Lecture 6 Ship motions in regular waves

1. Introduction

Water is a dense and viscous fluid. Suitable idealization of ship motions is inextricably linked with wave induced hydrodynamics and associated floating body accelerations. Hydrodynamic actions are facilitated in the equations of motion as an addition to the mass of the object. This is known as the **added mass effect**. The added mass represents the weighted integration of the entire fluid mass effected by the accelerating object. Accordingly, Newton's equation of motion can be simplified to read:

$$(a+m)\ddot{x} + b\dot{x} + cx = F_0 sin(\omega t) \qquad (6\text{-}1)$$

where *a* stands for added mass, *b* is the hydrodynamic damping, *c* is the stiffness, F is the excitation due to external environment (assumed herby sinusoidal) and the x - variables represent the response (acceleration $\ddot{x}$, velocity $\dot{x}$ and displacement x).

Both added mass and hydrodynamic damping coefficients are a function of the frequency of oscillation. However, the **added mass** depends primarily on the shape of the object, the type of motion (linear or rotational), and the direction of the motion. In this way, it differs from mass which is a quantity independent of motion. **Hydrodynamic damping** is related to the viscosity of the fluid (and hence the frictional drag). However, when a free surface is involved the damping is dominated by the generation of waves. The larger the waves generated, the larger the hydrodynamic damping.

Each degree of freedom that has a restoring force has an associated natural frequency. So, for a ship, there is a natural frequency in heave, roll, and pitch. These natural frequencies depend on the mass and stiffness properties of the system. For a ship with port-starboard symmetry (e.g. typical ocean going or naval vessel) the coupled motions of heave – pitch and sway – roll – yaw can be examined separately during seakeeping analysis. Of these five motions only heave pitch and roll have a restoring force or moment. The forces provided due to the effects of added mass and damping are referred to as hydrodynamic forces. They arise from pressure distribution around the oscillating hull.

In the following sections the equations of motion heave and pitch and coupled heave pitch and roll are discussed. The aim is to provide an introduction to the seakeeping problem. The lecture also discusses aspects of relevance to the basic mathematical modelling of roll motions and ship stabilisation. The material presented is based on the references by Betram (2000), Matusiak (2021), personal communication with Temarel (2018), and the author's personal knowledge. Students with keen interest on the influence of motion coupling may refer to the papers by Ancafora (2017), Matusiak (2011), Matusiak (2000), Ruponen et al. (2009) included in the list of references (see section 8).

2. Uncoupled heave motion

Let us consider the case of a ship in still water which is subject to a mechanical excitation in the form of an upward force $F_z(t)$ leading to heave displacement $z(t)$. According to the theory explained in Lecture 5 the linear equation of motion for this 1-DOF system in naval architecture terms can be expressed as:

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z(t) \tag{6-2}$$

For a sinusoidally varying mechanical excitation $F_z(t) = F_1 e^{j\omega t}$. Assuming F_1 is a force vector of constant amplitude the response will also be sinusoidal namely $z(t) = Ze^{j(\omega t - \varepsilon)}$ where Z is the amplitude of excitation and ε the phase lag of the response. Accordingly,

$$Z = \frac{F_1}{\sqrt{(C_{zz} - \omega^2 M_{zz})^2 + (\omega N_{zz})^2}} \quad \text{and} \quad tan\varepsilon = \frac{\omega N_{zz}}{(C_{zz} - \omega^2 M_{zz})} \tag{6-3}$$

where :

- $C_{zz}z = \rho g A_w z$ is the hydrostatic heave restoring force (see Figure 6-1) with ρ representing the water density (kg/m^3) ; g the acceleration of gravity (m/s^2) and A_w the still water lane area (m^2).'
- $N_{zz}\dot{z}$ is the heave damping force provided by the surrounding water
- N_{zz} is the heave damping coefficient.
- $M_{zz} = m + m_{zz}$ is the virtual mass of the ship where the mass of the ship is $m = \rho\nabla$ and m_{zz} is the heave added mass.

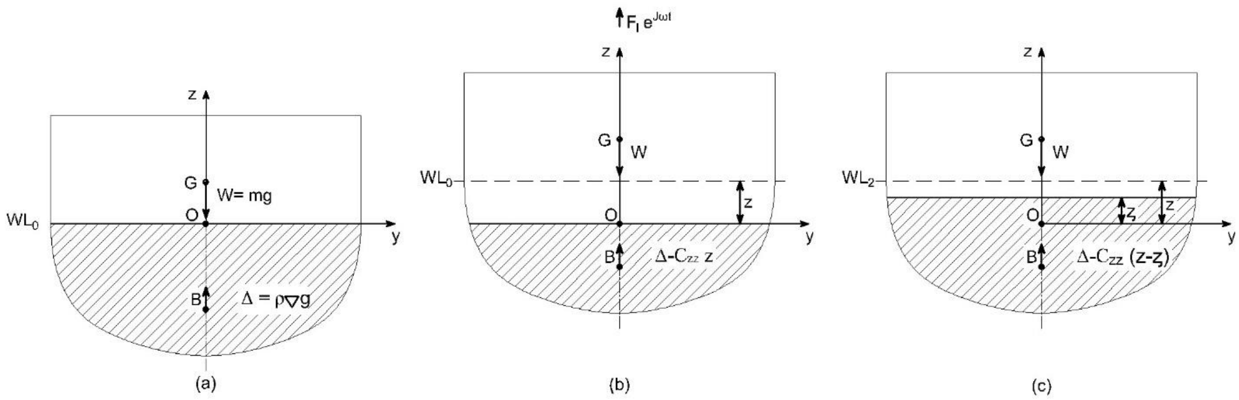

Figure 6-1 Demonstration of uncoupled heave motion. Underwater shaded areas indicate portion of hull underwater section.

