

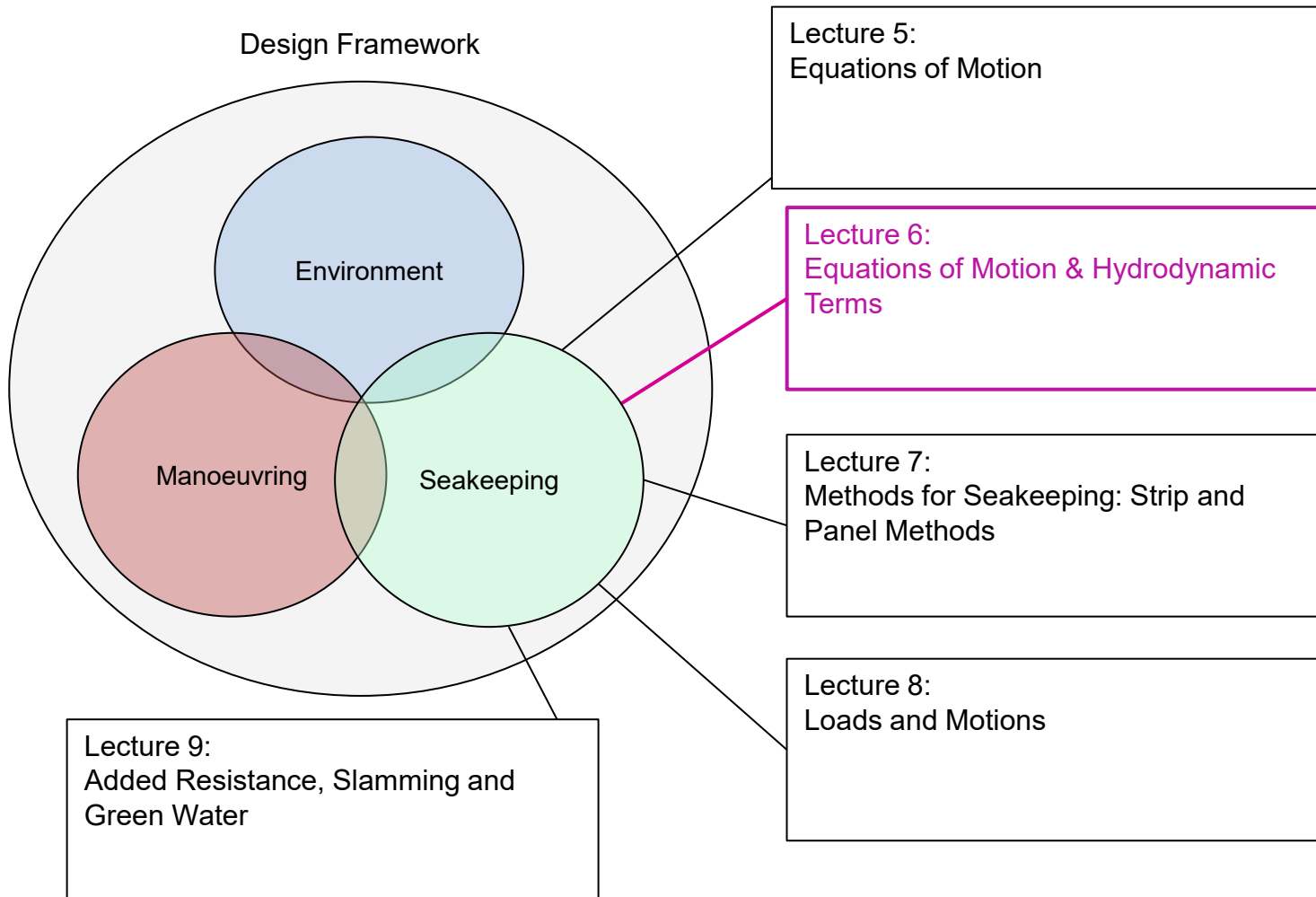
Aalto University

School of Engineering

MEC-E2004 Ship Dynamics (L)

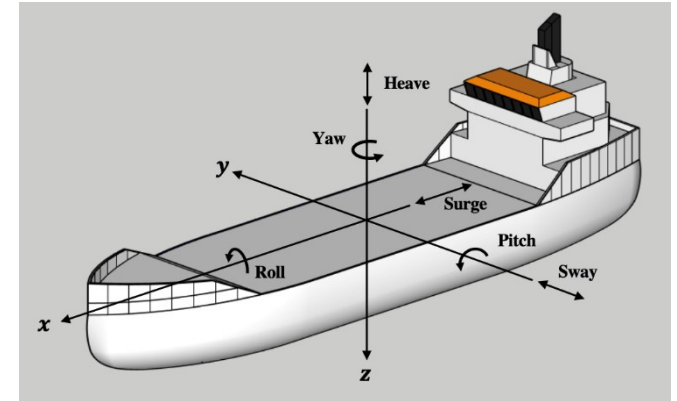
Lecture 6 – Equations of Motion (Part II)

Where is this lecture on the course?



Contents

- **Aim: To understand the physics of the components of rigid ship equations of motion in waves**
 - Basic Hydrodynamic phenomena (Radiation/Diffraction)
 - Components of Hydrodynamic EoM
 - Special cases
 - Hydrodynamic modelling
 - The experimental determination of motion components
- **Literature**
 - Journee, J.M.J., "Introduction to Ship Hydromechanics"
 - Lloyd, A.R.J.M, "Seakeeping – Ship Behavior in Rough Weather", John Wiley & Sons
 - Bertram, V., "Practical Ship Hydrodynamics", Butterworth-Heinemann, Ch. 4.
 - Matusiak, J., "Ship Dynamics", Aalto University
 - Lewis, E. V. Principles of Naval Architecture. Vol. 3, "Motions in waves and controllability"
 - Rawson, K. J., "Basic Ship Theory. Volume 2, Ship dynamics and design - ch.12 Seakeeping & ch.13 Manoeuvrability"



Motivation

❑ Ship behaviour in the seaway is affected by

- Sea state
- Control with rudders, propulsors etc.
- Cargo and general arrangement

❑ Motions and loads are evaluated by

- Spectral methods
- Linear dynamics
- Potential flow methods

❑ Nonlinear methods are useful when motions are excessive

- Time-domain hydrodynamics
- Specific sea states/time-frames by using different time histories (with same spectrum, phase shift)
- CFD methods are still under development

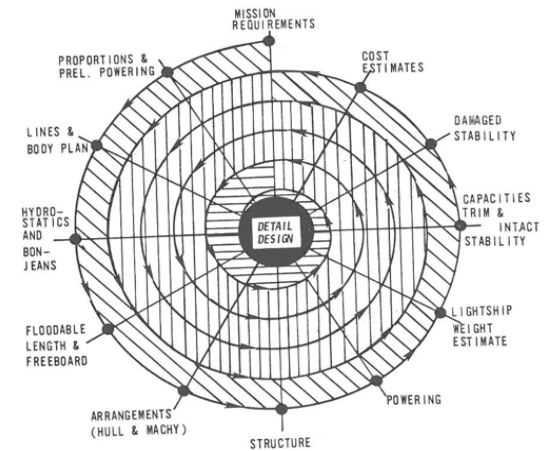
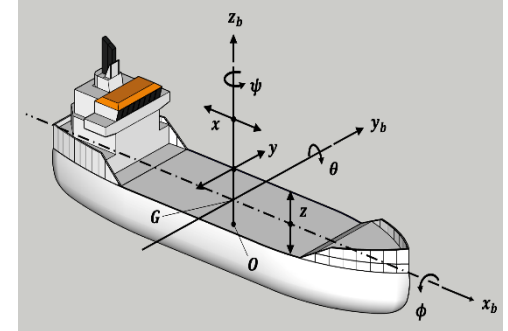


Fig. 1 Basic design spiral



Assignment 3

- **Grades 1-3:**
 - ✓ Select a book-chapter related to the ship equations of motion and read it
 - ✓ Identify the main components associated to equations of motion of your ship. How do they relate with the ship's mission (*think in operational safety terms*)
 - ✓ Discuss how the general arrangement, hull form and operational profile of your ship affect the equations of motion (*think in design for safety terms*).
 - ✓ Start using motions and loads design software (e.g. MaxSURF, Napa, etc.) and reflect how the software is related to the theory of ship motions.
- **Grades 4-5:**
 - ✓ Read 1-2 scientific journal articles related to Ship Equations of Motion
 - ✓ Reflect these in relation to knowledge from books and lecture slides
- Report and discuss the work.



