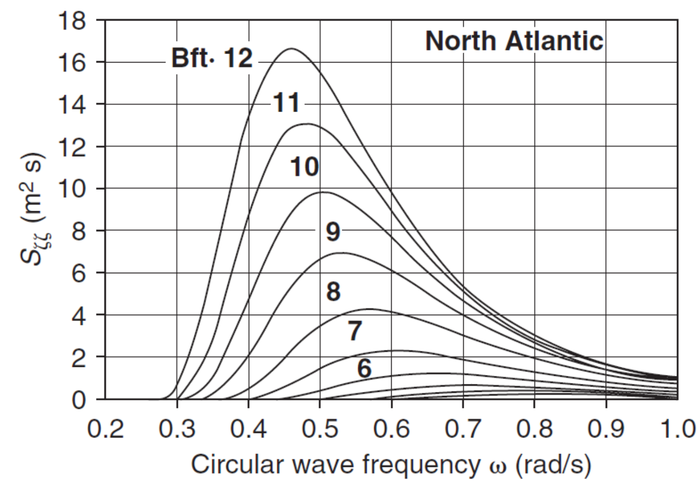
Spectral moments

- The spectral moments link the spectra to statistical characteristics of the time history:

$$m_n = \int_0^{\infty} \omega^n S_{\zeta}(\omega)d\omega$$

  - Wave variance $= m_0$
  - Mean Frequency: $\bar{\omega} = \frac{m_1}{m_0}$
  - Mean period: $\bar{T} = \frac{2\pi}{\bar{\omega}} = 2\pi\frac{m_1}{m_0}$
  - Mean peak period: $\bar{T}_P = 2\pi\sqrt{\frac{m_2}{m_4}}$
  - Mean zero-crossing period: $\bar{T}_z = 2\pi\sqrt{\frac{m_0}{m_2}}$



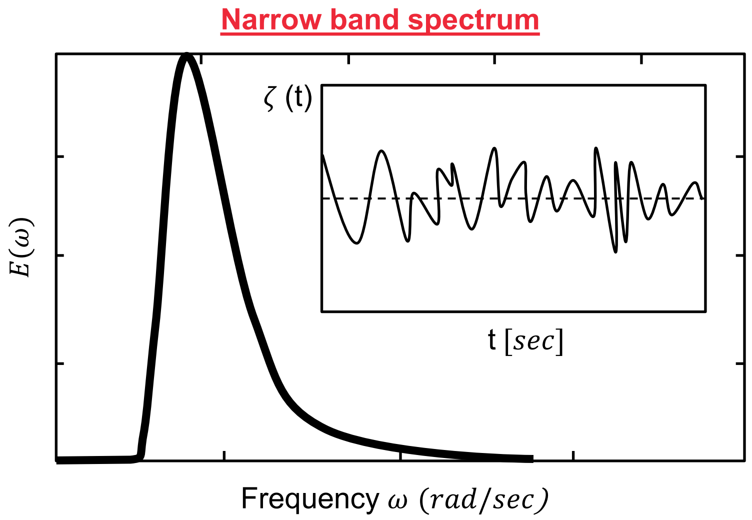


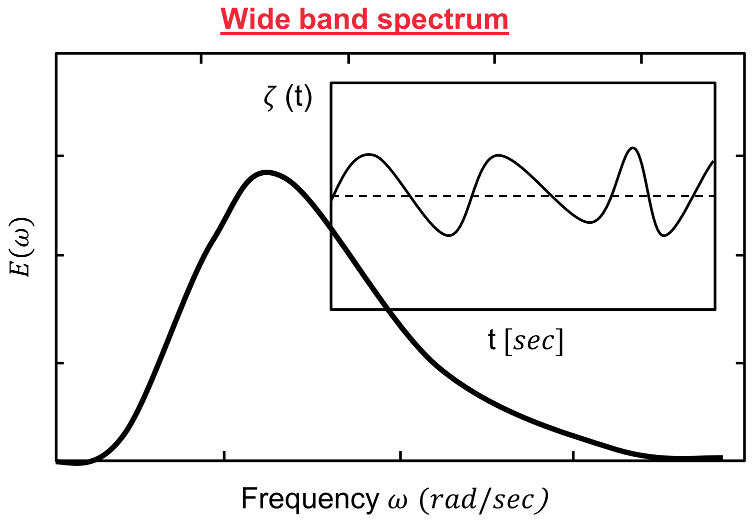
Spectrum bandwidth

- Describes the relative width of the wave energy spectrum compared to the height:
- Can be measured by the bandwidth parameter ε

$$\varepsilon = \sqrt{1 - \frac{\bar{T}_P^2}{\bar{T}_Z^2}} = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}$$