In linear hydrodynamic theory the force has a component proportional to the acceleration (the added mass) and a component proportional to the velocity (the damping coefficient). To understand the effect of waves we must consider the effect of the relative position of the ship with respect to waves. If we ignore the hydrodynamic effects and apply Newton's second law of motion (see Lecture 5) then for the uncoupled heave case we get

$$m\ddot{z} = -W + \Delta - C_{zz}(z - \varsigma) \; or \; m\ddot{z} + C_{zz}z = C_{zz}\varsigma \tag{6-4}$$

where ς is the wave profile defined with respect to the still water line and z-ς is called the relative displacement.

If we assume that the hydrodynamic effects are proportional to the relative velocity and acceleration the equation of motion in waves becomes:

$$m\ddot{z} = -m_{zz}(\ddot{z} - \ddot{\varsigma}) - N_{zz}(\dot{z} - \dot{\varsigma}) - C_{zz}(z - \varsigma) \tag{6-5}$$

or

$$(m + m_{zz})\ddot{z} + N_{zz}\dot{z} + C_{zz}z = m_{zz}\ddot{\varsigma} + N_{zz}\dot{\varsigma} + C_{zz}\varsigma = F_z(t) \tag{6-6}$$

Eq. (6-6) shows that for a ship in waves, the surrounding fluid not only provides the hydrostatic and hydrodynamic terms but also the wave excitation which is a function of the wave acceleration, velocity and displacement. For a stationary ship in a regular wave train of frequency ω the excitation term becomes

$$F_z(t) = F_{zs}sin(\omega t) + F_{zc}cos(\omega t) = \bar{F}_z sin(\omega t + \psi) \tag{6-7}$$

The amplitude of the wave excitation is defined as

$$\bar{F}_z = \sqrt{F_{zs}^2 + F_{zc}^2} \text{ for } F_{zs} = \bar{F}_z cos\psi \quad \text{and} \quad F_{zc} = \bar{F}_z sin\psi \tag{6-8}$$

Thus Eq. (6-6) can be re-written as

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z sin(\omega t + \psi) = \bar{F}_z e^{j(\omega t + \psi)} \tag{6-9}$$

The response can be expressed as $z(t) = Zsin(\omega t + \psi - \varepsilon)$ or in complex notation $z(t) = Ze^{j(\omega t + \psi - \varepsilon)}$. By back-substitution to Eq. (6-3) we obtain

$$Z[(C_{zz} - \omega^2 M_{zz})sin(\omega t + \psi - \varepsilon) + \omega N_{zz}cos(\omega t + \psi - \varepsilon)] = F_z sin(\omega t + \psi) \tag{6-10}$$

An easy way to obtain the heave amplitude (Z) and phase (ε) from Eq. (6-10) is to consider the following two cases

$$\omega t + \psi - \varepsilon = 0 \Rightarrow Z\omega N_{zz} = \bar{F}_z \sin(\varepsilon) \tag{6-11}$$

$$\omega t + \psi - \varepsilon = \frac{\pi}{2} \Rightarrow Z(C_{zz} - \omega^2 M_{zz}) = \bar{F}_z \cos(\varepsilon) \tag{6-12}$$

If we square Eqs.(6-11) and (6-12) and add them we produce the amplitude of the response, namely

$$Z = \bar{F}_z / \sqrt{(C_{zz} - \omega^2 \, M_{zz}\,)^2 + (\omega N_{zz})^2} \tag{6-13}$$

If we divide Eq. (6-11) by Eq. (6-12) we define the phase lag as

$$tan(\varepsilon) = \omega N_{zz} / (C_{zz} - \omega^2 M_{zz}) \tag{6-14}$$

For a ship progressing in regular waves with forward speed U and heading χ the variation of the wave elevation (and wave velocity and acceleration) with time is sinusoidal with the wave encounter frequency (ω_e). Thus, Eq. (6-9) becomes

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z = F_z sin(\omega_e t + \psi) = \bar{F}_z e^{j(\omega_e t + \psi)} \tag{6-15}$$

In the above equation the amplitude and phase of the wave sinusoidal wave excitation is proportional to the wave amplitude and is a function of the wave frequency and the ship's forward speed and heading.

