Lecture 7 Seakeeping methods

1 Introduction

Modern ship seakeeping analysis was introduced in the mid 20th Century as demonstrated by the landmark papers of (Ursell 1949a, Ursell 1949b), and (St Denis and Pierson Jr., 1953). In the late 20th century continuous refinements of analysis methods and mathematical techniques combined with the availability of high-performance desktop computers made routine seakeeping analysis possible in design offices. Today designers have several seakeeping tools to choose from and apply at preliminary design stage. This lecture primarily discusses some of the background of relevance to the basic hydrodynamic modelling methods for the evaluation of seakeeping responses using two- and three-dimensional (2D and 3D) potential flow models. The lecture concludes with a brief reference on nonlinear hydrodynamic methods.

2 Evaluation of hydrodynamic forces

In traditional seakeeping the numerical approximation of linear ship motions in waves is understood as the result of three different types of environmental forces, in addition to the restoring forces of hydrostatic origin. Those are,

- **Radiation forces** (or moments) where the ship is assumed to oscillate in calm seas and accordingly the hydrodynamic added inertia and damping coefficients are determined in still water conditions
- **Incident wave or Froude - Krylov forces** (or moments) where the wave is considered in the absence of the ship and the corresponding wave forces (or moments) acting on the ship are determined. In linear hydrodynamics we assume small displacements, i.e. "true", wetted surface is not considered.
- **Diffraction forces** (or moments) where the effects of the presence of the ship on the waves are considered and the corresponding diffracted wave forces (or moments) are determined.

The evaluation of these force components using linear hydrodynamics is achieved under the assumption that a ship is subject to an incident wave that is **progressive, regular and harmonic** (see Lecture 3). Progressive means that it has a translation speed known as wave celerity (*c*). Regular means that the spatial variation of the wave component is repetitive and is expressed by the wavelength λ. Accordingly the spatial frequency is the wave number $k=\frac{2\pi}{\lambda}$. Harmonic means that the variation of the waveform repeats itself after a time interval *T* known as the wave period. The associated circular frequency to this wave period is defined as $\omega=\frac{2\pi}{T}$. In potential flow hydrodynamics, the velocity potential associated with the incident wave is determined using linearized description of fluid structure interaction. This is achieved by utilizing the velocity potential function $\Phi(x,y,z,t)$ which describes the fluid flow arising from the existence of the incident wave system. The fluid is assumed to be inviscid and incompressible and fluid flow is assumed to be irrotational. Accordingly:

- $\nabla^2 \Phi = 0$ everywhere in the fluid as the flow is assumed to be incompressible and irrotational;
- $\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial \Phi}{\partial y} = 0$ on the undisturbed free surface (y = 0) due to requirements for continuity of pressure and velocity across the surface;
- $\frac{\partial \Phi}{\partial \eta} = 0$ on the impermeable seabed where η denotes the normal to the free surface.

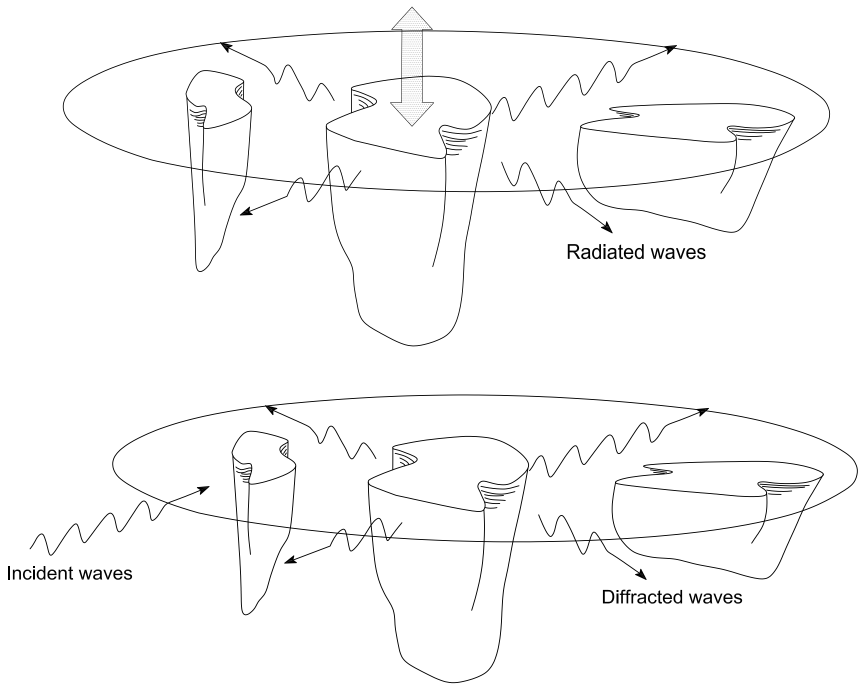

Figure 7-1 Radiation and diffraction idealization in way of adjustment arbitrary structures

Along the lines of classic seakeeping theory let us assume that the ship is a rigid body (see Lectures 5, 6). The incident wave will hit upon different parts of the hull at different times. It takes an initial period before the ship structure becomes aware of the existence of a **steady state** (i.e., the situation for which the loading and responses of the structure are harmonic). The period of time that lapses prior to the persistence of the steady state is known **transient**. This transient period gives rise to a phase shift between the harmonic incident wave and the diffraction (see Lecture 5, Section 5.2.4 and Figure 5.9). The fluid flow is manifested by the sum of incident and diffraction potentials,

$$\Phi = \Phi_{incident} + \Phi_{diffraction} \tag{7-1}$$

The linearized dynamic wave excitation pressure over an elemental area (dS) is defined as

$$F_k^{excitation} = -\int_{S_w} p^{excitation} n_k dS = -\rho \frac{\partial}{\partial t} \left[\Phi_{incident} + \Phi_{diffraction} \right] \tag{7-2}$$

where:

- k (= 1,2,…,6) are the six scalars corresponding to excitations in 6-DOF namely (surge, sway, heave, roll, pitch, yaw);
- n_k is the unit vector in way of the excitation in 6-DOF;
- S_w is the surface of the ship structure in way of which n_k applies.