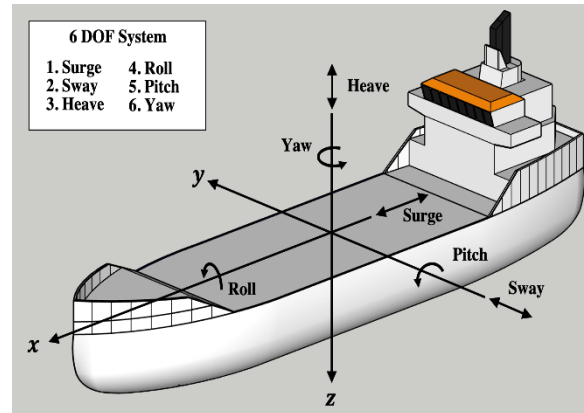
Key principles

Inputs



Fluid actions from
wind and / waves

System



6 DOF ship system

Output



Ship motions and
wave induced loads

- Water is denser and more viscous than air (i.e. damping is increased)
- Added mass** represents the amount of **total fluid** accelerated by the object

Ship Equations of motion - Regular waves

Hydro-damping

Sinusoidal excitation

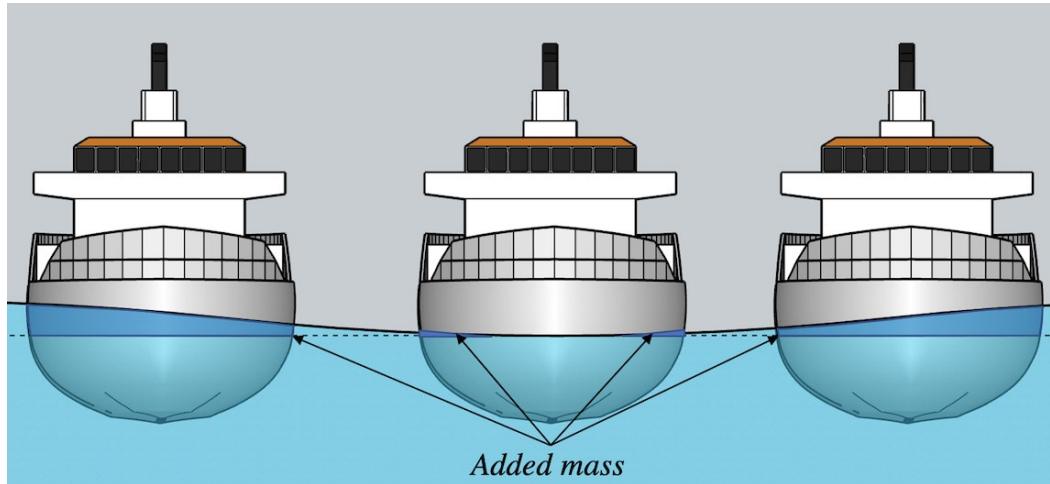
$$(a + m)\ddot{x} + b\dot{x} + cx = F_0 \sin(\omega t)$$

added mass

stiffness

- ❑ **Added mass & Hydro-damping** are functions of the frequency of oscillation.
- ❑ **Added mass** depends on the shape of the object and the type /direction of motion (linear or rotational)
- ❑ **Hydrodynamic damping** depends on fluid viscosity (frictional drag) and waves
- ❑ Each degree of freedom that has a restoring force has an associated natural frequency. These natural frequencies depend on the mass and stiffness properties of the system.

Hydrodynamic Forces



- The forces due to added mass and damping are **hydrodynamic forces**. They arise from pressure distribution around the oscillating hull.
- In linear hydrodynamic theory the hydrodynamic force has a component proportional to acceleration (i.e. added mass) and a component proportional to the velocity (i.e. damping coefficient).
- The following forces are used to obtain coefficients in the equations of motion :
 - **Radiation forces (or moments)**
 - **Incident wave or Froude - Krylov forces (or moments)**
 - **Diffraction forces (or moments)**

Hydrodynamic Forces

- ❑ **Radiation forces (or moments)** where the ship is assumed to oscillate in still water and the hydrodynamic added inertia and damping coefficients are determined
- ❑ **Incident wave or Froude - Krylov forces (or moments)** where the wave is considered in the absence of the ship and the corresponding wave forces (or moments) acting on the ship are determined. **NB:** In linear hydrodynamics we assume small displacements, i.e. “true”, wetted surface is not considered.
- ❑ **Diffraction forces (or moments)** where the effects of the presence of the ship on the waves are considered and the corresponding diffracted wave forces (or moments) are determined.
- ❑ For a ship with port-starboard symmetry the coupled motions of **heave – pitch** and **sway – roll – yaw** can be examined separately during seakeeping analysis. **Of these five motions only heave pitch and roll have a restoring force or moment.**

Fundamental EoM (Newton's Law)



$$(a + m)\ddot{x} + b\dot{x} + cx = F_0 \sin(\omega_e t)$$

The LHS of EoM contains the dynamic properties of the ship and thus the forces on the ship when it is forced to oscillate in still water conditions (**hydro-mechanical**)

- Structural Mass (M) and Added Mass (a)
- Damping (b)
- Restoring forces (c)



$$[-\omega_e^2(M + A) + i\omega_e N + S]\hat{u} = \hat{F}_e$$



The RHS of EoM contains the forces on the ship when it is restrained from motion and subjected to regular waves (**wave exciting**)

- Incident wave Froude-Krylov
- Diffraction

$$RAO = \frac{F_{e,0}}{N - (M + A(\omega_e)) \times \omega_e^2 + iB(\omega_e)\omega_e}$$

The RAO expresses the response. It is a frequency dependent function (also known as FRF). It is a complex number, i.e. it comprises of an amplitude and a phase; where i expresses the imaginary unit of this number. It is common to consider the absolute number of the RAO in linear seakeeping.

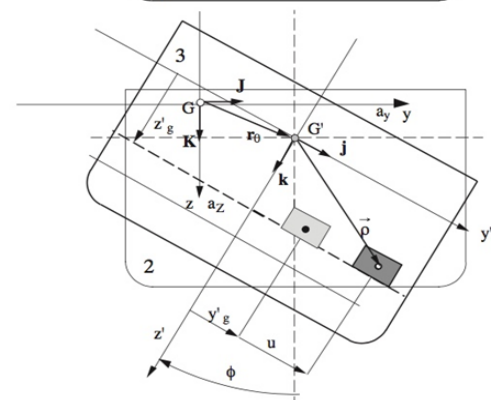
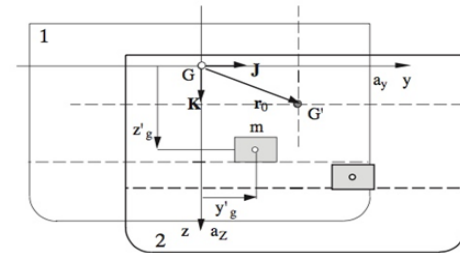
Structural Mass (M)

- ❑ Total mass consists of the ship structure, equipment, cargo, etc. and we assume perfectly symmetric hull along the longitudinal direction (y – axis system)
- ❑ 6 by 6 matrix corresponding to 6 dof
- ❑ Mass distributions couple with global ship motions and local actions such as sloshing liquid in tanks, openings such as swimming pools, cargo moving containers, cars.
- ❑ If we assume that the mass is perfectly attached to the CoG the cargo movement must be assessed with separate models.
- ❑ The mass moments of inertia θ relate to the origin of the ship fixed coordinate system and defined as:

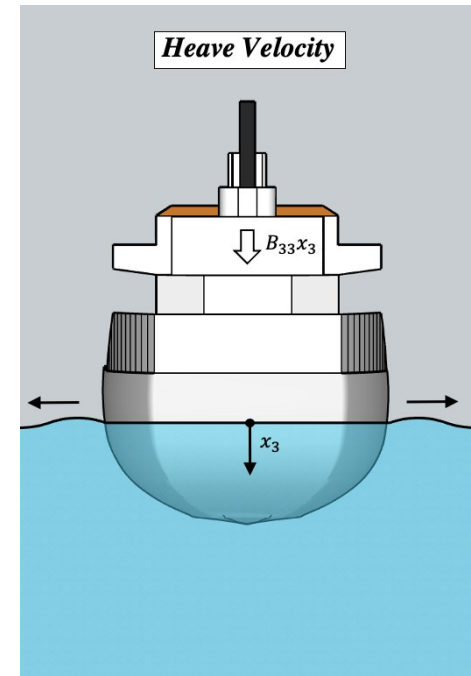
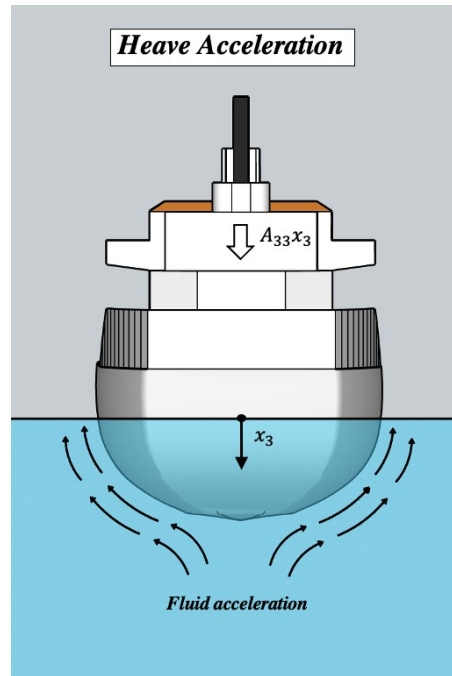
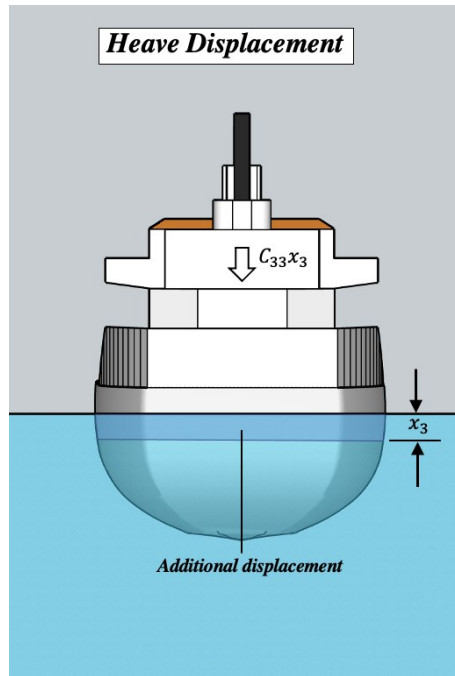
$$\theta_{xx} = \int (y^2 + z^2) dm; \quad \theta_{xz} = \int xz dm; \quad \text{etc.}$$

$$[-\omega_e^2(M + A) + i\omega_e N + S]\hat{u} = \hat{F}_e$$

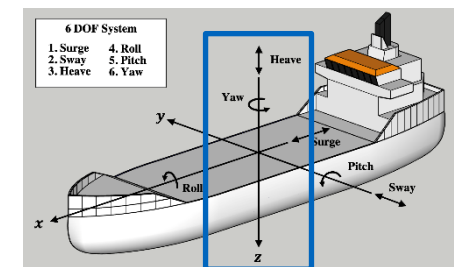
$$M = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & 0 \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & 0 & -mx_g & 0 \\ 0 & -mz_g & 0 & \theta_{xx} & 0 & -\theta_{xz} \\ mz_g & 0 & -mx_g & 0 & \theta_{yy} & 0 \\ 0 & mx_g & 0 & -\theta_{xz} & 0 & -\theta_{zz} \end{bmatrix}$$



Heave Motion



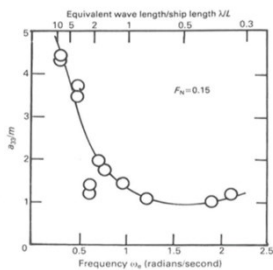
- Heave motion occurs in the z-direction and for the uncoupled case, the amount of additional displacement (water mass) during a wave-loading event causes the ship to accelerate along the z-axis due to buoyant forces
- Fluid acceleration around the hull cause waves to radiate



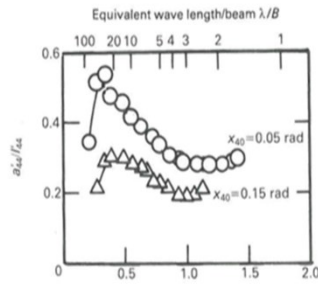
Added Mass (A)

- **Added mass** relates to the motion of fluids along with the ship. This affects the entire fluid domain, but the largest effect is at the neighborhood of ship.
- An easy way to define A : *the liquid that has the same acceleration and phase as the ship.*

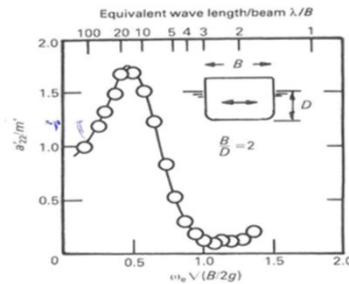
$$(A + M)\ddot{x} + B\dot{x} + Cx = F_0 \sin(\omega_e t)$$



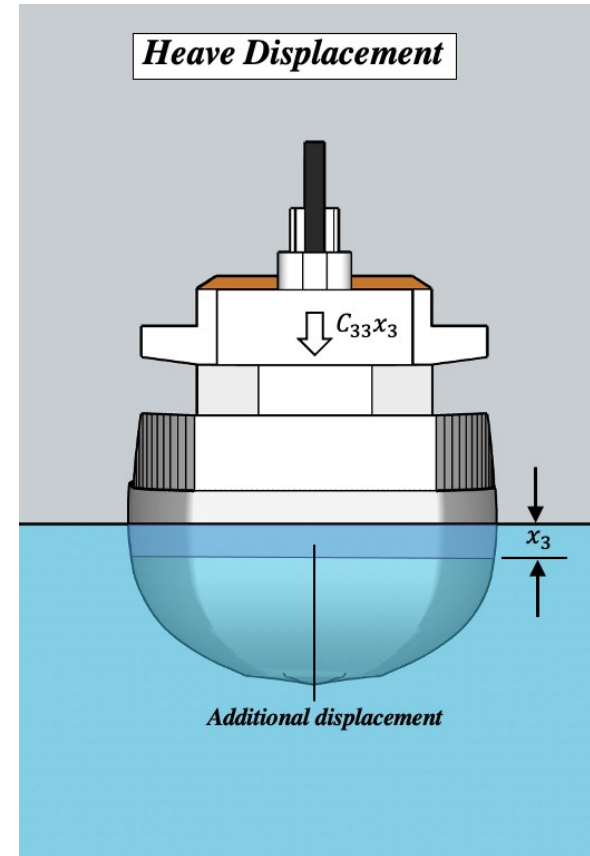
A_{heave}



A_{roll}



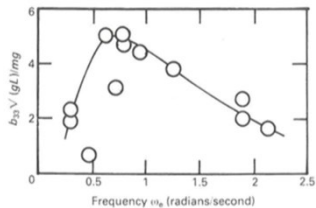
A_{sway}



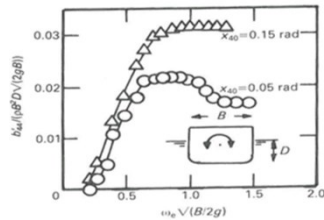
Damping (C)

- Damping (loss of energy) is caused by ship-induced waves. The waves decay at some distance from the ship
- Damping can be increased by appendices or the effects of viscosity
- The damping matrix mainly contains the effects of radiated waves.

