Added Resistance and Maneuvering

1. Introduction

Added resistance in waves is the part of a ship's total resistance that is caused by encountering waves. Calculations of added resistance can be used as an addition to the calm water resistance to predict the total resistance of a ship in a seaway. There will always be waves on the sea, so there will always be added resistance. A ship can experience a 15-30% resistance increase in a seaway (Arribas 2007), where the added resistance is the main reason for this increase. Being able to predict added resistance due to waves is therefore a vital part of the prediction of a ship's resistance. Prediction of added resistance can for instance be used in the following problems:

- **Weather margin**: the so-called Weather Margin for new ship designs can be decided, where the maximum resistance increase due to weather can be predicted, to decide engine install and so on.
- **Weather Routing**: Weather Routing is very important due to its economic effect on ship exploitation. It is for instance very important to make good estimations of the time it will take for a ship to travel a route, so the cargo owners know when the ship will arrive in port, minimizing the costs of storage and so on. It is also very important to be able to optimize routes to reduce the fuel consumption and emissions. A good prediction of added resistance in waves is important for both these tasks.
- **Performance analysis: t**he previous two problems use the prediction of added resistance to get the total resistance, the reversed problem is however also of interest. Being able to get rid of the influence of the stochastic waves in a seaway, can be used to calculate a ship's "real" calm water resistance. This "real" calm water resistance can be used as a measurement of the ships performance over time. The ship owners could use this information to determine the value of a ship, and how often it should be docked for antifouling and so on.

2. Ship added resistance in waves

When a ship is oscillating due to waves, it supplies energy to the surrounding water, energy that will increase the resistance. This energy is primarily transmitted with the waves radiating from the ship Figure 0-1. The supplied energy is due to damping of the oscillatory motions. Hydrodynamic damping is dominating for heave- and pitch motions, which are the biggest contributors to added resistance. The viscous damping can therefore be neglected, which means that added resistance can be considered as a non-viscous phenomenon (Ström-Tejsen 1973). This means that potential theory can be used. The radiation induced resistance is dominating when the ship motions are big. This happens in the region of the resonance frequency of heave and pitch motions Figure 0-2. The reflection of incident waves is also causing added resistance. The so-called diffraction induced resistance is dominating for high wave frequencies Figure 0-2, where the ship motions are small.

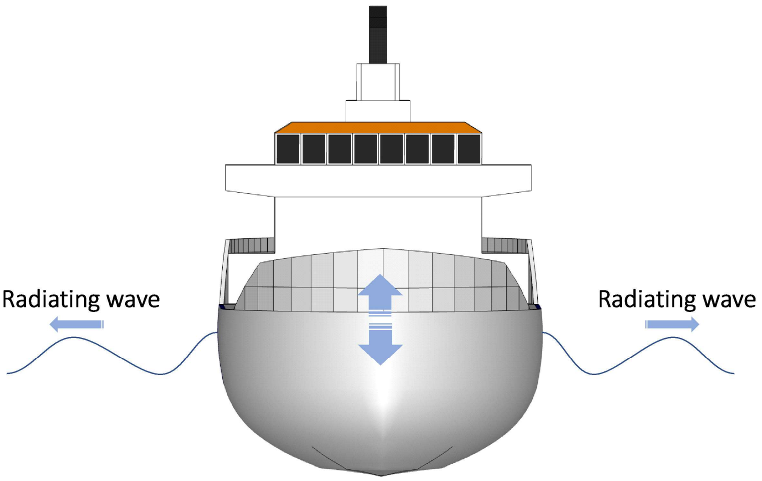

Figure 0-1 Radiating waves due to oscillation.

Energy is also transmitted to the surrounding water by waves generated by the forward speed of the ship. But this is referred to as the calm water resistance, which is not handled in this lecture. The added resistance in a seaway is considered to be independent of the calm water resistance (Ström-Tejsen 1973).