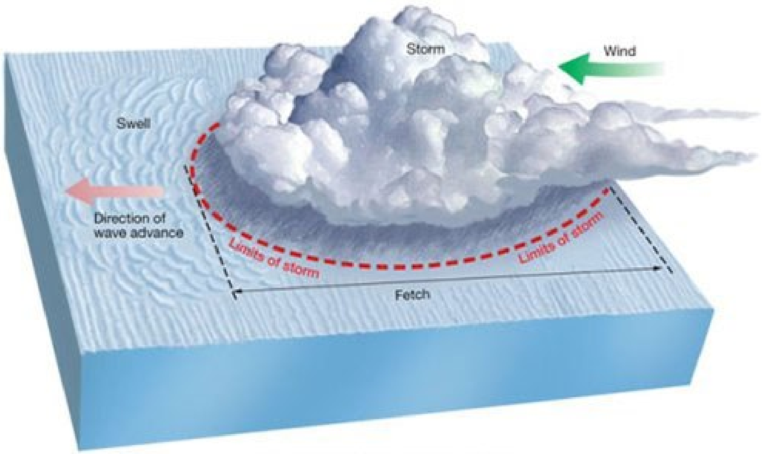
  - Narrow band spectrum is concentrated in a narrow range of frequencies. ε approaches zero
  - wide band spectrum, the energy is distributed among wide range of frequencies. ε approaches 1

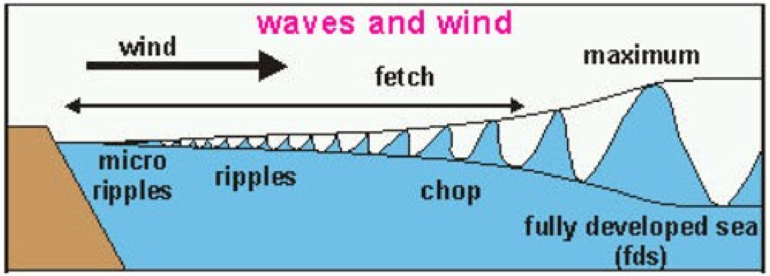




Idealized wave spectra

- Idealized wave spectra are used for assessing the vessel's performance in frequency domain:
  - Ship motions.
  - Structural response (shear force, bending moment, stresses).
- The spectrum can be given as a function of wind speed for fully developed seas.
- Energy contained in a sea state depends on:
  - Wind velocity
  - Wind duration
  - Fetch (length of water over which a given wind has blown without obstruction)
- Fully developed seas:
  - Occur in open oceans with infinite fetch
  - In equilibrium, the amount of energy transferred from the wind maintains the wave heights
  - Viscosity and wave breaking prevent additional amplitude growth

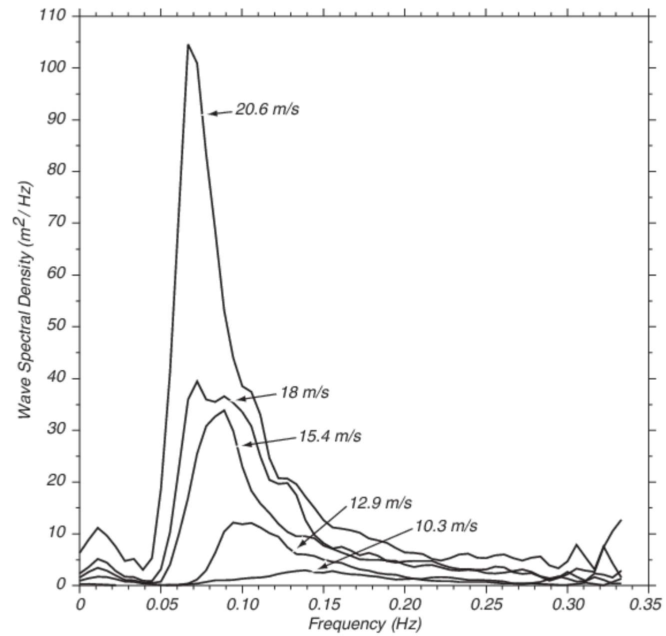




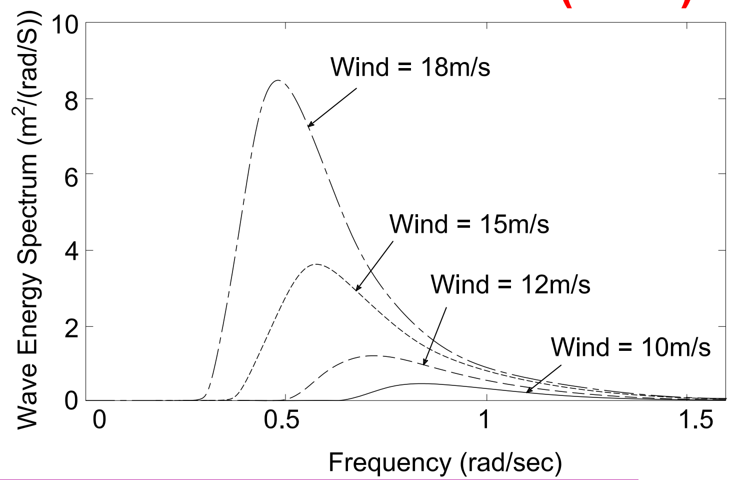
Idealized wave spectra

- There are a number of wave spectrum descriptions that depend on correlation of observation data and assumptions.
- These idealized wave spectra are used in ship design, such as, P-M, Bretschneider's, JONSWAP .
- Pierson Moskowitz (1964)
  - One parameter wave spectrum:
  - Describe fully developed seas
  - Input parameter is only the wind speed at 19.5 m above the sea surface $W_{19.5}$