As explained in Lecture 5 the hydrodynamic damping and added mass coefficients are not constant values. They vary with the frequency of the ship's oscillation, i.e. $m_{zz} = m_{zz}(\omega_e)$; $N_{zz} = N_{zz}(\omega_e)$ and $\bar{F}_z = aF_z(\omega, \omega_e)$.On this basis Eq. (6-15) can be expressed in the following more explicit form

$$[m + m_{zz}(\omega_e)]\ddot{z} + N_{zz}(\omega_e)\dot{z} + C_{zz}z = aF(\omega, \omega_e) sin(\omega_e t + \psi) = F_z(\omega, \omega_e) e^{j(\omega_e t + \psi)} \tag{6-16}$$

The heave amplitude and phase lag are

$$Z = \frac{aF_z(\omega, \omega_e)}{\sqrt{[C_{zz} - \omega_e^2(m + m_{zz}(\omega_e))]^2 + [\omega_e N_{zz}(\omega_e)]^2}} \text{ and } tan\varepsilon = \frac{\omega_e N_{zz}(\omega_e)}{C_{zz} - \omega_e^2(m + m_{zz}(\omega_e))} \tag{6-17}$$

If we assume free motions in waves then Eq. (6-16) becomes

$$[m + m_{zz}(\omega_e)]\ddot{z} + N_{zz}(\omega_e)\dot{z} + C_{zz}z = 0 \tag{6-18}$$

Analytical solution of this equation is not possible due to the presence of coefficients which are not constants but functions of the encounter frequency. Nevertheless, the free heave displacement will be exponentially decaying oscillatory function of time. For an undamped motion

$$[m + m_{zz}(\omega_e)]\ddot{z} + C_{zz}z = 0 \tag{6-19}$$

leading to the characteristic equation

$$C_{zz}z - \omega_e^2[m + m_{zz}(\omega_e)] = 0 \qquad (6\text{-}20)$$

The above equation cannot be solved analytically. However, it is possible to assume a constant value of added mass namely $\overline{m}_{zz} = m_{zz}(\omega_e \rightarrow \infty)$ and therefore the heave natural frequency in water can be approximated as

$$\omega_{3n} = \sqrt{\frac{c_{zz}}{m + m_{zz}}} \qquad (6\text{-}21)$$

Variation of the nondimensionalized heave added mass, damping coefficient and excitation amplitude (for head waves of amplitude 1m) are shown in Figure 6-2 (a,b,c) as a function of the encounter frequency for a naval ship. Note that the variations of added mass and damping coefficients with speed are very small whilst those of the wave excitation are significant. The corresponding heave amplitude (per unit wave amplitude, i.e. the Response amplitude Operator) is sown in Figure 6-2(d) for head waves. Both amplitudes and phases of the wave excitation terms are functions of wave and wave encounter frequencies.

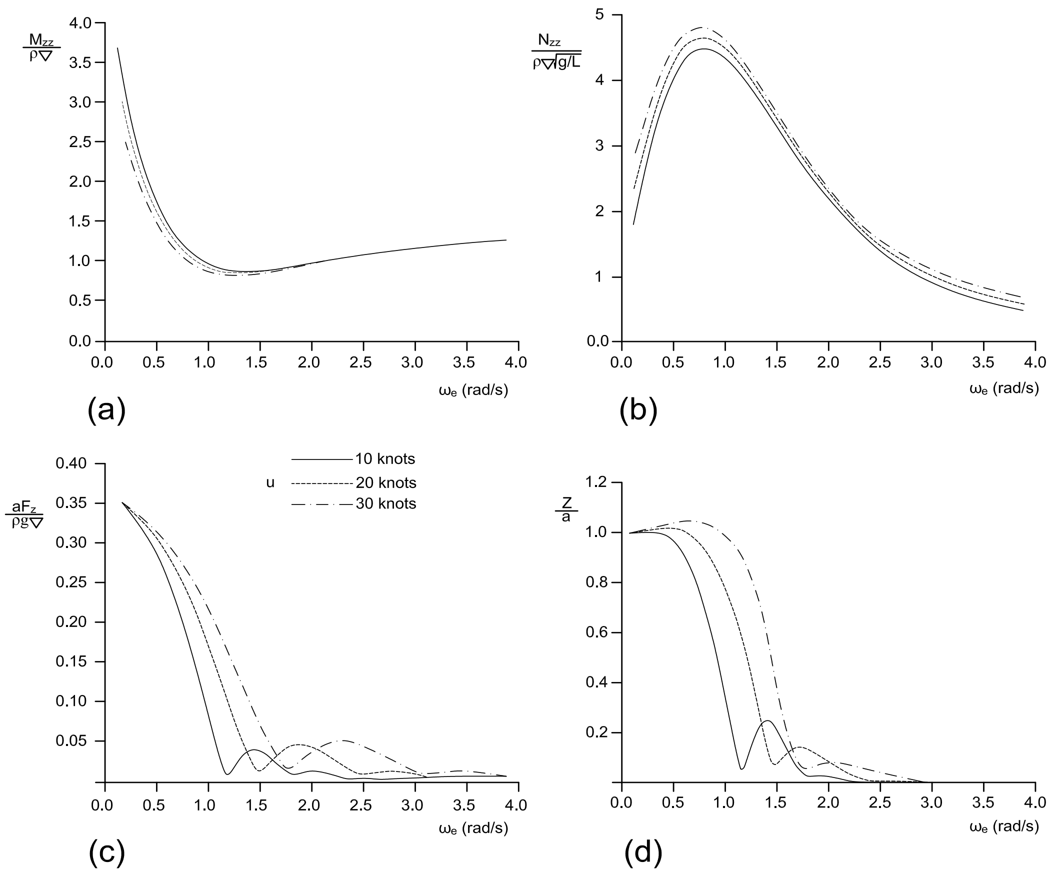

Figure 6-2: Typical nondimensionalized heave added mass, damping coefficients and excitation amplitude for a naval ship at different speeds (Mzz= added mass; Nzz = hydrodynamic damping; Fz = excitation amplitude; Z = heave amplitude; ρ = density; g = acceleration of gravity).