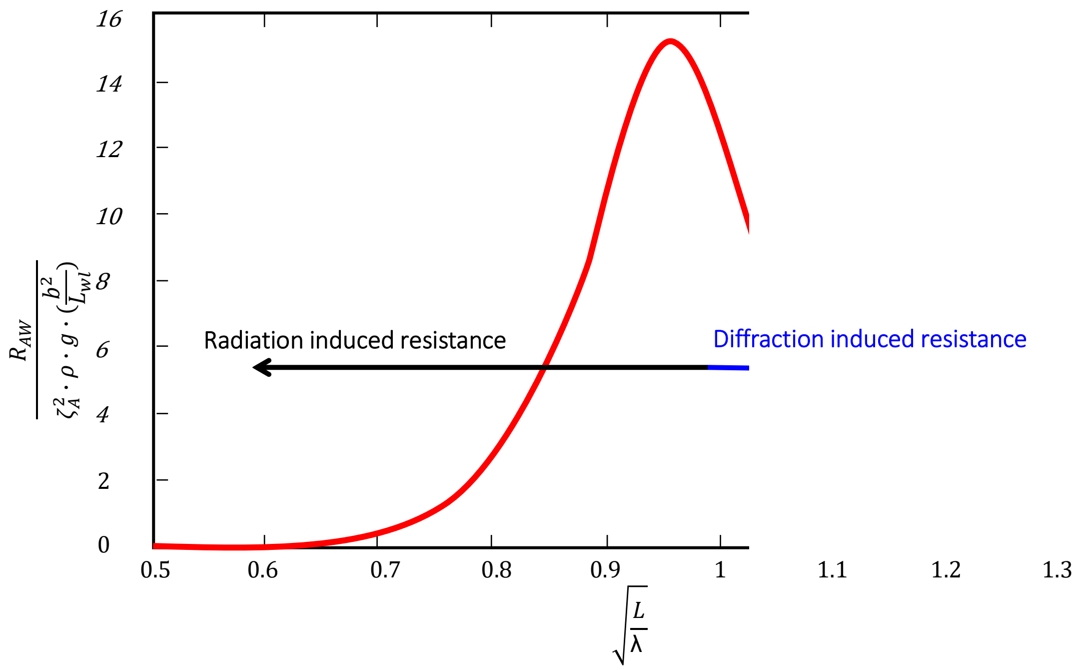

Figure 0-2 Radiation induced resistance and diffraction induced resistance, for different wave frequencies.

3. Added resistance in regular waves

Usually ship motions and forces are modeled as a so called LTI system (Linear Time Invariant system). This means that a ship is considered as a system which uses a linear sine-wave, representing the water wave, as input signal and delivers a linear sine-wave, representing for instance a motion or a force, as response to this signal. The LTI system is allowed to respond with a phase lag on the input signal and a linear change of the amplitude. These restrictions give a very advantageous property of the LTI system in that the superposition principle can be used. This means that if a signal $x(t)$ can be expressed as the sum of sub signals $x_k(t)$, the response to this signal $y(t)$ can be expressed as the sum of the responses of the sub signals $y_k(t)$:

$$x(t) = \sum_k x_k(t)$$
$$\rightarrow y(t) = \sum_k y_k(t) \quad (0\text{-}1)$$

This means that ship motions and forces in irregular waves can be expressed as the sum of the responses in regular waves, which is a very powerful property of a LTI system. In reality ships do not respond linearly to the waves. In order to model the responses as a LTI system, the responses have to be linearized. This linearization gives good accuracy according to (Faltinsen 1993), since the linear part is dominating the responses. Ship motions are therefore considered to be a first order problem.

The added resistance is the mean force in the heading direction of the ship. Calculating the mean force using a linear force from section 3 will give a zero mean value. This is because the time mean value of an arbitrary sine wave with an arbitrary amplitude A and period time T_e is zero:

$$\frac{1}{T_e} \cdot \int_0^{T_e} A \cdot cos(\omega \cdot t + \varepsilon) \cdot \partial t = 0 \quad (0\text{-}2)$$

A second order sine wave however, will give a non zero time mean value:

$$\frac{1}{T_e} \cdot \int_0^{T_e} (A \cdot cos(\omega \cdot t + \varepsilon))^2 \cdot \partial t = \frac{A^2}{2} \quad (0\text{-}3)$$