$$S_\zeta(\omega) = \frac{0.0081g^2}{\omega^5} e^{-0.74\left(\frac{g}{W_{19.5}\,\omega}\right)^4}$$



Pierson Moskowitz (1964)



Idealized wave spectra

- Bretschneider's spectrum:
  - Two-parameter spectrum
  - Depends on significant wave height $H_{1/3}$ and modal period ω_0

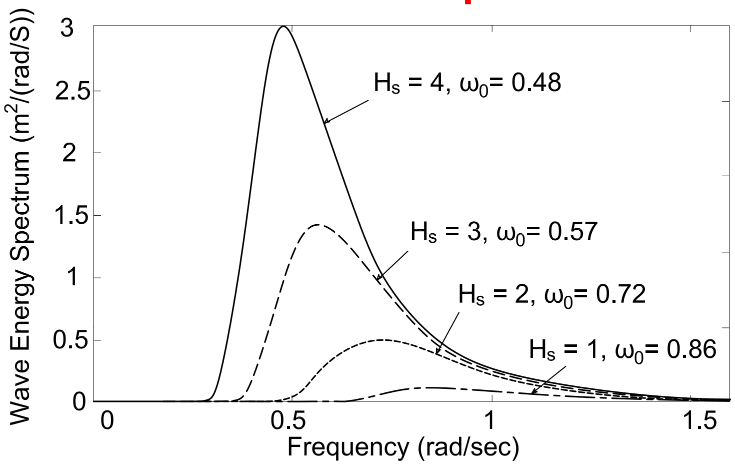
$$S_\zeta(\omega) = \frac{1.25}{4}\left(\frac{\omega_0}{\omega}\right)^4 \frac{\bar{H}_{1/3}^2}{\omega} e^{-1.25\left(\frac{\omega_0}{\omega}\right)^4}$$

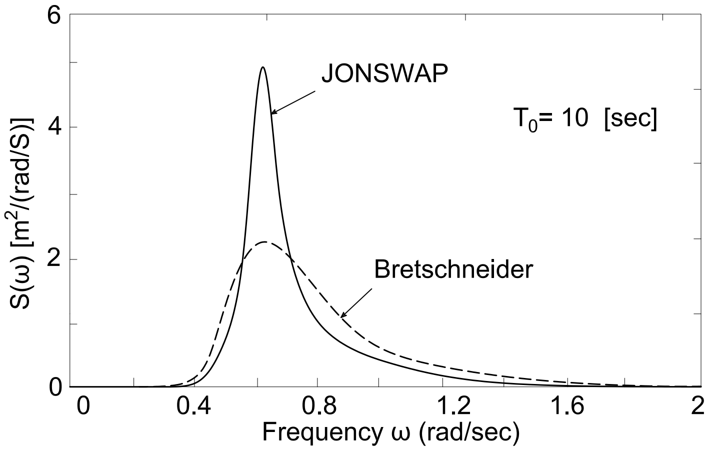
- JONSWAP (Joint North Sea Wave Observation Project) spectrum
  - Usually used in North Atlantic
  - For limited fetch conditions
  - Narrower spectrum
  - Three-parameter spectrum
  - Depends on speed, fetch and steepness factor (γ)

$$S_\zeta(\omega) = B_j \bar{H}_{1/3}^2 \frac{2\pi}{\omega}\left(\frac{\omega_0}{\omega}\right)^4 e^{-\left[\frac{5}{4}\left(\frac{\omega_0}{\omega}\right)^4\right]} \gamma^r$$

$$B_J = \frac{0.06238}{0.230 + 0.0336\gamma - \frac{0.0185}{1.9+\gamma}}[1.094 - 0.01915 ln\gamma]$$