3. Uncoupled pitch motion

If we consider that the ship is an 1-DOF system subject to pitch excitation namely *ϑ(t)* then the corresponding mathematical expression to Eq. (6-16) is

$$[I_{yy} + I_{\vartheta\vartheta}(\omega_e)]\ddot{\vartheta} + N_{\vartheta\vartheta}(\omega_e)\dot{\vartheta} + C_{\vartheta\vartheta}\vartheta = \bar{M}_{\vartheta}(\omega, \omega_e)e^{j(\omega_e t+u)} \tag{6-22}$$

where :

I_{yy} is the mass moment of inertia about axis Oy

$I_{\vartheta\vartheta}$ is the pitch added mass moment of inertia

$N_{\vartheta\vartheta}$ is the pitch damping coefficient

$C_{\vartheta\vartheta} = \rho g I_{long}$ for $I_{long} = \text{longitudinal } 2^{\text{nd}} \text{ moment of water plane area}$

$\bar{M}_{\vartheta}$ is the amplitude of the wave excitation vector

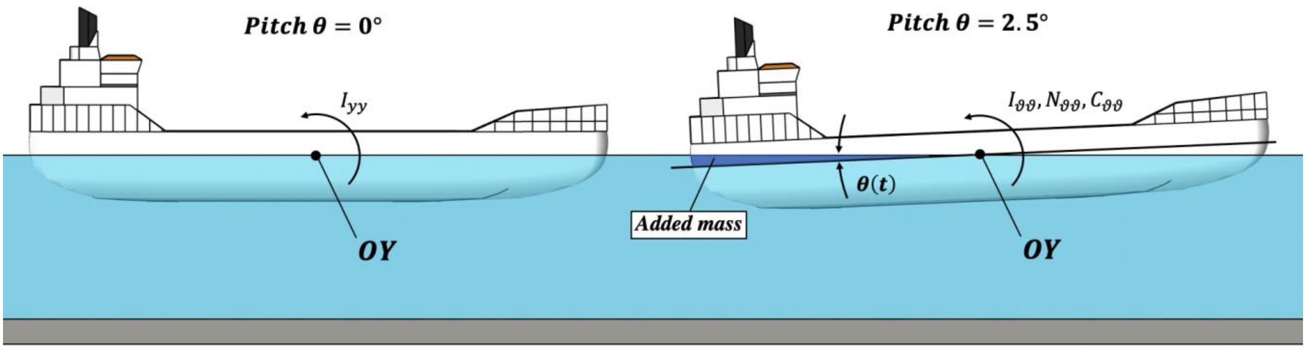

Figure 6-3: Uncoupled pitch motion

The solution of this equation is similar to the one presented in Eq.(6-17). The pitch natural frequency in water can be approximated as

$$\omega_p = \sqrt{\frac{c_{\theta\theta}}{I_{yy}+\bar{I}_{\theta\theta}}} \tag{6-23}$$

where $\bar{I}_{\theta\theta} = I_{\theta\theta} \rightarrow \infty$.

4. Coupled heave and pitch motions

The coupled equations of motion for heave and pitch can be expressed in matrix format as

$$\begin{bmatrix} m+m_{zz} & m_{z\theta} \\ m_{\theta z} & I_{yy}+I_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} N_{zz} & N_{z\theta} \\ N_{\theta z} & N_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_{zz} & C_{z\theta} \\ C_{\theta z} & C_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} z \\ \theta \end{bmatrix} = \begin{bmatrix} F_z(\omega,\omega_e)e^{j(\omega_e t+\psi)} \\ \overline{M}_\vartheta(\omega,\omega_e)e^{j(\omega_e t+u)} \end{bmatrix} \tag{6-24}$$

In this matrix equation in addition to heave added mass m_{zz} and pitch added inertia $I_{\theta\theta}$ we have the additional terms namely **heave into pitch** and **pitch into heave** for added mass and damping terms namely $m_{z\theta}, N_{zz}$ and $m_{\theta z}, N_{\theta\theta}$. There are no terms in the form of first moments of mass in the inertia matrix. The heave into pitch restoring terms are defined as

$$C_{z\theta} = C_{\theta z} = \rho g M_l \tag{6-25}$$

where $M_l = \int_L xB(x)dx$ represents the longitudinal first moment of water plane area and $B(x)$ is the beam in way of the water line.