Therefore, the quadratic term in the response has to be included in the problem. The quadratic term is small compared to the linear term but has to be included to obtain a mean value. (Ström-Tejsen 1973) has shown in experiments that the added resistance in regular waves varies linearly with the wave height squared at a constant wave length, added resistance is therefore considered to be a second order problem. It is unfortunately hard to get good predictions of added resistance, since it is a second order problem. If the motions are predicted with an accuracy of approximately $10 - 15\%$, the second order added resistance can not be expected to be of accuracy better than $20 - 30\%$ (Salvesen 1978). The wave is usually expressed with a velocity potential function. The velocity potential function is derived from boundary conditions that can be linearized. This is referred to as linear wave theory, which will give a linear wave velocity potential. The linear theory is applicable until the wave steepness becomes sufficiently large, that non-linear effects become important. Although added resistance is a second order problem, the linear wave velocity potential is the only one needed. Higher order velocity potentials are not needed, to study the added resistance (Faltinsen 1993).

4. Added resistance in irregular waves

Added resistance is the time mean value of a second order force. Consider a signal $x(t)$ consisting of two signals $x_1(t)$ and $x_2(t)$:

$$x_1(t) = A_1 \cdot cos(\omega_1 \cdot t + \varepsilon_1)$$
$$x_2(t) = A_2 \cdot cos(\omega_2 \cdot t + \varepsilon_2) \quad (0\text{-}4)$$
$$x(t) = x_1(t) + x_2(t)$$

The quadratic response to this signal:

$$x(t)^2 = \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{A_1^2}{2} \cdot cos(2\omega_1 \cdot t + 2\varepsilon_1) + \frac{A_2^2}{2} \cdot cos(2\omega_2 \cdot t + 2\varepsilon_2)$$
$$+A_1 \cdot A_2 \cdot cos\big((\omega_1 - \omega_2)t + \varepsilon_1 - \varepsilon_2\big) + A_1 \cdot A_2 \cdot cos\big((\omega_1 + \omega_2)t + \varepsilon_1 + \varepsilon_2\big) \quad (0\text{-}5)$$

The second order force in an irregular wave can therefore not be expressed with superposition, because of the trigonometric cross terms $A_1 \cdot A_2 \cdot cos(\dots)$. But added resistance is the time mean value of this second order force, where the trigonometric terms from (0-5) disappears, so that the time mean value of (0-5) can be expressed as:

$$\overline{x(t)^2} = \frac{A_1^2}{2} + \frac{A_2^2}{2} \quad (0\text{-}6)$$

The added resistance in irregular waves can therefore be expressed with superposition of the regular wave responses. (Ström-Tejsen 1973) has shown this relation in experiments and that the average added resistance $\bar{R}_{AW}$ in irregular waves with good accuracy can be expressed as:

$$\bar{R}_{AW} = 2\int_0^\infty R(\omega) \cdot S_\zeta(\omega) \cdot \partial\omega$$
$$R(\omega) = \frac{\boldsymbol{R}_{\mathrm{AW}}(\omega)}{\zeta_a^2} \quad (0\text{-}7)$$
$$S_\zeta(\omega) = \frac{1}{2} \cdot \zeta_a(\omega)^2$$

$R(\omega)$ is the mean response curve, and $S_\zeta(\omega)$ is the wave energy spectrum. The evaluation of (0-7), made by Ström-Tejsen, was done by inserting $R(\omega)$ and $S_\zeta(\omega)$ from regular wave experiments into (0-7), and compare that to the corresponding irregular wave experiment. The usual way to calculate added resistance in irregular waves, is therefore to first calculate the added resistance in regular waves for different wave frequencies and then use (0-7). This is why almost all available methods to calculate added resistance in waves, focus on regular waves. The added resistance for different wave frequencies can be presented in a transfer function like the schematic one in Figure 0-2. It is also important to be aware that the choice of wave energy spectrum $S_\zeta(\omega)$, will have a big influence on the integrated mean added resistance $\bar{R}_{AW}$. The relation between the spectral peaks in the wave energy spectrum $S_\zeta(\omega)$ and the mean response curve $R(\omega)$ will have a big impact on the result. So it is reasonable to conclude that to find an accurate wave energy spectrum $S_\zeta(\omega)$, is as important as to find an accurate prediction of the added resistance in regular waves $R(\omega)$.