The **total excitation force** is then expressed as

$$F_k^{excitation} = F_k^{Froude-\,Krylov} + F_k^{Diffraction} \tag{7-3}$$

where:

$$F_k^{Froude-\,Krylov} = -j\omega\rho \int_{S_w} \varphi_{incident} n_k dS e^{-j\omega t} \tag{7-4}$$

$$F_k^{diffraction} = -j\omega\rho \int_{S_w} \varphi_{diffraction} n_k dS e^{-j\omega t} \tag{7-5}$$

The first term on the right-hand side of Eq.(7-3) is equivalent to summing the pressure due to the progression of regular harmonic waves acting on a virtual structure of the same shape and position as the actual structure. The second term represents the extent of the interaction of the incident wave with the ship. Having investigated the interaction of the incident wave with the fixed structure (namely diffraction) we can next consider the forced oscillation of the structure in calm waters. The reactive or radiation forces and moments are expressed as

$$F_{kj} = -\int_{S_w} p^{radiation} n_k dS = \int_{S_w} \rho \frac{\partial}{\partial t}(\varphi_{radiation}^{j}) n_k dS = -j\omega\rho \int_{S_w} \varphi^j n_k dS e^{-j\omega t} \tag{7-6}$$

Given that we have 6 force components (k) in 6 different directions of motion (*j*) there would be 36 values of F_{kj} at each incident wave frequency ω. If we resolve the radiation forces into added mass and fluid damping then

$$F_{kj} = -A_{kj}\ddot{s}_j - B_{kj}\dot{s}_j \tag{7-7}$$

leading to,

$$A_{kj} = \frac{\rho}{a_j\omega} \int_{S_w} \varphi^j n_k dS \tag{7-8}$$

$$B_{kj} = -\frac{\rho}{a_j} \int_{S_w} \varphi^j n_k dS \tag{7-9}$$

3 Seakeeping in 6-DOF

According to Newton's 2nd law of motion the rate of change of linear or angular momentum is equal to the sum of the external forces and moments acting on the ship structure. Thus, for translation and rotation in the j^{th} degree of freedom

$$\frac{dMS_j}{dt} = F_{excitation} + F_{radiation} + F_{hydrostatic} + F_{other} \tag{7-10}$$

$$\frac{dI_{jj}S_j}{dt} = M_{excitation} + M_{radiation} + M_{hydrostatic} + M_{other} \tag{7-11}$$

The arbitrary shape of the structure means that all motions are coupled. Consequently, the radiation forces in the j^{th} direction will have contributions from motions in all 6-DOF. The hydrostatic restoring forces based on *"Archimedes Principle"* involve only the vertical plane motions for heave roll and pitch. Those arguments allow us to write down the six equations of motion as

$$-\omega^2 M s_k = F_{kexcite} - \sum_{j=1}^{6}\left(-\omega^2 A_{kj} - i\omega B_{kj}\right)s_j - \sum_{j=3,4,5} C_{vj} s_j \tag{7-12}$$

where $k = 1,2,\ldots,6$ correspond to 6 – DOF (surge, sway, heave, roll, pitch, yaw) and v notations next to restoring term *C* correspond to roll, pitch and yaw. These arguments collectively allow us to write down the six equations of motion in the format

Surge
$$-\omega^2 M s_1 = F_{1excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{1j} - i\omega B_{1j}\right)s_j \tag{7-13}$$

Sway
$$-\omega^2 M s_2 = F_{2excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{2j} - i\omega B_{2j}\right)s_j \tag{7-14}$$

Heave
$$-\omega^2 M s_3 = F_{3excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{3j} - i\omega B_{3j}\right)s_j - \sum_{j=3,4,5} C_{3j} s_j \tag{7-15}$$

Roll
$$-\omega^2 I_{44} s_4 = F_{4excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{4j} - i\omega B_{4j}\right)s_j - \sum_{j=3,4,5} C_{4j} s_j \tag{7-16}$$

Pitch
$$-\omega^2 I_{55} s_5 = F_{5excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{5j} - i\omega B_{5j}\right)s_j - \sum_{j=3,4,5} C_{5j} s_j \tag{7-17}$$

Yaw
$$-\omega^2 I_{66} s_6 = F_{6excite} - \sum_{j=1}^{6}\left(-\omega^2 A_{6j} - i\omega B_{6j}\right)s_j \tag{7-18}$$