The above equations indicate that coupling takes place through hydrodynamic and hydrostatic actions. All added masses and damping coefficients are dependent on the frequency of oscillation. Examples of heave (m/m) and pitch (rad/m) RAOs for a naval ship are shown in Figure 6-4. In these figures three different axes were used to illustrate the variation of the RAOs namely ω_e, L/λ and λ/L; where L : ship length and λ : wave length. It is interesting to note that the peaks of heave and pitch occur in the vicinity of $L = \lambda$. This phenomenon is called **ship wave matching**.

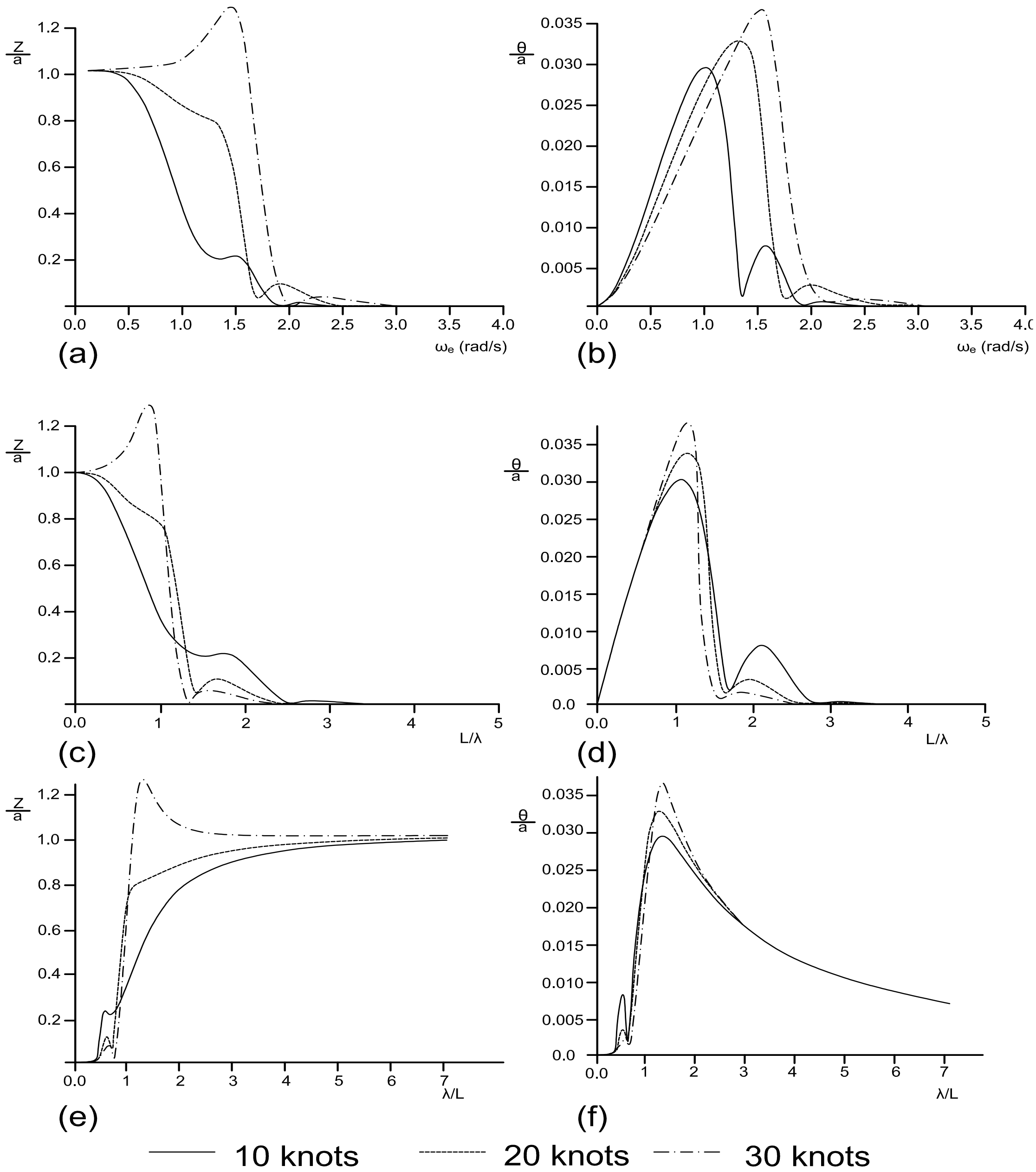

Figure 6-4 Examples of heave (m/m) and pitch (rad/m) RAOs for a naval ship at different forward speeds (Figures a,c,d denote heave; Figures b,d,f denote pitch).