Bretschneider's spectrum



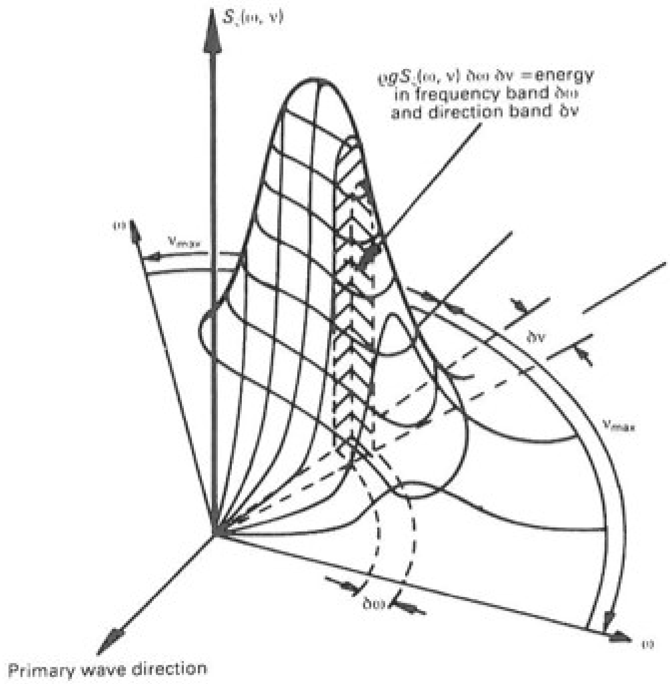


Directionality of Spectra

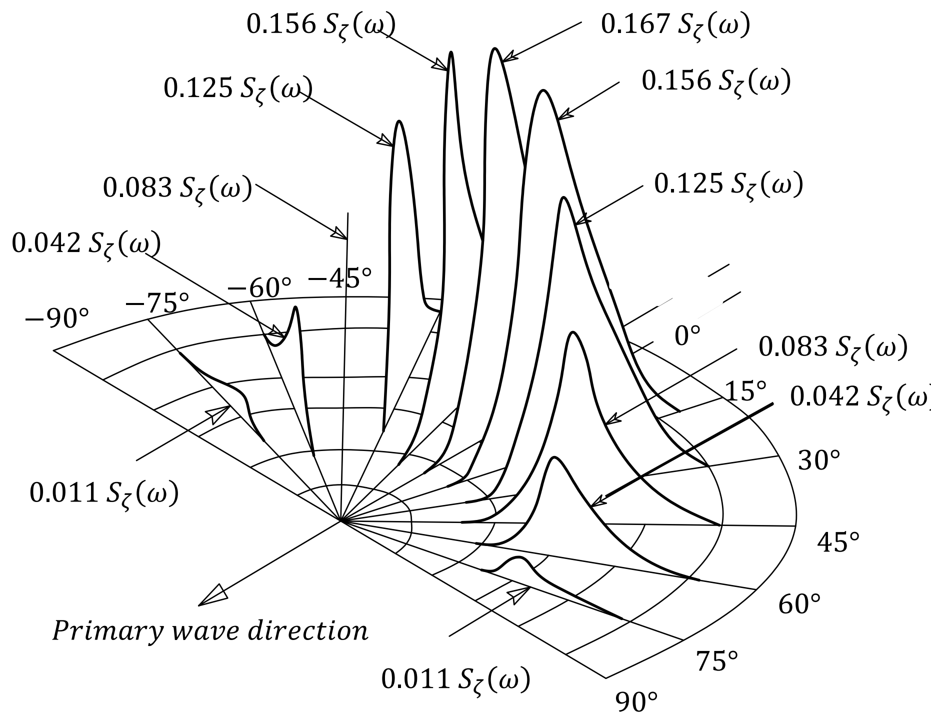
$$S(\omega, \theta) = S(\omega)D(\theta, \omega) = S(\omega)D(\theta)$$

$$\int_{-\pi/2}^{\pi/2} D(\theta, \omega)\mathrm{d}\theta = 1$$

Continuous

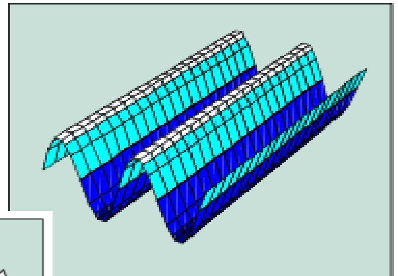


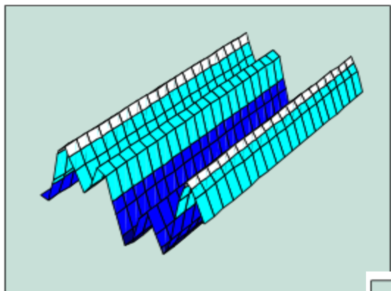
Discrete spreading



From 2D to 3D wave description – spreading

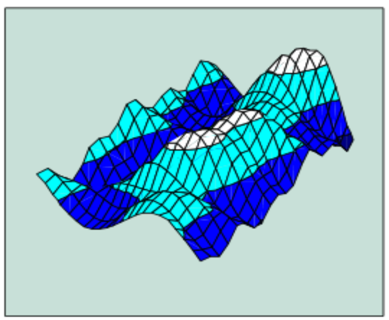
Long crested regular wave





Long crested irregular wave (M = 4)

Short crested (or confused seas) irregular wave (M=4)



In short crested or confused seas spreading function is used to express the waves in different directions