If we transfer the non-wave excitation terms to the left hand side of Eqs.(7-13) to (7-18) and we re-arrange the motion dependent terms in a strict order format we get the following system of 6 coupled algebraic equations

$$\begin{aligned} F_{1,excite} = {} & [-\omega^2(M + A_{11}) - i\omega B_{11}]s_1 + (-\omega^2 A_{12} - i\omega B_{12})s_2 \\ & + (-\omega^2 A_{13} - i\omega B_{13})s_3 \\ & +(-\omega^2 A_{14} - i\omega B_{14})s_4 + (-\omega^2 A_{15} - i\omega B_{15})s_5 + (-\omega^2 A_{16} - i\omega B_{16})s_6 \end{aligned} \tag{7-19}$$

$$\begin{aligned} F_{2,excite} = {} & (-\omega^2 A_{21} - i\omega B_{21})s_1 + [-\omega^2(M + A_{22}) - i\omega B_{22}]s_2 \\ & + (-\omega^2 A_{23} - i\omega B_{23})s_3 \\ & +(-\omega^2 A_{24} - i\omega B_{24})s_4 + (-\omega^2 A_{25} - i\omega B_{25})s_5 + (-\omega^2 A_{26} - i\omega B_{26})s_6 \end{aligned} \tag{7-20}$$

$$F_{3,excite} = (-\omega^2 A_{31} - i\omega B_{31})s_1 + (-\omega^2 A_{32} - i\omega B_{32})s_2 + [C_{33} - \omega^2(M + A_{33}) - i\omega B_{33}]s_3 + (C_{34} - \omega^2 A_{34} - i\omega B_{34})s_4 + (C_{35} - \omega^2 A_{35} - i\omega B_{35})s_5 + (-\omega^2 A_{36} - i\omega B_{36})s_6 \quad (7\text{-}21)$$

$$F_{4,excite} = (-\omega^2 A_{41} - i\omega B_{41})s_1 + (-\omega^2 A_{42} - i\omega B_{42})s_2 + [C_{43} - \omega^2 A_{43} - i\omega B_{43}]s_3 + [C_{44} - \omega^2(I_{44} + A_{44}) - i\omega B_{44}]s_4 + (C_{45} - \omega^2 A_{45} - i\omega B_{45})s_5 + (-\omega^2 A_{46} - i\omega B_{46})s_6 \quad (7\text{-}22)$$

$$F_{5,excite} = (-\omega^2 A_{51} - i\omega B_{51})s_1 + (-\omega^2 A_{52} - i\omega B_{52})s_2 + [C_{53} - \omega^2 A_{53} - i\omega B_{53}]s_3 + (C_{54} - \omega^2 A_{54} - i\omega B_{54})s_4 + +[C_{55} - \omega^2(I_{55} + A_{55}) - i\omega B_{55}]s_5 + (-\omega^2 A_{56} - i\omega B_{56})s_6 \quad (7\text{-}23)$$

$$F_{6,excite} = (-\omega^2 A_{61} - i\omega B_{61})s_1 + (-\omega^2 A_{62} - i\omega B_{62})s_2 + (-\omega^2 A_{63} - i\omega B_{63})s_3 + (-\omega^2 A_{64} - i\omega B_{64})s_4 + (-\omega^2 A_{65} - i\omega B_{65})s_5 + (-\omega^2(I_{66} + A_{66}) - i\omega B_{66})s_6 \quad (7\text{-}24)$$

In conclusion, the original coupled differential equations of motion become six algebraic equations which must be solved the complex quantities $(s_1, \dots, s_6)$. In this way we can derive the corresponding Response Amplitude Operators (RAOs) at each wave frequency and heading.

4 Linear seakeeping analysis methods

Solving the linearized equations of motion requires evaluation of the coefficients and the excitation amplitudes and phases. In computational marine hydrodynamics considerable effort has been devoted into the development of theoretical methods that can be used to determine hydrodynamic coefficients and excitations. The ultimate goal of this effort is to allow ship motions to be calculated without recourse to experiments. To date methods have been developed on the basis that seakeeping analysis method evolved on the basis of the following facts: (a) the analysis should be based on fluid actions reflecting the environmental conditions encountered by the vessel , (b) prediction of the response characteristics of the vessel is fundamental output of the analysis and (c) specification of the criteria used to assess the vessel's seakeeping behavior should be achieved within the context of ship design requirements and operational practice. Comparison of different designs or assessment of a single design against specified criteria is dependent on accurate information for all three items listed above. Evaluation of seakeeping performance depends heavily on the environment (wave spectra) that the vessels are being subjected to and the criteria which are being used to compare the designs. Two classic types of analyses are in use to obtain hydrodynamic forces using potential flow analysis. The first is known as **strip theory** and the other as **panel method**. Details related with the basic assumptions associated with each of these methods follow.

4.1 Strip theory

Strip theory is a two-dimensional analysis whereby the hull is divided into a number of uniform sections. The hydrodynamic properties (i.e., added mass, damping and stiffness) obtained for each section reflect the flow around an infinitely long uniform cylinder. The sectional added inertia and damping coefficients are obtained for heave and coupled sway-roll motions of each section. There are limitations concerning what assumptions can be made to use strip theory depending on the problem specifics. The global hydrodynamic values for the complete hull are then computed by integrating the two - dimensional values of the strips over the length of the ship. A linear strip theory assumes that the vessel's motions are linear and harmonic. In this case, the response of the vessel in both pitch and heave, for a given wave frequency and speed, will be proportional to the wave amplitude and slope, respectively. The basic of linear strip theory are:

- The fluid is inviscid and accordingly viscous damping is ignored or implemented independently via an empirical coefficient usually associated with roll damping;
- The ship is slender (i.e., the length is much greater than the beam or the draft, and the beam is much less than the wave length);
- The hull is rigid so that no flexure of the structure occurs;
- The speed is moderate so there is no appreciable planing lift;
- The motions are small (or at least linear with wave amplitude);
- The ship hull sections are wall—sided;
- The water depth is much greater than the wave length so that deep water wave approximations may be applied;
- The presence of the hull has no effect on the waves (Froude -Krylov hypothesis).