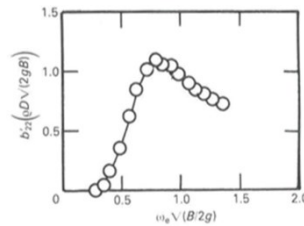
$$(A + M)\ddot{x} + B\dot{x} + Cx = F_0 \sin(\omega_e t)$$



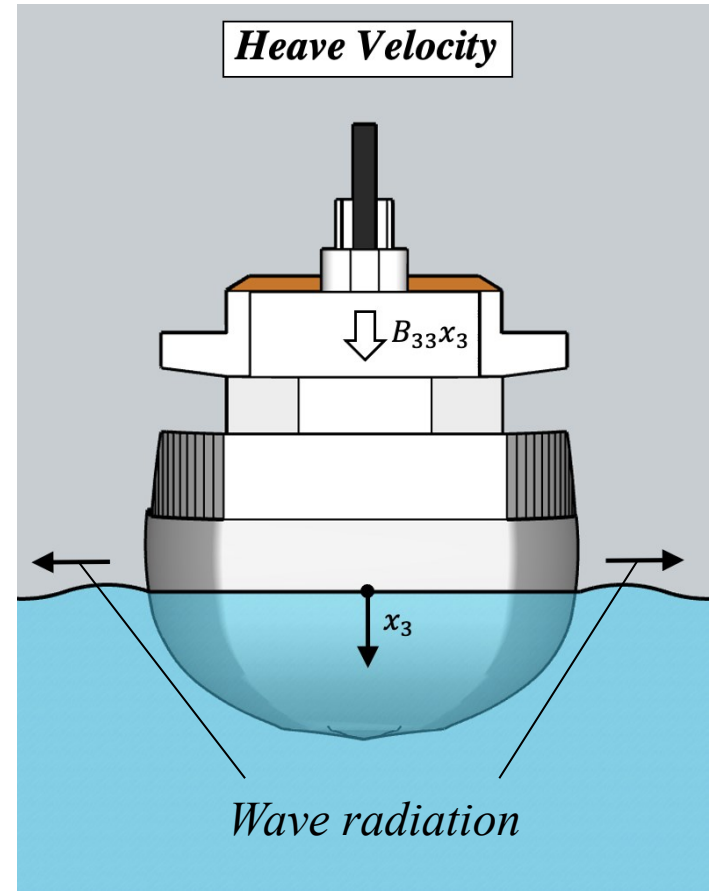
C_{heave}



C_{roll}



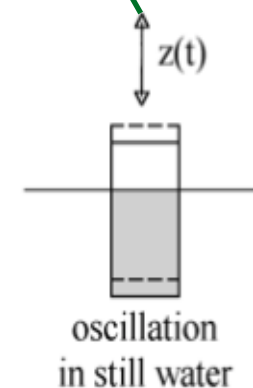
C_{sway}



Restoring matrix (C)

- ❑ The restoring forces are analogous to spring stiffness in mechanical systems
- ❑ Symmetry of the ship → main terms are zero
- ❑ When small displacements are considered, these can be derived from the hydrostatics
- ❑ The idealization presented assumes no contributions from transom stern and zero forward speed
- ❑ \overline{GM} : metacentric height ; \overline{GM}_L : longitudinal metacentric height calculated w.r.t. the origin of the coordinate system (usually amidships and not at WL), x_w the x coordinate of the center of the WL, A_w the WL area , ω_g the circular eigenfrequency of the control motions

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g A_w & 0 & -\rho g A_w x_w & 0 \\ 0 & 0 & 0 & gm\overline{GM} & 0 & 0 \\ 0 & 0 & -\rho g A_w x_w & 0 & gm\overline{GM}_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{zz}\omega_g^2 \end{bmatrix}$$