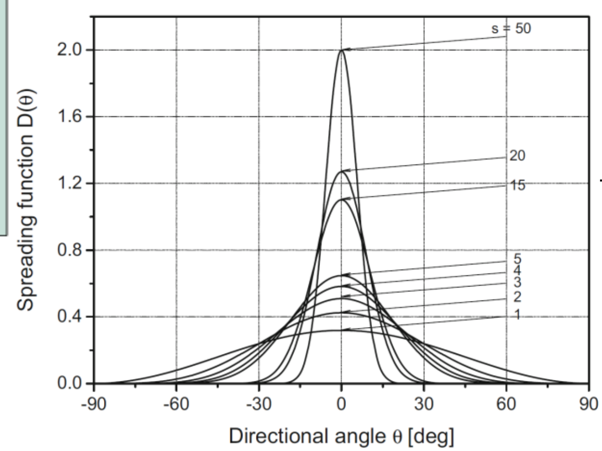
The spreading function is :

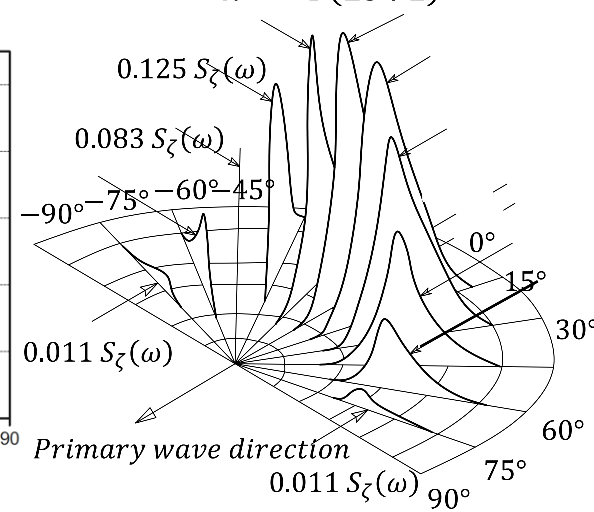
$$S(\omega, \theta) = S(\omega) D(\theta)$$

$$D(\theta) = G(s) \cos^{2s}(\theta - \theta_0)$$

θ_0=0 degree

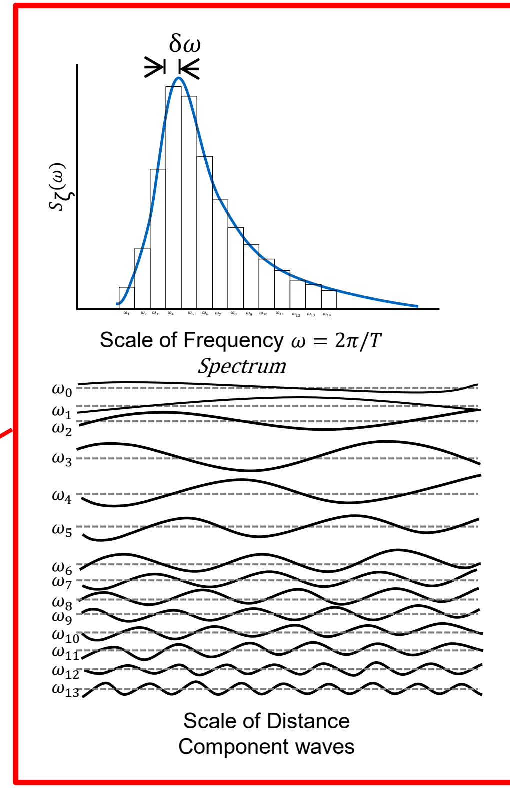
normalization factor $G(s) = \frac{2^{2s-1}}{\pi} \frac{\{\Gamma(s+1)\}^2}{\Gamma(2s+1)}$



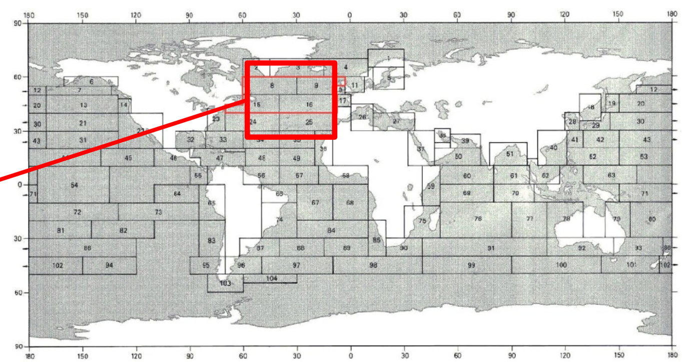


Statistics on Sea States

- For certain *operational area*, certain sea states occur with certain probability
- Probability for certain *sea states* with *T* and *H* is known from the measurements, (scatter diagram), **$p_1(H,T)$**
- Sea state then can be described with *wave spectrum, like* Pierson-Moskowitz for fully developed sea and JONSWAP for developing sea.
- which includes *energy contribution of certain wave components*
- For each wave spectrum *average, extreme elevation,* amplitude etc. can be calculated), **$p_2(H_{max})$=>$ptot=p_1p_2$**