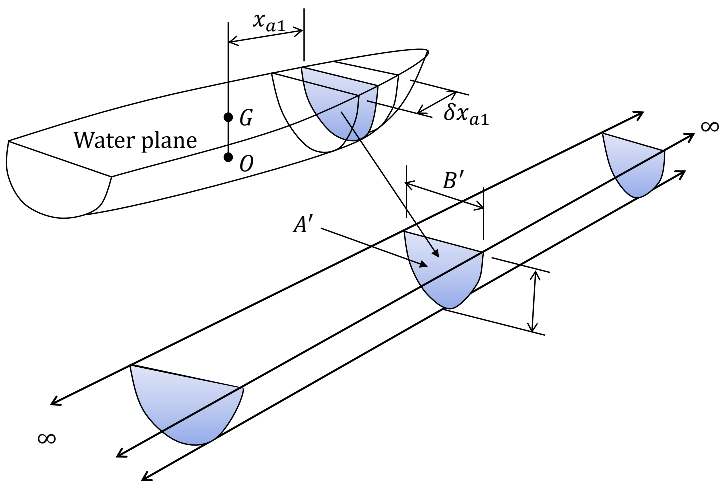

Figure 7-2 *Strip theory idealisation*

The theory presented below is based on the frequency domain strip theory introduced by Salvesen, Tuck and Faltinsen (1970). In this approach the surge motion is not considered. This is because for the case of a slender body hydrodynamic forces associated with surge are considered much smaller than those associated with the other five degrees of freedom. The equation of motion in the frequency domain is expressed as

$$\sum_{k=1}^{6}[-\omega_e^2(M_{jk}+A_{jk})+i\omega_e B_{jk}+C_{jk}]\,\zeta_k = F_{j,excite} \qquad (7\text{-}25)$$

$$\text{or } \sum_{k=1}^{6}(M_{jk}+A_{jk})\ddot{s}_k + B_{jk}\dot{s}_k + C_{jk}s_k = F_j e^{i\omega_e t} \qquad \text{for } j = 2,3,\ldots.6$$

where $F_{j,excite}$ is the excitation force at each degree of freedom (see Equations (7-19) to (7-24)); M_{jk} is the generalized mass matrix; for free motions the non-zero hydrostatic coefficients C_{jk} correspond to the restoring terms C_{33},C_{44},C_{55}, and C_{35}= C_{53}. If the ship is assumed to be with lateral symmetry (symmetric about the *x-z* plane), and the center of gravity is located at (0,0,z_g) then the generalized mass matrix is given by

$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_g & 0 \\ 0 & M & 0 & -Mz_g & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_g & 0 & I_4 & 0 & -I_{46} \\ Mz_g & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix} \quad (7\text{-}26)$$

The added mass and damping matrix are given by

$$A_{jk}(orB_{jk}) = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad (7\text{-}27)$$

By substituting the damping matrix, the added mass matrix and the restoring force matrix in the equation of motion for lateral symmetry, the six coupled equations of motion reduce to (a) three coupled equations for surge, heave, and pitch and (b) three coupled equations for sway, roll, and yaw. For example, the coupled equations of heave and pitch are

$$(M + A_{33})\ddot{s}_3 + B_{33}\dot{s}_3 + C_{33}s_3 + A_{35}\ddot{s}_5 + B_{35}\dot{s}_5 + C_{35}s_5 = F_3 e^{i\omega_e t} \quad (7\text{-}28)$$

$$A_{53}\ddot{s}_3 + B_{53}\dot{s}_3 + C_{53}s_3 + (A_{55} + I_5)\ddot{s}_5 + B_{55}\dot{s}_5 + C_{55}s_5 = F_5 e^{i\omega_e t} \quad (7\text{-}29)$$

The relationship between added mass and damping coefficients are given in detail by Salvesen et al. (1970). For example, added mass, damping and hydrostatic restoring coefficients of heave and pitch are defined as

$$A_{33} = \int a_{33} d\xi - \frac{U}{\omega_e^2} b_{33} \quad (7\text{-}30)$$

$$B_{33} = \int b_{33} d\xi - U a_{33} \quad (7\text{-}31)$$

$$C_{33} = \rho g \int_L B d\xi = \rho g A_{wp} \quad (7\text{-}32)$$

$$C_{35} = C_{53} = -\rho g \int_L \xi B d\xi = -\rho g M_{wp} \quad (7\text{-}33)$$

$$C_{55} = \rho g \int_L \xi^2 b d\xi = -\rho g I_{wp} \quad (7\text{-}34)$$

In Eqs. (7-32) to (7-34) A_{wp},M_{wp}, and I_{wp} represent the waterplane area and the first moment and moment of inertia of this waterplane; U is the speed, ρ is the density of water and g the acceleration of gravity.

4.1.1 Conformal Mapping

The solution of local (two dimensional) hydrodynamic coefficients $a_{jk}\ and\ b_{jk}$ in the previous equations is mathematically challenging and typically computers are used. Once we know the added mass, hydrodynamic damping, etc. we can use the method of "*conformal mapping"* to estimate the properties of ship-like sections in the two-dimensional plane. The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrarily shape of a cross-section in a complex plane can be derived from the more convenient circular cross-section in another complex plane. In this manner hydrodynamic problems can be solved directly with the coefficients of the mapping function. This method begins by defining the properties (Sway, Heave and roll) of an infinitely long semicircular cylinder with radius a as illustrated in Figure 7-3. The cylinder is assumed to oscillate with small motion amplitudes and the resultant radiated waves in ± x directions are calculated using the potential flow theory. Most of the ship sections are not circular, hence conformal transformation techniques are required to extend the circular plane results into solution for more realistic hull shapes. A simple illustration of one of these techniques is shown in Figure 7-4. In this method the circle parameters and the flow around it (stream and potential functions) are defined in the complex circle plane, z plane:

$$z = x + iy = ire^{-i\theta} \tag{7-35}$$

Then the obtained results from the z plane are mapped into the flow around a hull section in the complex ship cross-section plane ζ:

$$\zeta = x_{B2} + ix_{B3} \tag{7-36}$$

Both planes (circle and ship's cross-section planes) are related by the transformation function (7-48):

$$\zeta = f(z) \tag{7-37}$$

The definition of this equation depends on the shape and size of the ship's cross-section; hence it is different for each ship's section. In this lecture, only the commonly used Lewis transformation equation (7-38) is explained in order to understand the broad nature of the calculations and to appreciate their limitations. The method is suitable for most of the conventional hull cross- sections and gives the expressions for the added mass and damping coefficients.