5. Non dimensional added resistance

The full scale added resistance $\boldsymbol{R}_{\mathrm{AW}}$ in regular waves can be made non dimensional using the following expression:

$$R_{aw} = \frac{\boldsymbol{R}_{\mathrm{AW}}}{\zeta_A^2 \cdot \rho \cdot g(b^2/L_{wl})} \tag{0-8}$$

This relation has been confirmed by (Ström-Tejsen 1973) in model tests, using models of the same ship with varying scale.

The peak of the added resistance transfer function Figure 0-2 usually occurs at a frequency where the wavelength is about the same size as the ships length. This is due to the big influence of pitch motion, which has its peak here, according to Figure 0-3.

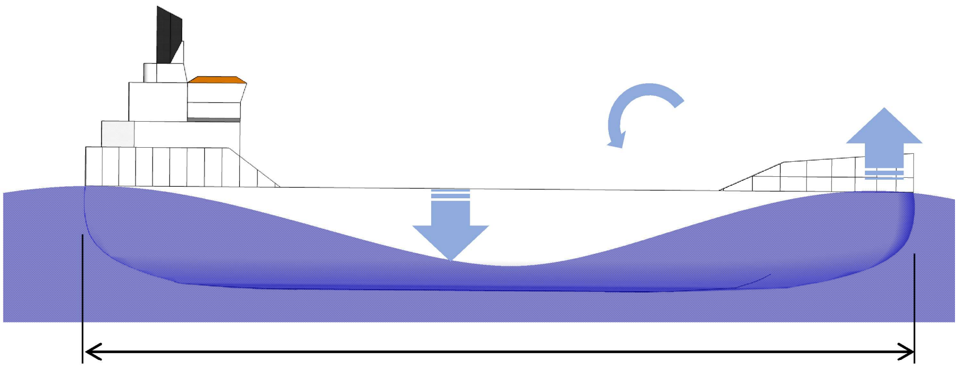

Figure 0-3 wavelengths near the ship length will produce heavy pitching, and added resistance.

This means that the length of the ship will have a big influence on where the peak of the added resistance will be. To capture this relation it is usual to present the transfer functions with a non dimensional frequency, normalized with the ships length in some way. This can be done in a variety of ways, and different authors tend to invent their own way of normalizing the frequency. The non dimensional frequencies can be expressed by:

$$\omega_{\mathrm{norm}} = \sqrt{\frac{L}{\lambda}} \tag{0-9}$$

Which can be related to the wave frequency on deep water as:

$$\omega_{\mathrm{norm}} = \sqrt{\frac{L}{\lambda}} = \omega \cdot \sqrt{\frac{L}{2 \cdot \pi \cdot g}} \tag{0-10}$$

A non-dimensional frequency of encounter:

$$\omega_{\mathrm{norm}} = \omega_e \cdot \sqrt{\frac{L}{2 \cdot \pi \cdot g}} \tag{0-11}$$

6. Methods to calculate added resistance in waves

Three methods to calculate added resistance in waves are considered in this section: **Gerritsma and Beukelman's** method, **Boese's** method and **Faltinsen's** asymptotic method. **Gerritsma and Beukelman's** method is a so-called radiated energy method. This problem starts out by trying to describe the energy that the oscillating ship transmits to the surrounding water. It is assumed that to maintain a constant forward ship speed, this energy should be delivered by the ship's propulsion plant. Boese's method is a so-called pressure integration method, which basically means that the linear pressure in the undisturbed wave is integrated over the ship hull, to obtain a mean force in the heading direction of the ship. It may seem strange that the linear pressure would give a mean force, but it does in this case since the ship hull, where the integration is performed, is moving. Both these methods primarily deal with radiation induced resistance. Faltinsen's asymptotic method on the other hand, only deals with diffraction induced resistance, and neglects the ship motions.

Relative velocity: Both Gerritsma and Beukelman's method and Boeses method to calculate added resistance use Relative velocity. The relative velocity is the vertical velocity of the water related to a point on the ship Figure 0-4.