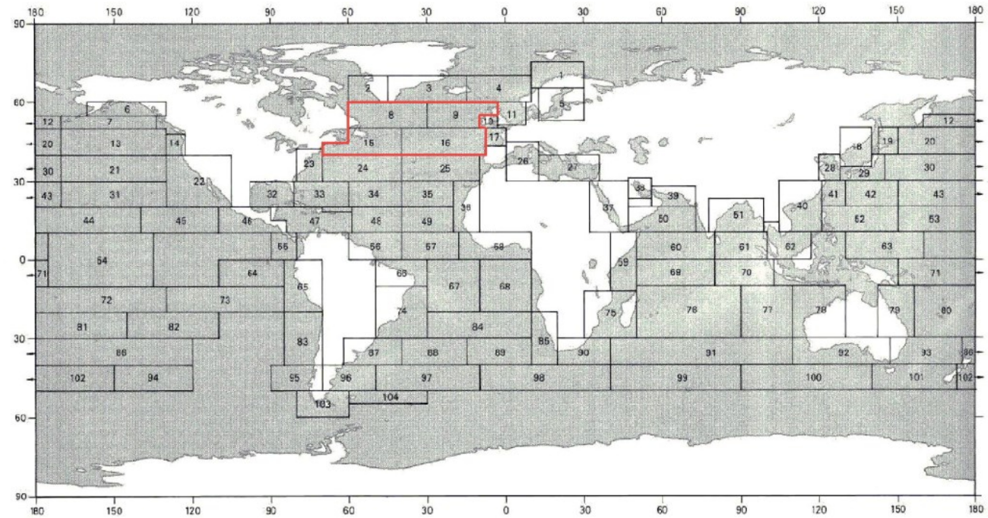


Hs (m) /Tz (s)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5	17.5	18.5	Totals
0.5	1.30	133.70	865.60	1186.00	634.20	186.30	36.90	5.00	0.70	0.10							3050
1.5		29.30	986.00	4976.00	7738.00	5569.70	2375.70	703.50	160.70	30.50	5.10	0.80	0.10				22575
2.5		2.20	197.50	2158.80	6230.00	7449.50	4860.40	2066.00	644.50	160.20	33.70	6.30	1.10	0.20			23810
3.5			34.90	695.50	3226.50	5675.00	5099.10	2838.00	1114.10	337.70	84.30	18.20	3.50	0.60	0.10		19128
4.5			6.00	196.10	1354.30	3288.50	3857.50	2685.50	1275.20	455.10	130.90	31.90	6.90	1.30	0.20		13289
5.5			1.00	51.00	498.40	1602.90	2372.70	2008.30	1126.00	463.60	150.90	41.00	9.70	2.10	0.40	5.00	8328
6.5			0.20	12.60	167.00	690.30	1257.90	1268.60	825.90	386.80	140.80	42.20	10.90	2.50	0.50	0.10	4806
7.5				3.00	52.10	270.10	594.40	703.20	524.90	276.70	111.70	36.70	10.20	2.50	0.60	0.10	2586
8.5				0.70	15.40	97.90	255.90	350.60	296.90	174.60	77.60	27.70	8.40	2.20	0.50	0.10	1309
9.5				0.20	4.30	33.20	101.90	159.90	152.20	99.20	48.30	18.70	6.10	1.70	0.40	0.10	626
10.5					1.20	10.70	37.90	67.50	71.70	51.50	27.30	11.40	4.00	1.20	0.30	0.10	285
11.5					0.30	3.30	13.30	26.60	31.40	24.70	14.20	6.40	2.40	0.70	0.20	0.10	124
12.5					0.10	1.00	4.40	9.90	12.80	11.00	6.80	3.30	1.30	0.40	0.10		51
13.5						0.30	1.40	3.50	5.00	4.60	3.10	1.60	0.70	0.20	0.10		21
14.5						0.10	0.40	1.20	1.80	1.80	1.30	0.70	0.30	0.10			8
15.5							0.10	0.40	0.60	0.70	0.50	0.30	0.10	0.10			3
16.5								0.10	0.20	0.20	0.20	0.10	0.10				1
Totals	1	165	2091	9280	19922	24879	20870	12898	6245	2479	837	247	66	16	3	1	100000



Sea States for ship structures (Long Term)

- For unlimited operation the North-Atlantic (Area 25 of BSRA statistics)
- For restricted service at the discretion of the Class Society Service Factor Analysis can be employed
- Some Key References :
  - IACS URS 11A, Rec. 34 ;
  - Lloyd's Register Rules (Part 4 Ship Structures) and



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Service Factor Assessment of a Great Lakes Bulk Carrier Incorporating the Effects of Hydroelasticity