$$\zeta = f(z) = a_0 a\left(\frac{z}{a} + \frac{aa_1}{z} + \frac{a^3a_3}{z^3} + \frac{a^5a_5}{z^5} + \frac{a^7a_7}{z^7} + \cdots\right) \tag{7-38}$$

The equation maps any point on a semicircle of radius a in ($z\ plane$), Figure 7-4, into a corresponding point on ship's cross-section ($\zeta\ plane$). The odd terms a_1, a_3, a_5, … only describe one side of the ship (assuming the ship is symmetric around CL). The a_0 is a scale factor governing the overall size of the Lewis form. It is usual to truncate the transformation series to only three terms:

$$\zeta = f(z) = a_0 a\left(\frac{z}{a} + \frac{aa_1}{z} + \frac{a^3a_3}{z^3}\right) \tag{7-39}$$

The coefficients a_1, a_3 can be defined by the section area coefficient $\sigma = A'/(B'.D')$ and Beam/Draft ratio $B'/\,D'$ of the section, where A' is the cross-section area:

$$H = \frac{B'}{D'} = \frac{2(1+a_1+a_3)}{1-a_1+a_3} \tag{7-40}$$

$$\sigma = \frac{A'}{B'D'} = \frac{\pi}{4}\left(\frac{1-a_1^2-3a_3^2}{1-a_1^2+2a_3+a_3^2}\right) \tag{7-41}$$

Solving equations (7-40) and (7-41) we get:

$$a_1 = (1+a_3)\left(\frac{H-2}{H+2}\right) \tag{7-42}$$

$$a_3 = \frac{3-C+\sqrt{9-2C}}{C} \tag{7-43}$$

Where

$$C = 3 + \frac{4\sigma}{\pi} + \left(1-\frac{4\sigma}{\pi}\right)\left(\frac{H-2}{H+2}\right)^2 \tag{7-44}$$

Substituting Equations (7-46) and (7-47) into equation (7-50) and separating real and imaginary parts we obtain a pair of parametric equations in θ describing the shape of the Lewis form in the ζ plane:

$$\begin{aligned} x_{B2} &= a_0 a[(1+a_1)\sin\,\theta - a_3 \sin\,(3\theta)] \\ x_{B3} &= a_0 a[(1-a_1)\cos\,\theta + a_3 \cos\,(3\theta)] \end{aligned} \tag{7-45}$$

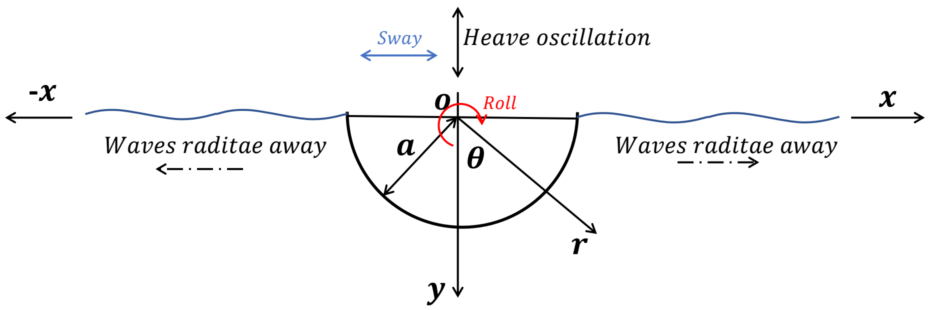

Figure 7-3 Circular cylinder oscillating in the free surface

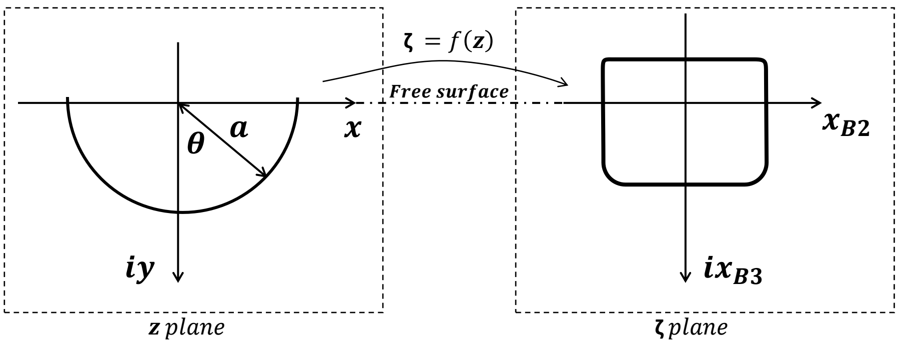

Figure 7-4 Conformal transformation from the z (circle) plane to the ζ (ship) plane

Examples of Lewis forms of realistic ship sections of different H ratios and section area coefficients σ are illustrated in Figure 7-5. In Lewis forms there are no limits for the H ratios but the section area coefficient σ is valid only for $\sigma \leq \frac{\pi}{64H}(H^2 + 20H + 4)$ when the term underneath the square root of equation (7-43) $9 - 2C \geq 0$. It is noteworthy that, large section area coefficients that exceed the limits of Lewis forms will have unconventional shapes and the effect of flow separation will be dominant, hence the strip theory will be no longer accurate. While very small section area coefficients will render physically impossible shapes of Lewis forms, see Figure 7-6. Therefore, Lewis conformal mapping is limited to conventional forms completely within the circumscribing rectangle.