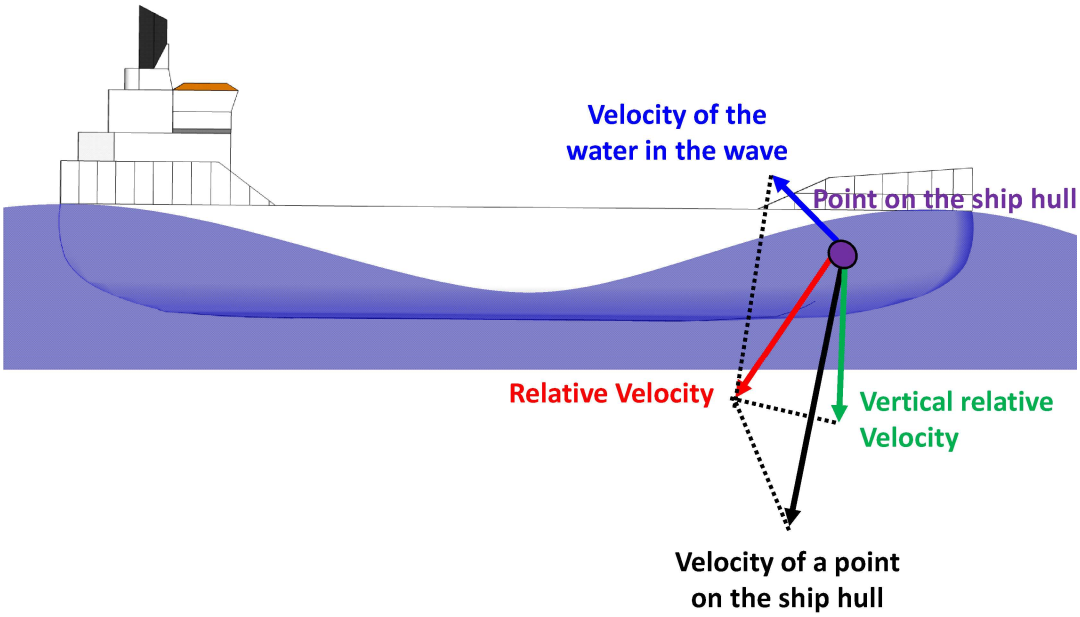

Figure 0-4 definition of vertical relative velocity

Gerritsma and Beukelman's method (Gerritsma and Beukelman 1972) for calculation of added resistance is a so-called radiated energy method. The added resistance is calculated with the following expression:

$$R_{aw} = \frac{-k \cdot cos(\beta)}{2 \cdot \omega_e} \int_0^L b^{'} \left|V_{z_b}\right|^2 \cdot \partial x_b \quad (0\text{-}12)$$

This method is very much related to the Strip theory, where (0-12) is an integration along the ships length, over the strips. $b^{'}$is the sectional damping coefficient for speed, for the different strips (Gerritsma and Beukelman 1972):

$$b^{'} = b_{33} - V \cdot \frac{\partial a_{33}}{\partial x_b} \quad (0\text{-}13)$$

is the amplitude of the relative velocity, which is the water velocity related to the strip:

$$V_{z_b} = -V \cdot \eta_5 - \dot{\eta}_3 + x_b \cdot \dot{\eta}_5 + i \cdot \omega \cdot \zeta_a \cdot e^{k \cdot Z} \cdot e^{i \cdot (\omega_e t - k \cdot x_b \cdot cos(\beta))} \quad (0\text{-}14)$$

This is an equation for various strips (different x_b), but it is also an equation for various values of Z, representing the depth where the water velocity is evaluated. In (Gerritsma and Beukelman 1972) the water velocity is evaluated at a mean depth $\bar{D}$ for every strip:

$$\bar{D} = \frac{A^{'}}{B^{'}} \quad (0\text{-}15)$$

Where $A^{'}$ is the area of the “wet” part of the strip, and $B^{'}$ is the beam of the strip in the waterline. The relative velocity can now be written:

$$V_{z_b} = -V \cdot \eta_5 - \dot{\eta}_3 + x_b \cdot \dot{\eta}_5 + i \cdot \omega \cdot \zeta_a \cdot e^{-k \cdot \bar{D}} \cdot e^{i \cdot (\omega_e \cdot t - k \cdot x_b \cdot cos(\beta))} \quad (0\text{-}16)$$

The damping coefficient (0-13) and the relative velocity (0-16) only contain heave $N_v^{'}$ and pitch motion $N_v^{'}$, so Gerritsma and Beukelman’s method does not account for roll η_4 or yaw motion η_6.