Spyridon E. Hirdaris,[1] Norbert Bakkers,[2] Nigel White,[2] and Pandeli Temarel[3]

This paper presents a summary of an investigation into the effects of hull flexibility when deriving an equivalent service factor for a single passage of a Great Lakes Bulk Carrier from the Canadian Great Lakes to China. The long term wave induced bending moment predicted using traditional three-dimensional rigid body hydrodynamic methods is augmented due to the effects of springing and whipping by including allowances based on two-dimensional hydroelasticity predictions across a range of headings and sea states. The analysis results are correlated with full scale measurements that are available for this ship. By combining the long term "rigid body" wave-bending moment with the effects of hydroelasticity, a suitable service factor is derived for a Great Lakes Bulk Carrier traveling from the Canadian Great Lakes to China via the Suez Canal.

Keywords: Great Lakes; hydrodynamics; longitudinal strength

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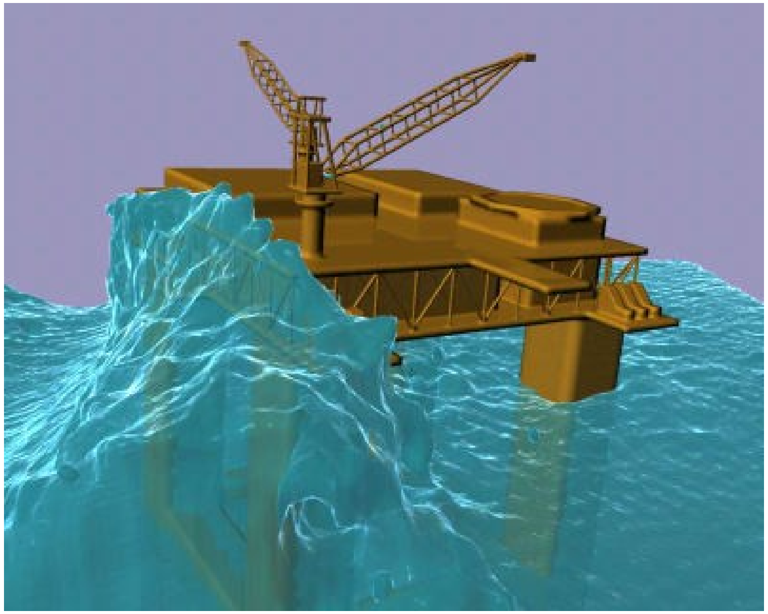
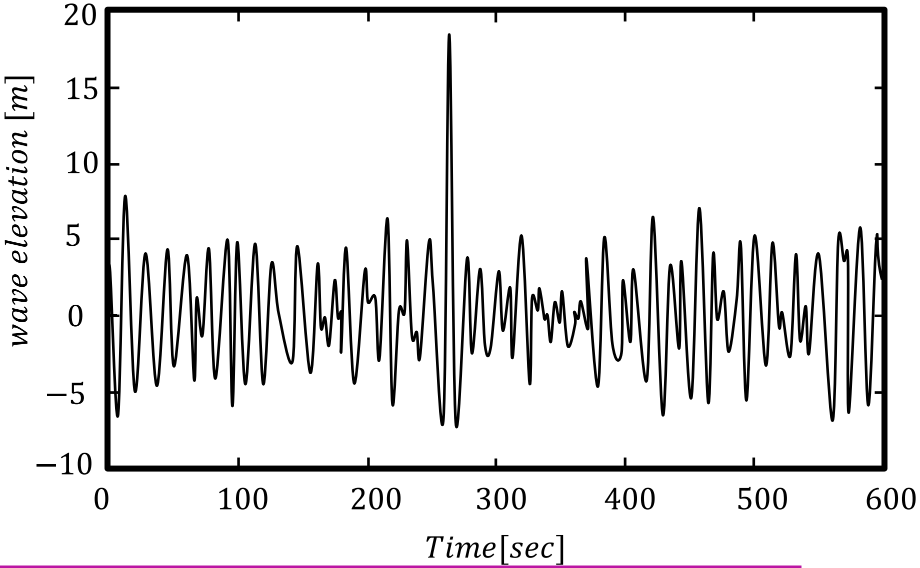
Freak Waves

- Rogue waves
  - Also known as freak waves, monster waves, episodic waves, killer waves, extreme waves and abnormal waves
  - Large, unexpected and suddenly appearing surface waves
  - Phase matching of component waves result in single wave which is much larger than surrounding waves
- The Draupner wave
  - a single giant wave measured on New Year's Day 1995
  - confirmed the existence of freak waves, which had previously been considered near-mythical
- Withstanding a Rogue Wave (2 min 20 sec) [YouTube link]