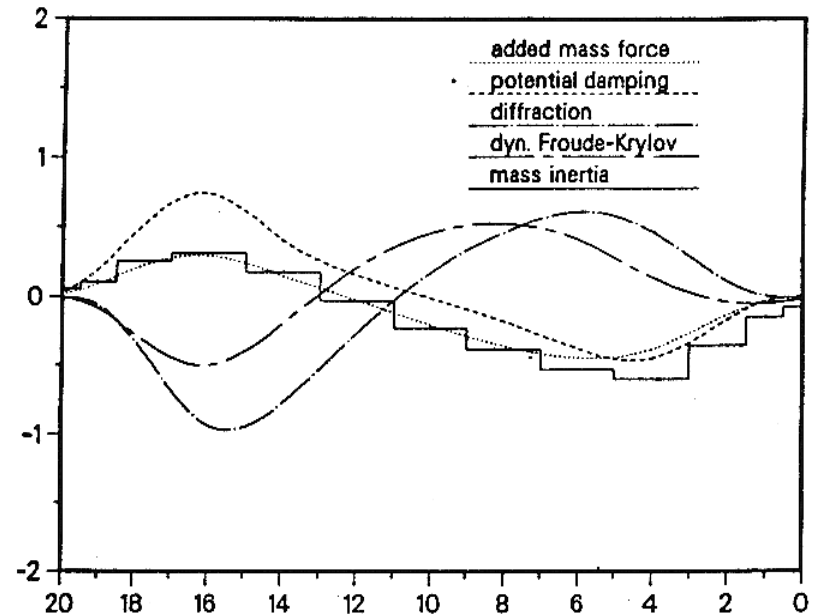


$$\theta_{xx} = \int (y^2 + z^2) dm$$

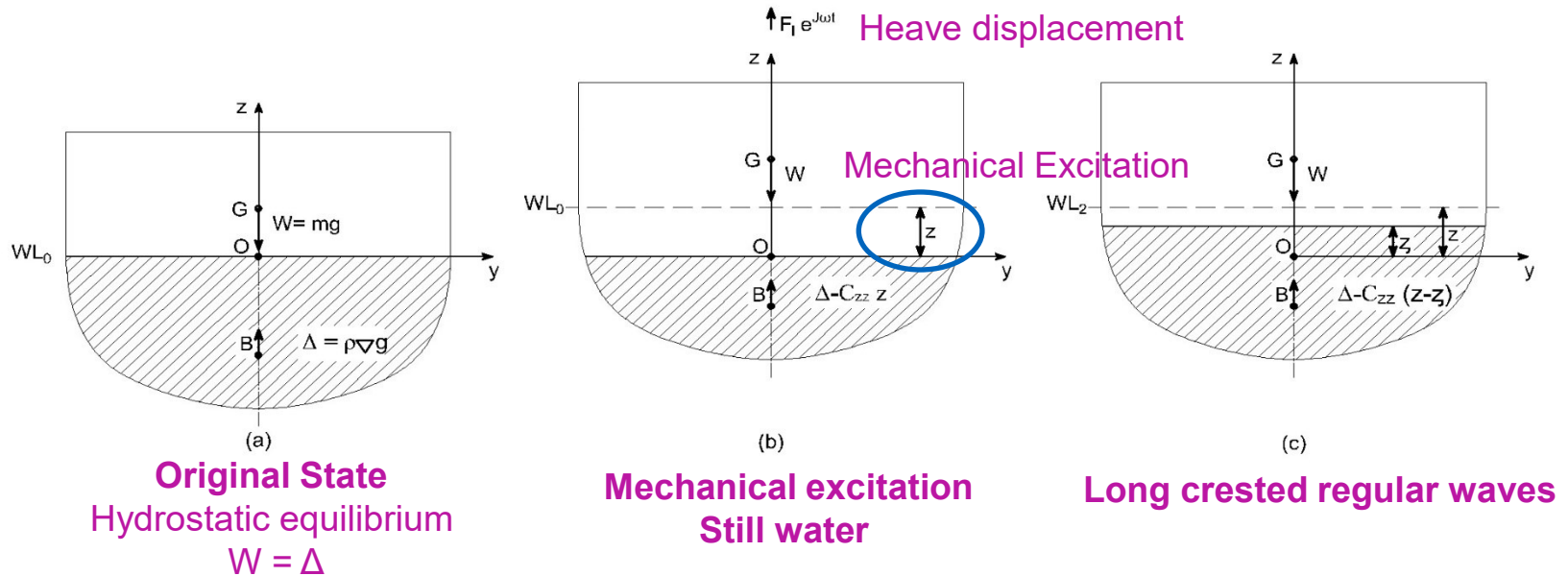
$$\theta_{xz} = \int xz dm$$

Froude-Krylov and Diffraction forces

- **Froude–Krylov force** : Pressure in way of the undisturbed waves integrated over the wetted surface of the ship
- **Diffraction forces** : Pressures that occur due to the disturbances in the water because of the presence of the ship
- We assume small displacements, i.e. “true”, wetted surface is not considered in linear hydrodynamics.
- J. Matusiak. (2011). **On the non-linearities of ship’s restoring and the Froude-Krylov wave load part**. International Journal on Naval Architecture and Ocean Engineering. pp: 111-115. [Journal Link](#)



Uncoupled Heave Motion



Let us assume that a ship in still water is subject to a mechanical excitation in the form of an upward force $F_z(t)$ leading to heave displacement $z(t)$. The linear equation of motion for this 1 DOF system will be:

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z(t)$$

Uncoupled Heave Motion

For a sinusoidally varying mechanical excitation $F_z(t) = F_1 e^{j\omega t}$ Assuming F_1 is a force vector of constant amplitude the response will also be sinusoidal namely $z(t) = Z e^{j(\omega t - \varepsilon)}$ where Z is the amplitude of excitation and ε the phase lag of the response. Accordingly:

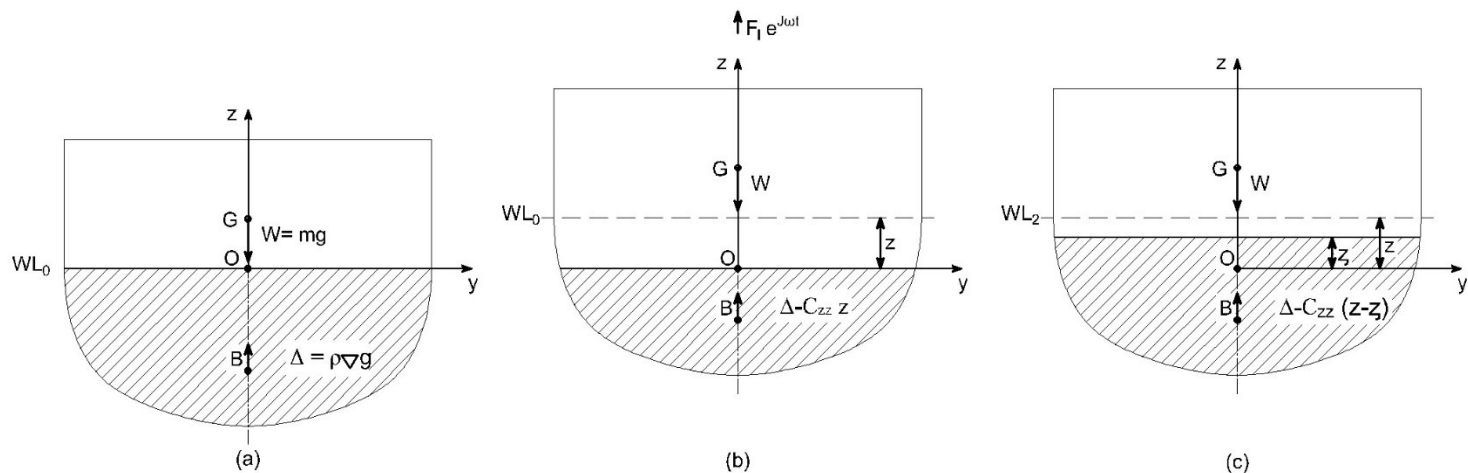
$$Z = \frac{F_1}{\sqrt{(C_{zz} - \omega^2 M_{zz})^2 + (\omega N_{zz})^2}} \quad \text{and} \quad \tan(\varepsilon) = \frac{\omega N_{zz}}{(C_{zz} - \omega^2 M_{zz})}$$

- ✓ $C_{zz} = \rho g A_w z$ is the hydrostatic heave restoring force with ρ representing the water density (kg/m^3);
- ✓ g the acceleration of gravity (m/s^2)
- ✓ A_w the still water lane area (m^2).
- ✓ N_{zz} is the heave damping force
- ✓ $M_{zz} = m + m_{zz}$ is the virtual mass of the ship ($m = \rho \nabla$ and m_{zz} = heave added mass)

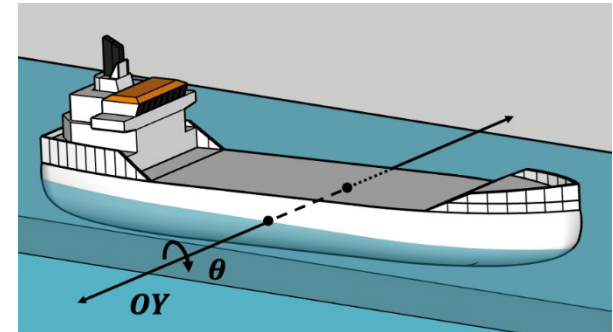
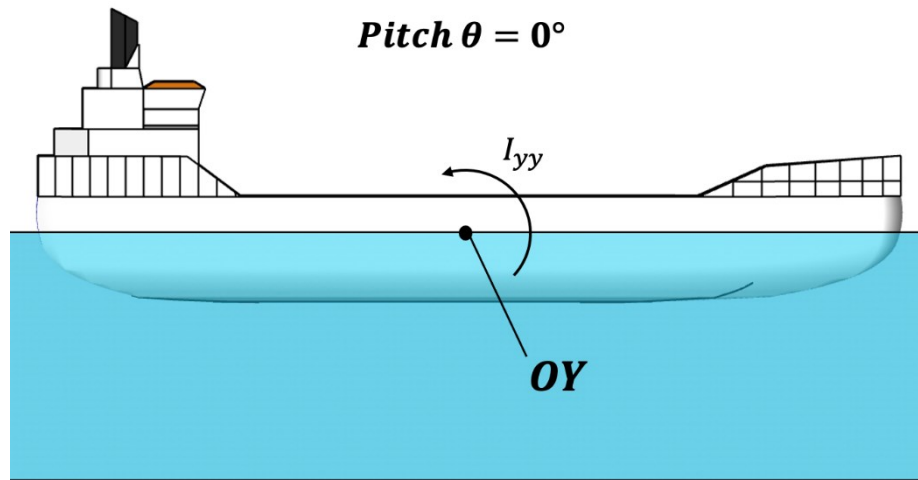
Uncoupled Heave Motion

If we assume free motions in waves it is possible to assume a constant value of added mass namely $\bar{m}_{zz} = m_{zz}(\omega_e \rightarrow \infty)$ and therefore the heave natural frequency in water is:

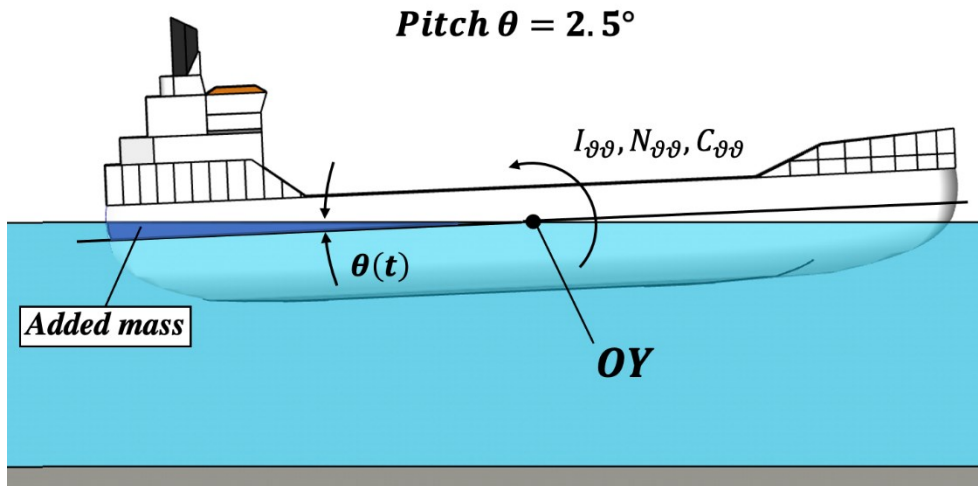
$$\omega_{3n} = \sqrt{\frac{c_{zz}}{m + m_{zz}}}$$



Uncoupled Pitch Motion (1/2)



Rotation about OY is only degree of freedom



When the ship rotates and introduces added mass there is a corresponding effect to the EoM coefficients

Uncoupled Pitch Motion (2/2)

- If we consider that the ship is an 1DOF system subject to pitch excitation namely $\theta(t)$

$$[I_{yy} + I_{\theta\theta}(\omega_e)]\ddot{\theta} + N_{\theta\theta}(\omega_e)\dot{\theta} + C_{\theta\theta}\theta = \bar{M}_{\theta}(\omega, \omega_e)e^{j(\omega_e t + u)}$$

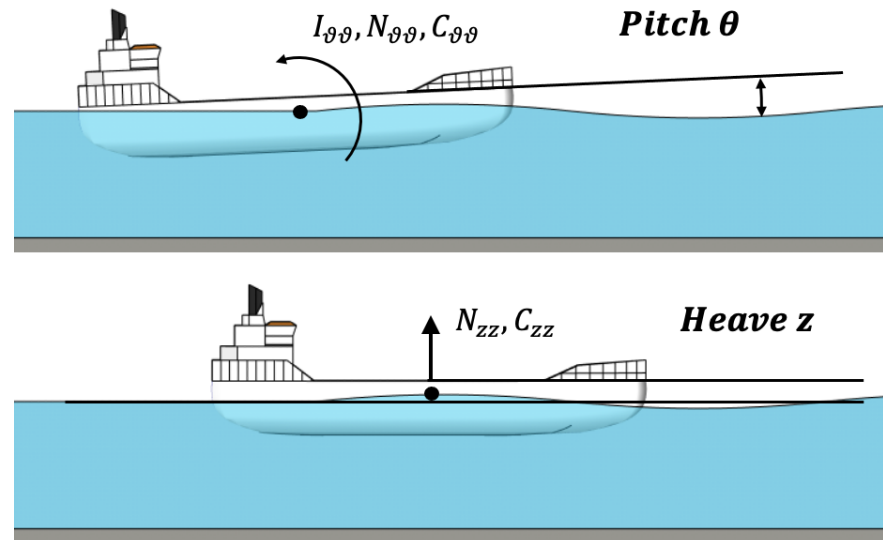
- ✓ I_{yy} is the mass moment of inertia about axis Oy
 - ✓ $I_{\theta\theta}$ is the pitch added mass moment of inertia
 - ✓ $N_{\theta\theta}$ is the pitch damping coefficient
 - ✓ $C_{\theta\theta} = \rho g I_{long}$ for $I_{long} =$ longitudinal 2nd moment of water plane area
 - ✓ \bar{M}_{θ} is the amplitude of the wave excitation vector
- The pitch natural frequency in water can be approximated as

$$\omega_p = \sqrt{\frac{c_{\theta\theta}}{I_{yy} + \bar{I}_{\theta\theta}}} \quad \bar{I}_{\theta\theta} = I_{\theta\theta} \text{ as } t \rightarrow \infty$$

Coupling Heave & Pitch

$$\begin{bmatrix} m + m_{zz} & m_{z\theta} \\ m_{\theta z} & I_{yy} + I_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} N_{zz} & N_{z\theta} \\ N_{\theta z} & N_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_{zz} & C_{z\theta} \\ C_{\theta z} & C_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} z \\ \theta \end{bmatrix} = \begin{bmatrix} F_z(\omega, \omega_e) e^{j(\omega_e t + \psi)} \\ \bar{M}_\theta(\omega, \omega_e) e^{j(\omega_e t + u)} \end{bmatrix}$$

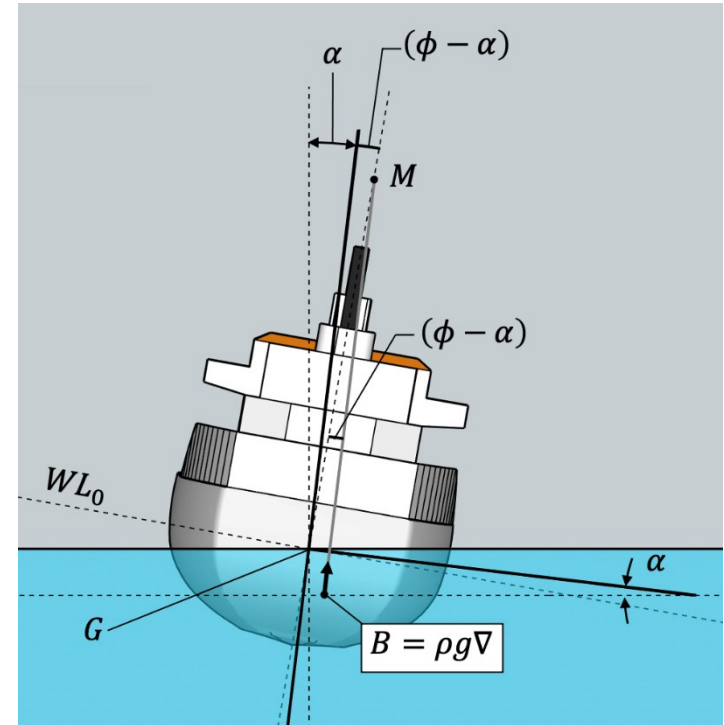
- In addition to heave added mass m_{zz} and pitch added inertia $I_{\theta\theta}$ we have **heave into pitch** (and **pitch into heave**) added mass terms namely $m_{z\theta}$ and $m_{\theta z}$
- In addition to heave damping N_{zz} and pitch damping $N_{\theta\theta}$ coefficients we have the heave into pitch (and pitch into heave) terms defined as $N_{z\theta}$ and $N_{\theta z}$ respectively.
- The heave into pitch restoring terms are defined as $C_{z\theta} = C_{\theta z} = \rho g M_l$
- $M_l = \int_L xB(x)dx$ represents the longitudinal first moment of water plane area and $B(x)$ is the beam in way of the water line.