5. Small Amplitude Roll

As explained in Lecture 5 in rigid body ship dynamics there are three rotational degrees of freedom namely roll, pitch and yaw (see Figure 5-2). For typical ship shapes, the radii of gyration (k_i) have a relationship to the ship's geometry. So, the roll, itch and yaw radii of gyration are defined as $k_4 = 0.3 \times B_{WL}$; $k_5 = 0.25 \times L_{pp}$; $k_6 = 0.25 \times L_{pp}$ respectively (where B_{WL} : the waterline beam and L_{pp} is the length between perpendiculars)

If we assume that a ship undertakes small amplitude roll oscillations about her center of mass (usually close to the undisturbed water line) then dynamics are described by the equation :

$$\left[J_{xx} + I_{\varphi\varphi}(\omega)\right]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(t) \tag{6-26}$$

where :

$K_{\varphi}(t)$ is a sinusoidal mechanical excitation producing a rolling moment $K_{\varphi}(t) = K_1 e^{j\omega t}$;

ϕ is the angle of roll;

J_{xx} is the mass moment of inertia about the longitudinal axis through the center of mass;

$C_{\varphi\varphi} = \Delta GM_T = \rho g \nabla GM_T$ is the hydrostatic roll restoring coefficient;

$I_{\varphi\varphi}$ = roll added inertia (frequency of oscillation dependent);

$N_{\varphi\varphi}$ = roll damping coefficient associated with fin and tank stabilizers.

It is noted that roll damping increases with forward speed. The increase in damping results in a smaller maximum resonant peak, but also a slight reduction in the frequency at which the peak response may occur. The mass of a ship is determined by her total weight or displacement. Thus, the rotational inertia associated with roll is determined by the distance of each weight from center of gravity. The further the heaviest weights are from the center of gravity (COG) of the vessel, the larger the rotational moment of inertia. If all of the masses were located equidistantly from COG the moment of inertia would be easy to calculate and would be equal to the total mass times the distance from the COG squared. If the roll added inertia and damping coefficients are constant the free damped equation of motion becomes:

$$\left[J_{xx} + I_{\varphi\varphi}(\omega)\right]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = 0 \tag{6-27}$$

If we ignore damping,

$$\left[J_{xx} + I_{\varphi\varphi}(\omega)\right]\ddot{\varphi} + C_{\varphi\varphi}\varphi = 0 \tag{6-28}$$

Although the mass in a ship is never located equidistantly from the COG, we can find the representative distance the mass would need to be if the ship was idealized as a sphere. This representative distance is the radius of gyration, k_{xx}. If we have the radius of gyration, we can find the ship's moment of inertia. Thus, Eq.(6-28) leads to:

$$\rho\nabla(k_{xx}^2\ddot{\varphi} + gGM_T\varphi)\text{=}0 \quad \text{(6-29)}$$

where $k_{xx}$is defined as $I = I_{xx} + I_{\varphi\varphi} = mk_{xx} = \rho\nabla k_{xx}^2$

The roll natural frequency including the effects of added inertia becomes

$$\omega_\varphi = \sqrt{\frac{C_{\varphi\varphi}}{I_{xx}+I_{\varphi\varphi}}}\sqrt{\frac{gGM_T}{k_{xx}^2}} \quad \text{(6-30)}$$

If we define the constant roll damping coefficient as $N_{\varphi\varphi} = 2\varsigma(I_{xx} + I_{\varphi\varphi})\omega_\varphi$ the damped equation of motion in still water after dividing terms by the inertia term becomes

$$\varphi + \ddot{2}\zeta\omega_\varphi\dot{\varphi} + \omega_\varphi^2\varphi = 0 \quad \text{(6-31)}$$

where ζ is the damping ratio.

For a ship rolling in waves that are long compared to her beam, the instantaneous wave surface can be represented by the wave slope shown in Figure 6-5.

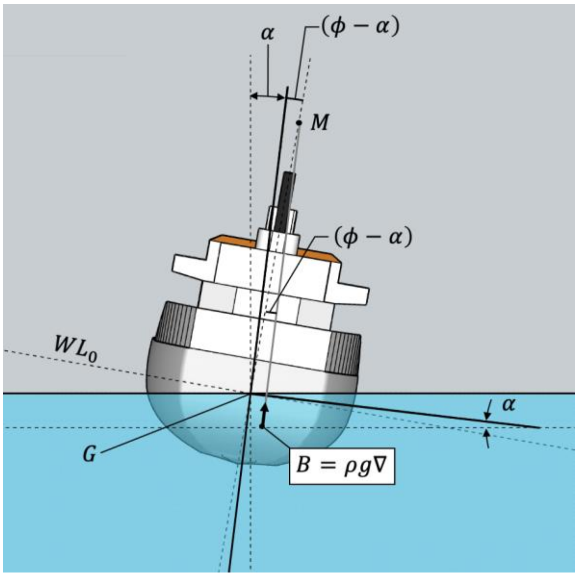

Figure 6-5 Illustration of Roll Motion

If we ignore the effects of roll damping associated hydrodynamic forces (and any other environmental forces such as current, wind etc.) and we only consider the hydrostatic buoyancy force acting perpendicular to the wave surface (i.e. the wave slope) then provided that the roll angle is small, taking moments about G (see Figure 6-5) lead to the equation of motion

$$I_{xx}\ddot{\varphi} = -\rho g\nabla GM_T sin(\varphi - \alpha) \rightarrow \rho g\nabla GM_T(\varphi - \alpha) \quad \text{(6-32)}$$