Tsunamis

- **Causes**
  - Seismic underwater activity such as earthquakes, volcanic eruptions and underwater explosions, landslides, glaciers, meteorite impacts, and other underwater disturbances
- **Seismic sea waves**
  - Do not resemble normal sea waves
  - Very long wavelength
  - Resemble a rapidly rising tide “tidal waves”
- They comprise of series of waves with periods ranging from minutes to hours, arriving in a "wave train” (L=10km, very shallow, Speed 800km/h)
- Japan Tsunami (3 min 34 sec) YouTube link

Although the impact of tsunamis is limited to coastal areas their destructive power can be enormous

Modelling of freak waves

- Freak waves of up to 35 m in height are much more common than the probability theory (Rayleigh distribution of wave peaks) could predict.
- Linear model introduced previously cannot account for the existence of “freak waves”.
- Three main types of freak wave:
  - **Walls of water** travelling up to 10 km over the ocean surface before become extinct.
  - **Three sisters** are groups of three waves.
  - **Single, giant storm waves** that build up to more than four times the average height of storm waves, but they collapse in a relatively short time.

Modelling of freak waves

- Nonlinear Schrödinger equation NLS used to explain the occurrence of the freak wave in deep water.
- The most commonly form is the cubic NLS

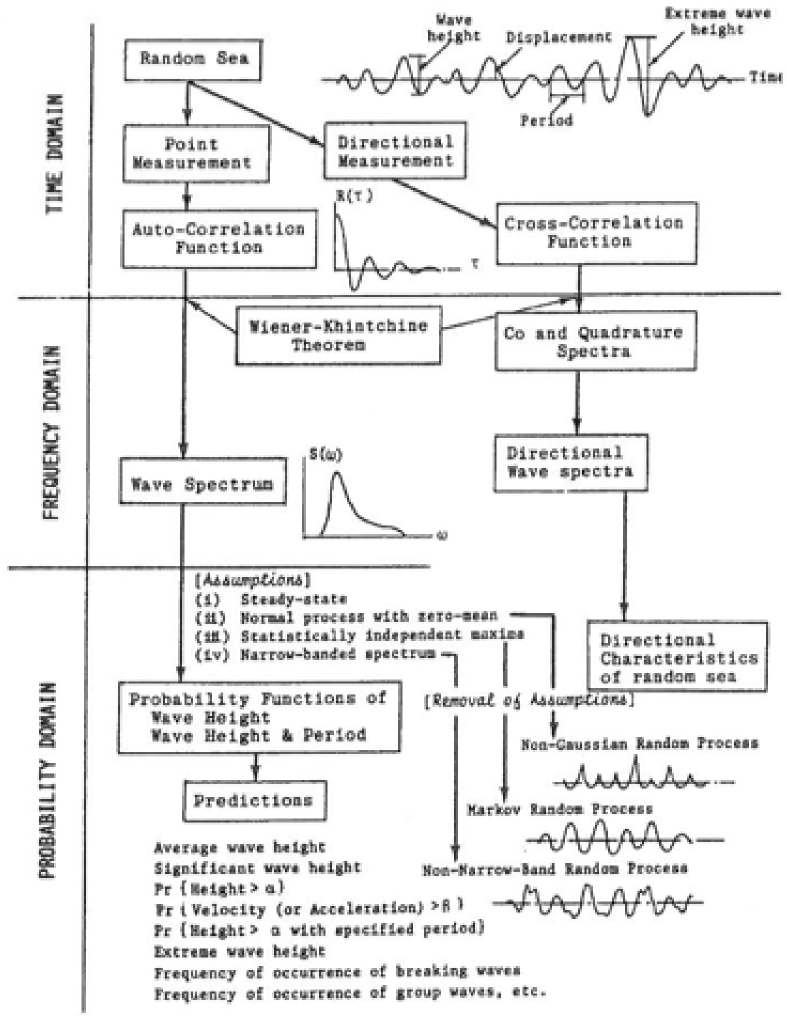
$$(\nabla^2 + i\partial_t)\Psi = \kappa|\Psi(\mathbf{r},t)|^2\Psi(\mathbf{r},t) + s(\mathbf{r},t).\ \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

  - where Ψ is the wave function, which is expressed by wave period t and space r≡(x,y,z),
- The equation represents a model in which a wave interacts with its own energy |Ψ|^2.
- The coefficient κ determines the strength of this “self-interaction”
- It is noteworthy that all classes of NLS are phenomenological in origin, and their results can be only confirmed experimentally
- The equation cannot be derived and its solution cannot be measured directly, but it provides one of the most accurate models for characterizing the freak waves

- NLS can’t describe the freak waves that are caused by
  - Diffraction effect: occurs when a collection of small diffracting waves coherently combine in phase to produce a freak wave.
  - Current focusing: occurs due to the interference between the storm force waves that are driven from opposite directions.

Summary

- Wave spectrum is needed to derive ship responses
- Stochastic loads can be assessed using spectral methods
- When you know the spectrum, you can define the maximum response (probability theory for stochastic processes)



Thank you !!