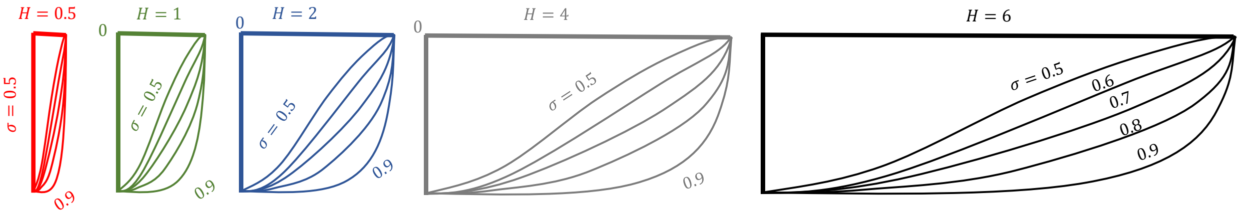

Figure 7-5 Examples of Lewis forms

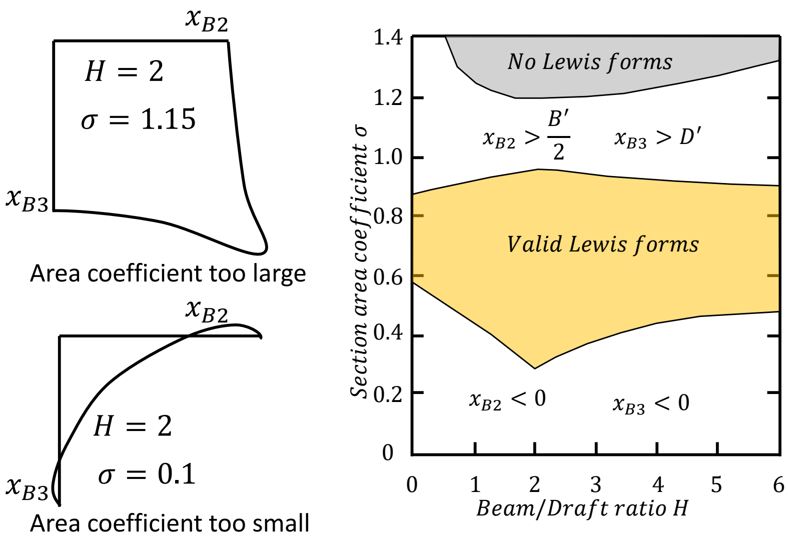

Figure 7-6 Permissible ranges of Lewis forms and examples of invalid ones

4.2 Panel methods

In the Green function panel method, the 3D flow around the ship is calculated in order to obtain the pressure, forces and moments acting on the wetted hull surface (see Figure 7-7). In the pulsating source method, the source density is computed on the center of each panel. A distribution of sources is applied, either on the panel (hull surface) or at some distance from it within the body, to smooth flow irregularities occurring at the boundaries of the panels. All these potentials fulfill the Laplace equation and the radiation, the bottom or infinite depth and linearized free-surface conditions.

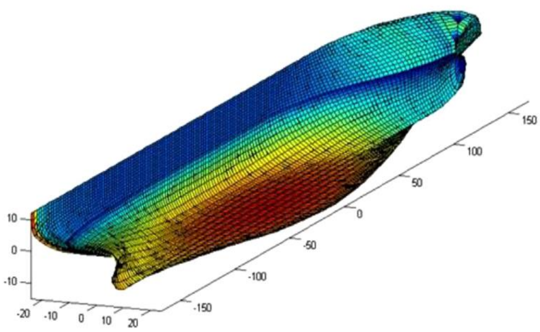

Figure 7-7 Green function idealization of a 10,000 TEU container ship (Hirdaris et al. 2016)

The ship response can be determined in the frequency domain, where the motions of the ship are defined for various regular waves frequencies. Accurate solutions using the zero-speed free-surface Green function method are obtained for problems with linearized free-surface boundary conditions at zero forward speed, but good or reasonable approximations are possible with moderate steady forward speed. The most important effect of forward speed can easily be taken into account by accounting for the influence of the encounter frequency ω_e which as explained in Lecture 3 can be expressed as

$$\omega_e = \omega - kU\,cos\,\mu \tag{7-46}$$

where $k = \frac{2\pi}{\lambda}k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, $U$is the ship speed, and μ is the wave direction. The amplitude of the incoming linear wave of a unit amplitude in deep water (for simplification only) is

$$\hat{\phi}_0 = -i\frac{\omega}{k}e^{-ikx\,cos\,\mu + iky\,sin\,\mu} \tag{7-47}$$

The remaining potentials $\hat{\phi}_{J=1,2,3,..,7}$ can be determined numerically by the panel method. To obtain these potentials the model should satisfy the Laplace equation in the fluid domain and the zero-speed linearized free surface boundary condition at the non-oscillating water surface $z = 0$. The diffraction potential (j = 7) and radiation potentials (j = 1,..,6) can be determined by superimposing the potentials of all panels as

$$\hat{\phi}_j = \sum_1^p q_{j,p}\hat{\phi}_p \tag{7-48}$$

where *p* is the number of panels and $q_{j,p}$is the source density of each panel. The source densities can be obtained by solving a linear equation system for j = 1,…,7 and then satisfy the body boundary conditions at the center of all panels. The radiation potentials are divided into two parts

$$\hat{\phi}_j = \hat{\phi}_j^0 + \frac{U}{i\omega_e}\hat{\phi}_j^U \tag{7-49}$$

where $\hat{\phi}_j^0$and $\hat{\phi}_j^U$are the speed-independent conditions that satisfy the body boundary condition. The fluid pressure (*p*) can be obtained using the Bernoulli equation for unsteady flow

$$\frac{p}{\rho} = -\dot{\phi} - \frac{1}{2}|\nabla\phi|^2 + gz + \frac{1}{2}U^2 \tag{7-50}$$