Roll – small amplitudes

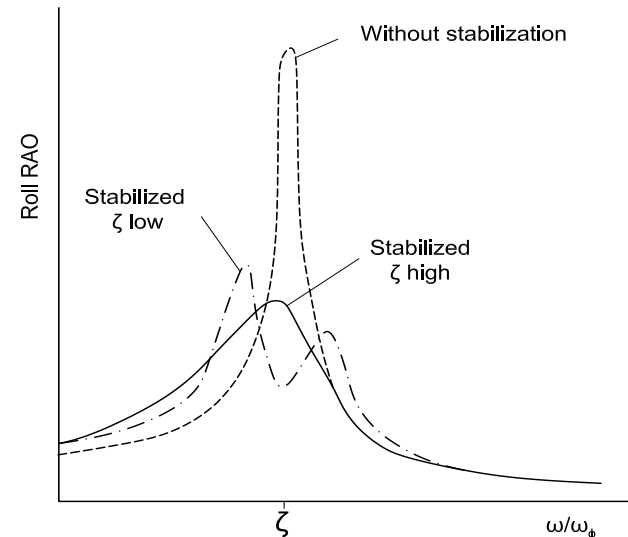
$$[J_{xx} + I_{\phi\phi}(\omega)]\ddot{\phi} + N_{\phi\phi}(\omega)\dot{\phi} + C_{\phi\phi}\phi = K_{\phi}(t) = K_1 e^{j\omega t}$$

- $K_{\phi}(t)$ is a sinusoidal mechanical excitation producing a rolling moment $K_{\phi}(t) = K_1 e^{j\omega t}$ ϕ is the angle of roll
- I_{xx} is the mass moment of inertia about the longitudinal axis through the center of mass
- $C_{\phi\phi} = \Delta GM_{\tau} = \rho g \nabla GM_{\tau}$ is the hydrostatic roll restoring coefficient
- $I_{\phi\phi}$ = roll added inertia (frequency of oscillation dependent)
- $N_{\phi\phi}$ = roll damping coefficient due to hydrodynamic effects associated with fin and tank stabilizers



Roll – large amplitudes

- Large amplitude of roll may cause discomfort compared to other motions.
- The amount of damping which is provided by the fluid is not always sufficient to reduce the roll amplitude to acceptable levels.
- Solution 1 : passive systems which make use of the roll motion and do not require any power source and control system
- Solution 2 : active systems which use power to move masses or control surfaces and a control system

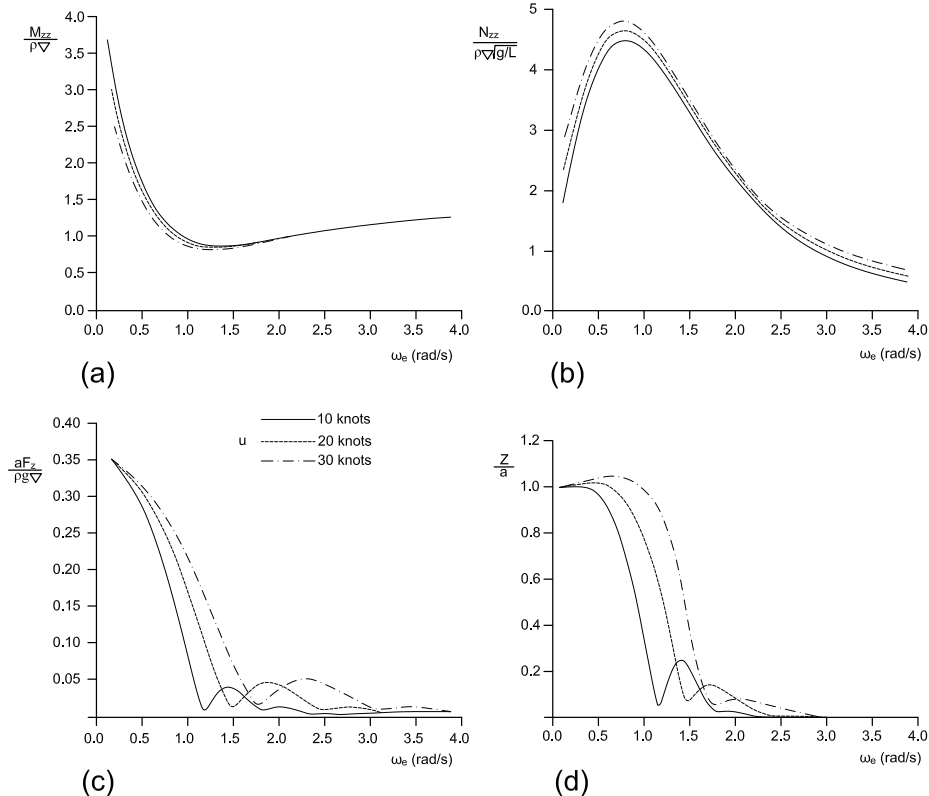


C_1 and C_3 decrease with virtual mass moment of inertia and restoring moment whilst C_2 increases the damping. C is associated with the lift generated by the fin stabilizers

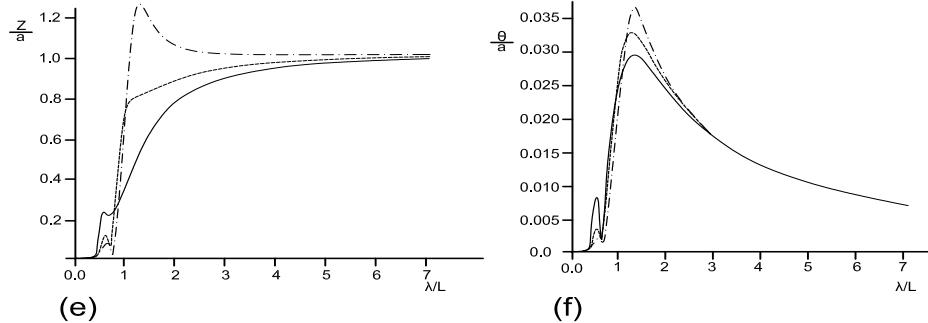
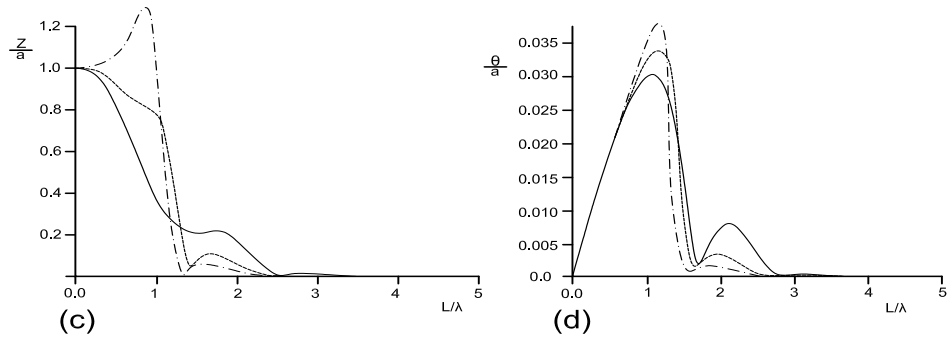
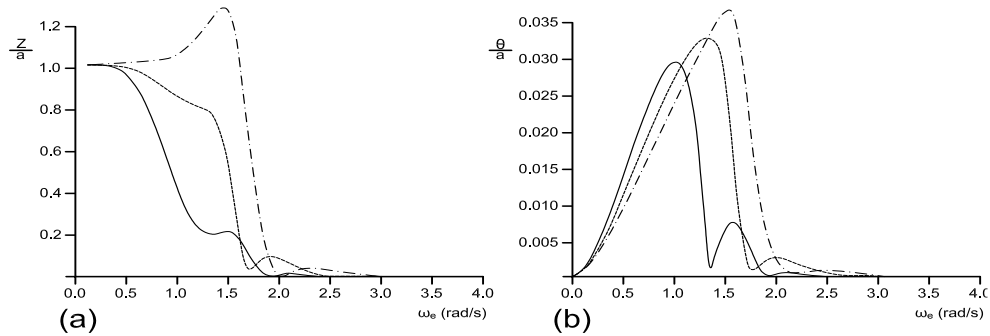
$$[I_{xx} + I_{\phi\phi}(\omega) - CC_1]\ddot{\phi} + (N_{\phi\phi} + CC_2)\dot{\phi} + (C_{\phi\phi} - CC_3)\phi = K_\phi(t)$$