The symbols of Eq. (6-32) are depicted in Figure 6-5. Experimental observations first presented by *“William Froude”* indicate that for the case idealized by Eq. (6-32) the use of maximum surface wave slope is recommended. This is because it technically represents an ideal wave profile. Having obtained a simplified form of wave excitation (valid essentially for long waves) we can generalize including the effects of damping and added inertia in the form of the equation

$$\left(J_{xx}+I_{\varphi\varphi}\right)\ddot{\varphi}+N_{\varphi\varphi}\dot{\varphi}+C_{\varphi\varphi}\varphi=K_{\varphi}cos(\omega t) \qquad (6\text{-}33)$$

The response to this sinusoidal excitation is $\varphi(t)=\Phi cos(\omega t-\varepsilon)$. To obtain the amplitude and phase we can back substitute in Eq (6-33) to obtain

$$\left[C_{\varphi\varphi}-\omega^2\left(I_{xx}+I_{\varphi\varphi}\right)\right]\Phi cos(\omega t-\varepsilon)-\omega N_{\varphi\varphi}\Phi sin(\omega t-\varepsilon)=K_{\varphi}cos(\omega t) \qquad (6\text{-}34)$$

An easy way to obtain the roll amplitude (Φ) and phase lag (ε) is to consider the following two cases

$$\omega t-\varepsilon=\frac{\pi}{2} leadingto-\Phi\omega N_{\varphi\varphi}=-K_{\varphi}\sin(\varepsilon) \qquad (6\text{-}35)$$

$$\omega t-\varepsilon=0 \quad \text{leading to} \quad \Phi\left[C_{\varphi\varphi}-\omega^2\left(I_{xx}+I_{\varphi\varphi}\right)\right]=K_{\varphi}\cos(\varepsilon) \qquad (6\text{-}36)$$

Squaring equations (6-35) and (6-36) and adding them produces the roll amplitude as follows

$$\Phi=\frac{K_{\varphi}}{\sqrt{\left[C_{\varphi\varphi}-\omega^2\left(I_{xx}+I_{\varphi\varphi}\right)\right]^2+\left(\omega N_{\varphi\varphi}\right)^2}} \qquad (6\text{-}37)$$

Dividing Eq. (6-35) by Eq. (6-36) defines the phase lag as

$$tan\varepsilon=\frac{\omega N_{\varphi\varphi}}{C_{\varphi\varphi}-\omega^2\left(I_{xx}+I_{\varphi\varphi}\right)} \qquad (6\text{-}38)$$

The amplitude and phase lag of the roll oscillation can be put into the following form

$$\Phi=\frac{a_m}{\sqrt{[1-\Lambda^2]^2+(2\varsigma\Lambda)^2}}=\mu a_m \qquad \text{and} \qquad tan\varepsilon=\frac{2\varsigma\Lambda}{(1-\Lambda^2)^2} \qquad (6\text{-}39)$$

where μ is referred to as the magnification factor and $\Lambda=\frac{\omega}{\omega_{\varphi}}$ is the tunning factor. Both μ and amplitude a_m vary with the frequency of oscillation. At resonance $\Lambda=1$ and $\Phi_{res}=\frac{a_m}{2\varsigma}$.

For a ship moving in regular waves with forward speed *U* and heading angle χ and equation of motion similar to Eqs. (6-16) and (6-22) can be written as

$$\left[I_{xx}+I_{\varphi\varphi}(\omega)\right]\ddot{\varphi}+N_{\varphi\varphi}(\omega)\dot{\varphi}+C_{\varphi\varphi}\varphi=K_{\varphi}(\omega,\omega_e)sin(\omega_e t+\delta)=\bar{K}_{\varphi}(\omega,\omega_e)e^{j(\omega_e t+\delta)} \qquad (6\text{-}40)$$

where the hydrodynamic coefficients and the wave excitation are evaluated from potential flow hydrodynamic theory using computational methods such as the ones explained in Lecture 7.

6. Roll in large amplitudes

Large amplitude roll motions may cause crew and passenger discomfort. In practice, the amount of damping provided by the fluid is not always sufficient to reduce the roll amplitude to acceptable levels. Figure 6-6 below shows a large roll angle of a ship denoted by $\phi(t)$.

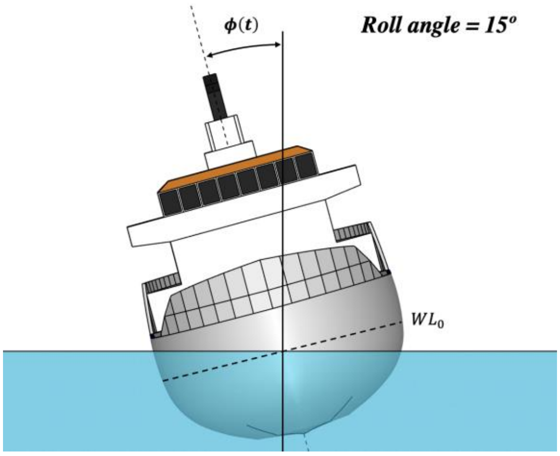

Figure 6-6: Large amplitude roll angle of ship

As explained in Lecture 2 of this course additional mechanisms are commonly used to increase the amount of roll damping. These can be grouped as:

- passive systems which make use of the roll motion and do not require any power source and control system
- active systems which use power to move masses or control surfaces and a control system. Typical passive systems are bilge keels, fixed fins, passive tanks and passive moving weights.