The pressure of a specific motion on each panel can be evaluated by solving the above equation separately for the wave potential ϕ_0, diffraction potential ϕ_7 and the six radiation potentials from ϕ_1 to ϕ_6. Consequently, the forces and moments can be evaluated by summing up the forces and moments on each panel namely

$$\hat{\boldsymbol{F}}_j = \sum^P \hat{p}_j \boldsymbol{n}_p \quad \text{and} \quad \hat{\mathbf{M}}_j = \sum^P \hat{p}_j \boldsymbol{x} \times \boldsymbol{n}_p \tag{7-51}$$

where $\boldsymbol{n}_p$ is a normal vector directed into the hull and its absolute equals the panel area; while the pressure $\hat{p}_j$ at the panel center equals the average pressure on each panel. Finally, the generalized motion vectors are evaluated by the equation

$$\left[-\omega_e^2 \boldsymbol{M} - \begin{pmatrix} \widehat{\boldsymbol{F}}_1 & \dots & \widehat{\boldsymbol{F}}_6 \\ \widehat{\boldsymbol{M}}_1 & \dots & \widehat{\boldsymbol{M}}_6 \end{pmatrix}\right] \begin{pmatrix} \widehat{\boldsymbol{u}} \\ \hat{\alpha} \end{pmatrix} = \begin{pmatrix} \widehat{\boldsymbol{F}}_0 + \widehat{\boldsymbol{F}}_7 \\ \widehat{\boldsymbol{M}}_0 + \widehat{\boldsymbol{M}}_7 \end{pmatrix} \tag{7-52}$$

This equation may be further complemented by adding corrections to account for the surge and roll damping, forces on fins, etc. However, this is not covered in this section. The complex amplitude of the translation $\widehat{\boldsymbol{u}}$ and rotation $\hat{\alpha}$ motions can be obtained by solving the above system of the six complex scalar linear equations. After obtaining the amplitude of the motion we can get the hull pressure, forces and moments in virtual cross-section, drift forces, etc.

5 Non-linear seakeeping analysis methods

Technical difficulties in the computations of modern hull ship motions are mainly related with understanding, simulating and validating the effects of nonlinearities. There are nonlinear phenomena associated with the fluid in the form of viscosity and the velocity squared terms in the pressure equation. The so-called free surface effect also causes nonlinear behavior due to the nature of corresponding boundary conditions (Bailey, 1997) and the nonlinear behavior of large amplitude incident waves (Mortola et al., 2011). Forward speed effects and the body geometry often cause nonlinear restoring forces and nonlinear behavior in way of the intersection between the body and the free surface (Chapchap et al. 2011).

A large variety of different nonlinear methods have been presented in the past three decades (Hirdaris et al., 2016). Clearly, as techniques become more sophisticated assumptions become more complex (Figure 7-8). Computational time and complexity may be an issue in the process of understanding, simplifying or validating the modelling assumptions. In this sense the accuracy of the solution must be balanced against the computational effort. A taxonomy of the methods available is presented in Table 7-1. From an overall perspective one may distinguish between methods based on linear potential theory (Level 1 methods) and those solving the Reynolds-Averaged Navier–Stokes (RANS) equations (Level 6 methods). The majority of methods currently used in practice is based on linear potential flow theory assumptions and account for some empirical forward speed corrections (Chapchap et al., 2011).

Within the group of weakly nonlinear potential flow methods (Levels 2–5) there is a large variety of partially nonlinear, or blended, methods, which attempt to include some of the most important nonlinear effects. For example, Level 2 methods present the simplest nonlinear approach where hydrodynamic forces are linear and all nonlinear effects are associated with the restoring and the Froude–Krylov forces. On the other hand, Level 3 and 4 methods refer to the so called body nonlinear and body exact methods. In these methods the radiation problem is treated as nonlinear and is solved partially in the time and frequency domains using a retardation function and a convolution integral.

The difference between these two levels is that the body nonlinear approach (Level 3) solves the radiation problem using the calm water surface and the body exact method (Level 4) uses the incoming wave pattern as in way of the free surface for the solution of the radiation problem. Level 5 methods are highly complex and computationally intensive. They have no linear simplifications and the solution of the equations of motion is carried out directly in the time domain. The hydrodynamic problem is solved using an MEL (Mixed Euler–Lagrange) approach. They are usually based on the assumption of smooth waves. Therefore, wave breaking phenomena that may be associated with large amplitude motions in irregular seaways cannot be modelled. Large advances in reducing computer processing times resulted in making basic RANS methods, excluding DES (Detached Eddy Simulations), URANS (Unsteady RANS) and DNS (Detached Navier Stokes).

Implementation of potential flow hydroelastic methods in the frequency - or time - domains may be possible irrespective to the type of hydrodynamic idealisation (e.g. Chapchap et al.,2011 and Mortola et al., 2011). More recent developments enabling full coupling between RANS with FEA software, may ensure the inclusion of hydroelasticity also within this more advanced CFD framework (Lakshmynarayanana and Hirdaris 2020).Nevertheless, there are quite a few issues to resolve even for the application of RANS methods to the conventional, rigid body, sea-keeping problem. For example, these include issues with the time efficiency for computations, the efficient and convergent meshing of the fluid domain associated with the movement of the body and the deforming free surface, as well as the influence of turbulence modelling.