Ship Response

Variation of the nondimensionalized heave added mass, damping coefficient and excitation amplitude (for head waves of amplitude 1m)



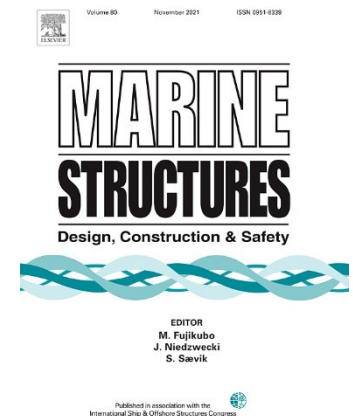
RAO demo for Heave & Pitch (Head waves)



— 10 knots - - - 20 knots - · - 30 knots

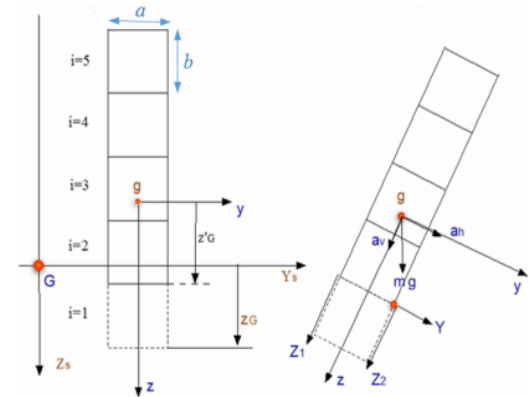
Cargo induced accelerations – Key publications

- P. Ruponen, J. Matusiak, J. Luukkonen, M. Ilus. (2009). **Experimental Study on the Behavior of a Swimming Pool Onboard a Large Passenger Ship.** Marine Technology, Vol 26, No. 1: pp. 27-33.
- J. Matusiak. (2000). **Dynamics of Cargo Shift Onboard a Ship in Irregular Beam Waves.** Int. Shipbuilding Program, Vol 47, No. 1: pp: 77-93.
- M. Acanfora, J. Montewka, T. Hinz, J. Matusiak. (2017). **On the Estimation of the Design Loads on Container Stacks Due to Excessive Acceleration in Adverse Weather Conditions.** Marine Structures Vol 53: pp105-123.
- BLU Code and BLU Manual- Code of practice for the safe loading and unloading of bulk carriers published by IMO

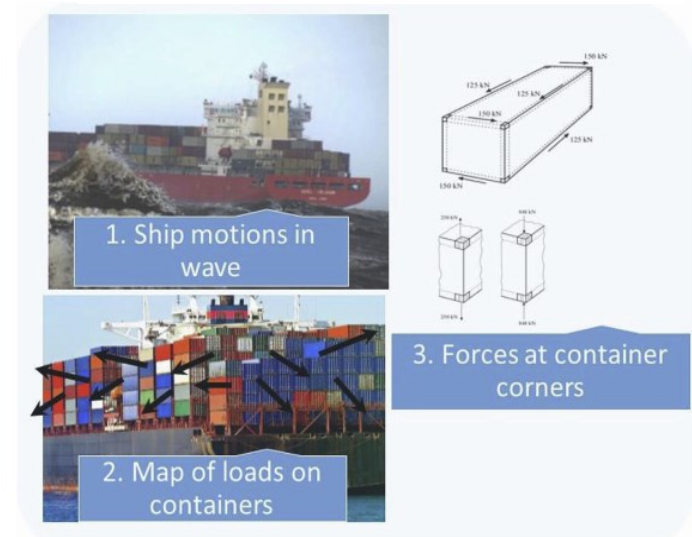


Cargo-induced accelerations - Practical example

- In recent years, many containership accidents have occurred that are coupled with violent ship motions.
- **Averse weather conditions** and **poor lashing** leading to domino effects of container stacks in waves.
- This is why the determination of accurate design accelerations and the evaluation of loads on container stacks are of primary importance for safety.
- Designers are interested to introduce safe container loadings and adequate dimensioning of the lashing equipment.
- This can be understood better by coupling seakeeping with cargo securing equations leading to avoidance of cargo shifting.

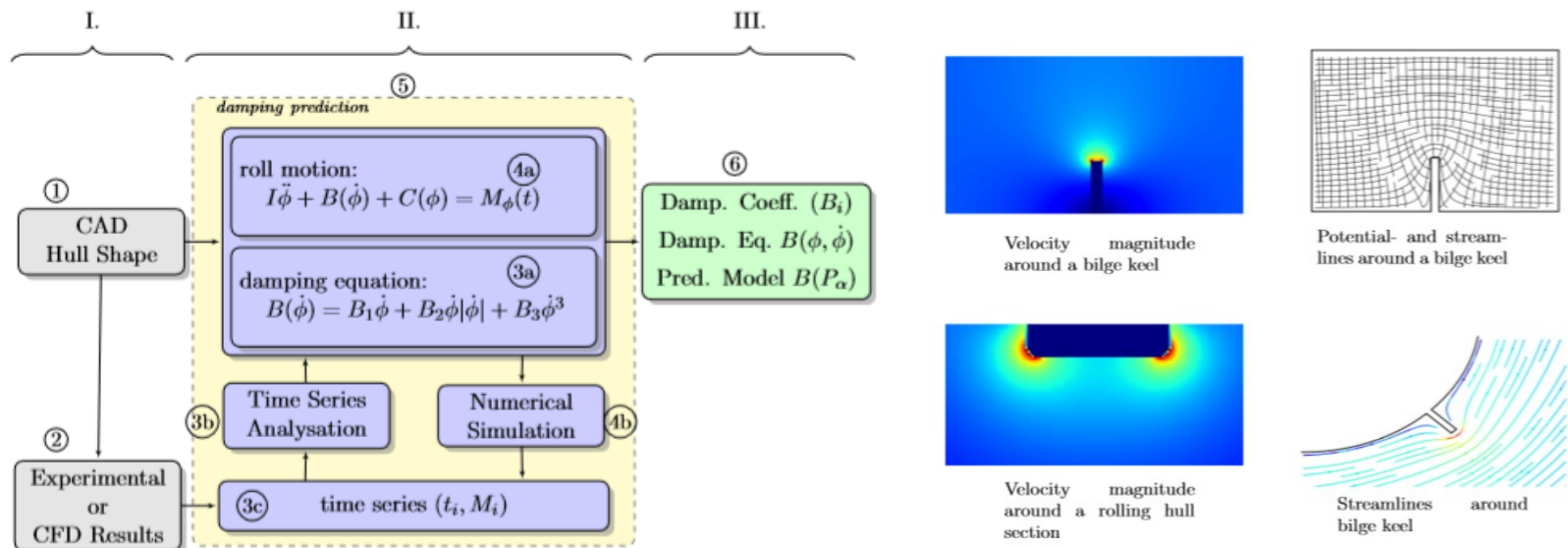


Forces at container stack – conventional arrangement.



The value of experimental methods

- The coefficients for the EoM need to be defined in order to obtain the solutions
- We can do this either computationally or experimentally
- Computational means are time efficient but not 100% accurate
- Model tests are an efficient way to determine the coefficients. However, they are expensive to carry out and sensitive to uncertainties



Summary

- ❑ The purpose of this lecture has been to give an overview on how the linear equations of motion for rigid ships are derived.
- ❑ For sinusoidal excitation we can obtain sinusoidal response with
 - ✓ Phase differences
 - ✓ Amplitude of response directly proportional to amplitude of input
- ❑ Coupling ship motions with cargo is a challenging problem that relates with ship safety
- ❑ The coefficients for equations of motion can be obtained computationally. However, using experimental techniques is important in terms of managing uncertainties
- ❑ Once we know the response for regular seas, we can move to irregular seas