Bilge keels are longer than fins (approximately 2/3 of a ship's length). Fins have longer chord length. Typical active systems are moving fins (retractable or not), active tans and moving weights. Typical damping ratios without any active or passive measures are 0.05 – 0.1. With the use of active stabilizers the damping ratio can be increased to 0.5 – 0.8. The mathematical background to the dynamics of roll stabilisation systems goes beyond the specifics of this course. As an indicative example, for roll stabilisation with active fins the equation of motion has some additional terms on the right hand side which are referred to as three term controller

$$[I_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + N_{\varphi\varphi}\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(t) + C(C_1\ddot{\varphi} - C_2\dot{\varphi} + C_3\varphi) \quad (6\text{-}41)$$

Moving the terms relating to φ to the left-hand side of the equation leads to

$$[I_{xx} + I_{\varphi\varphi}(\omega) - CC_1]\ddot{\varphi} + (N_{\varphi\varphi} + CC_2)\dot{\varphi} + (C_{\varphi\varphi} - CC_3)\varphi = K_{\varphi}(t) \quad (6\text{-}42)$$

C_1 and C_3 decrease with virtual mass moment of inertia and restoring moment whilst C_2 increases the damping; C is associated with the lift generated by the fin stabilizers and can be evaluated from the flow around airfoils as

$$C = \rho a A V^2 \frac{\partial C_L}{\partial a_L} \tag{6-43}$$

where

a is the distance from roll axis to center of pressure fin

A is the fin area (i.e. the product of fin chord and span)

V is the velocity into the fin usually assumed to be equivalent to U, i.e. the forward speed of the ship

a_L is the angle of attack

$\frac{\partial C_L}{\partial a_L}$ is the slope of the lift coefficient surve

Figure 6-4 shows a typical transfer function for roll motion. The response depends on the ship mass, added mass, hydrodynamic damping, buoyancy and excitation frequency in the direction of motion. For the zero forward ship speed case the excitation frequency would match the wave frequency. However, when the ship has forward speed, the excitation frequency depends on the wave frequency and the relative direction of the ship and waves. This resulting excitation frequency is the encounter frequency since it is the frequency at which the ship encounters the waves. Figure 6-7 demonstrates the influence of stabilizers on ship roll motion.

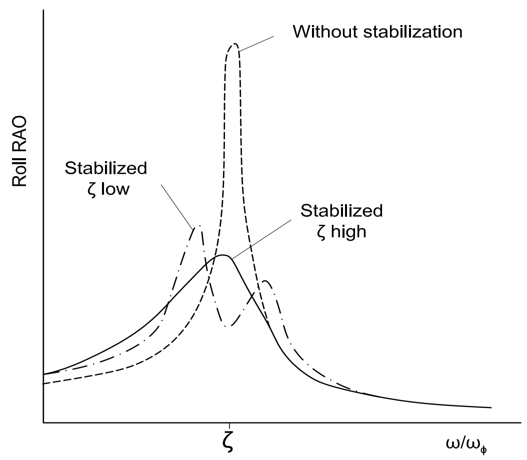

Figure 6-7 Influence of passive tank stabilization on ship roll RAO (roll/wave slope)

7. Questions

1. Consider a ship of displacement 1000 tonnes and metacentric height GM_T = 1m. For the initial undisturbed condition her natural roll period is 15 seconds. Determine the natural period for the loading case when two masses of 500 tonnes each are moved from 3 m to 6 m either side of the ship's centre line.

2. Show how the expressions for roll amplitude and phase lag given by Equation 6-39 are obtained.

3. For a ship rolling in beam waves the radius of gyration (k_{xx}) including the effects of added inertia is 4 m and the metacentric height GM_T = 0.62m. The magnification factor at resonance is 8. Assuming that the damping provided is constant, determine the range of wave lengths over which the magnification factor in excess of 2.5.

4. Assume that the above ship is fitted with fin stabilizers which reduce the magnification factor at resonance to unit value. If we neglect the effects of terms C_1 and C_3 as presented in Equation (6-42) find the value of C when C_2 = 5 s and the ship has a displacement of 1500 tonnes.

5. Briefly explain what hydrodynamic forces are and how they are important to the equations of motion for a ship.

6. Explain each of the terms for the characteristic equation 6-20. What assumption is needed in order to approximate the heave natural frequency?

7. Which two ship motions are often coupled together and their combined effect studied? What is unique about these motions that would make them important to study together?

8. A ship builder is studying the RAO of a newly designed cargo vessel which is 200 m long and with average design speed 12.5 knots. They notice that the pitch response is unacceptable when the wavelength of incoming waves is approximately 190 - 210 m. What is the explanation for the similarity in wavelength to ship length that produces this maximum pitch response? How could the ship builders reduce this response for a given wavelength?

9. What effect does the location of the heaviest pieces of onboard equipment from the center of gravity have on the sinusoidal mechanical excitation producing a rolling moment? What types of ships have heavy machinery located far from the center of gravity of the entire ship?

10. What is the difference between passive and active control systems with regards to roll damping? What are the advantages and disadvantages to each type and which types of ships would you expect to utilize one versus the other?

8. References

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