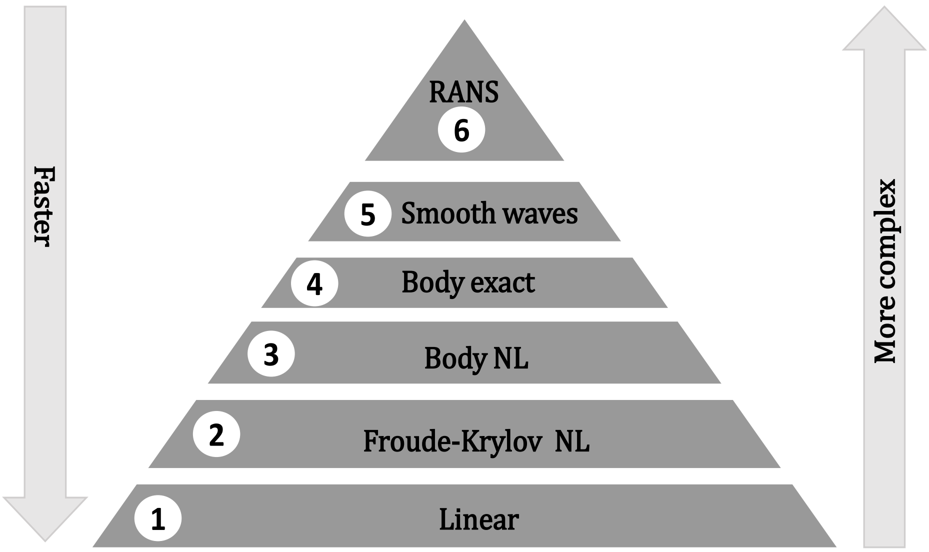

Figure 7-8 Level of idealisation for forward speed hydrodynamic solutions (Numbers 1–6 refer to Levels 1–6 of idealisation as per (Hirdaris et al., 2016).

Table 7-1 Taxonomy of hydrodynamic solution methods as per (Hirdaris et al., 2016).

Level/Description	Key features	Additional comments

1/Linear	• The wetted body surface is defined by the mean position of the hull under tile free surface • The free surface BC are applied in way of the internment wetted body surface • Hydrodynamics are solved in FD by strip theory or BEM using a range of GFM	• Computations are fast • Viscous forces are nor part or t11e solution and must be obtained by other methods, if important or required • The boundary integral methods cannot handle breaking waves. spray and water flowing onto and off the ship's deck.
2/Froude-Krylov NL	• The disturbance potential is determined as in Level I • Incident wave forces evaluated by integrating incident wave and hydrostatic pressures over tile wetted hull surface • The wetted hull surface is defined by the instantaneous position of the hull under the incident wave surface • Hydrodynamics are solved in FD or TD by Green Function method and convolution integrals are used for memory effects	• Computations are moderately fast • NL modification forces can be induded in addition to Froude-Krylov and restoring forces to account for slan1ming and green water
3/Body NL	• The disturbance potential is calculated for the wetted hull surface defined by the instantaneous position of the hull under the mean position of the free surface.	• Computations are slow since re-griddiog and re-calculation of the disturbance potential for each time step is required.
4/Body exact	• The disturbance potential is calculated for the welted hull surface defined by the instantaneous position of the hull under the incident wave surface • The disturbed, or scattered waves, caused by the ship are disregarded when the hydrodynamic boundary value problem is set up • The scattered waves arc considered small compared to the: incident waves and the steady waves	• Computations are mathematically complex and slow. This is because common GFM satisfies the free surface condition on the mean free surface and not on the incident wave surface.
5/Smooth waves	• Scattered waves are no longer assumed to be small, and they are induced when the boundary value problem is set up. • In MEL methods the Eulerian solution of a linear boundary value pro-blem and the Lagrangian time integration of the nonlinear free surface boundary condition is required at each time step. • Wave breaking or fragmentation of the fluid domain is ignored.	• Computations are typically forced to stop based on a wave breaking criterion • The stability of the free surface time-stepping can cause numerical problems
6/Fully NL	• The water/air volume is normally discrctiscd, and a finite difference. finite volume or a finite element technique is used to establish the equation system. • Particle methods, where no grid is used, can be applied to solve the Navier-Stokes equations. Examples are the Smoothed Particle Hydro-dynamics (SPH), the Moving Particle Semi-implicit (MPS) and the Con-strained Interpolation Profile (CIP) methods. with the latter believed to be more suitable for violent flows.	• Mathematics and computations are complex • There is no unification in the approaches used to solve sea-keeping problems, hence exten.sive efforts for validation of solution and the benefits of practical implementation are necessary.

6 Questions

1. Discuss the different types of hydrodynamic forces considered in linear seakeeping analysis.
2. What are the two theories used in the linear Seakeeping analysis to assess the ship motions? Explain briefly the assumptions, background theory and means of implementation for each of them.
3. What are the basic assumptions of strip theory?
4. What does conformal mapping mean? Elaborate your answer with a neat sketch.
5. What are the main assumptions of the Pulsating source green function method?
6. How does the forward speed can be simply taken into account in the Green Function method?

7. Explain briefly with equations how the diffraction and radiation potentials are determined using the Green Function method.
8. Explain briefly with equations how the pressure, forces and motions are determined using the Green Function method.
9. Why and when the nonlinear seakeeping analysis is important?
10. List and explain the different methods of non-linear seakeeping analysis. Sort these methods based on their computational complexity.

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