η_3 and $N_v^{'}$ can be expressed in a complex way:

$$\begin{aligned} \eta_3 &= \hat{\eta}_3 \cdot e^{i \cdot \omega_e \cdot t} \\ \eta_5 &= \hat{\eta}_5 \cdot e^{i \cdot \omega_e \cdot t} \end{aligned} \quad (0\text{-}17)$$

$\hat{\eta}_3$and $\hat{\eta}_5$ are complex amplitudes, which means that they contain both amplitude $|\eta_3|$, $|\eta_5|$ and phase ϕ_3, ϕ_5:

$$\begin{aligned} \hat{\eta}_3 &= |\eta_3| \cdot e^{i \cdot \phi_3} \\ \hat{\eta}_5 &= |\eta_5| \cdot e^{i \cdot \phi_5} \end{aligned} \quad (0\text{-}18)$$

This gives the final expression for the relative velocity:

$$V_{z_b} = \left[-V \cdot \eta_5 + i \cdot \omega_e (x_b \cdot \eta_5 - \eta_3) + i \cdot \omega \cdot \zeta_a \cdot e^{-k \cdot \bar{D}} \cdot e^{-i \cdot k \cdot x_b \cdot \cos(\beta)} \right] \cdot e^{i \cdot \omega_e \cdot t} \quad (0\text{-}19)$$

…and the amplitude:

$$\left| V_{z_b} \right| = \left| -V \cdot \eta_5 + i \cdot \omega_e (x_b \cdot \eta_5 - \eta_3) + i \cdot \omega \cdot \zeta_a \cdot e^{-k \cdot \bar{D}} \cdot e^{-i \cdot k \cdot x_b \cdot cos(\beta)} \right| \quad (0\text{-}20)$$

Note: This expression contains ω_e as well as ω .

In (Journée 2001) a derivation of Gerritsma and Beukelman’s method (0-12) is made. The basic idea with the method is to calculate the radiated wave energy during one period of oscillation, in regular waves. This would in other words be the energy required to create waves, when the ship is oscillating. And it is assumed that to maintain a constant forward ship speed, this energy should be delivered by the ship’s propulsion plant. According to (Gerritsma and Beukelman 1972) the radiated energy can be calculated with this equation:

$$E = \int_0^{T_e} \int_0^L b' \cdot V_{z_b}^2 \cdot \partial x_b \cdot \partial t \tag{0-21}$$

Studying the expression for V_{z_b}in (0-19) enables the possibility to express :

$$V_{z_b} = \left|V_{z_b}\right| \cdot cos\left(\omega_e \cdot t + \varepsilon_{V_{z_b}}\right) \tag{0-22}$$

$\varepsilon_{V_{z_b}}$is the phase lag of the relative velocity. The time integration in (0-21) can be performed:

$$E = \int_0^{T_e} \int_0^L b' \cdot V_{z_b}^2 \cdot \partial x_b \cdot \partial t = \frac{T_e}{2} \cdot \int_0^L b' \cdot \left|V_{z_b}\right|^2 \cdot \partial x_b = \frac{\pi}{\omega_e} \cdot \int_0^L b' \cdot \left|V_{z_b}\right|^2 \cdot \partial x_b. \tag{0-23}$$

The radiated energy during one period of oscillation can also be expressed in terms of added resistance R_{aw} (Journée 2001):

$$E = R_{aw} \cdot \lambda_\beta = R_{aw} \cdot \left(\frac{\lambda}{-\,cos(\beta)}\right) = R_{aw} \cdot \left(\frac{2\pi}{-k \cdot cos(\beta)}\right) \tag{0-24}$$

λ_β (Figure 0-5) is the wave length that the ship experiences when it is heading diagonally through the waves. (0-24) together with (0-23) gives Gerritsma and Beukelman's equation for added resistance (0-12).

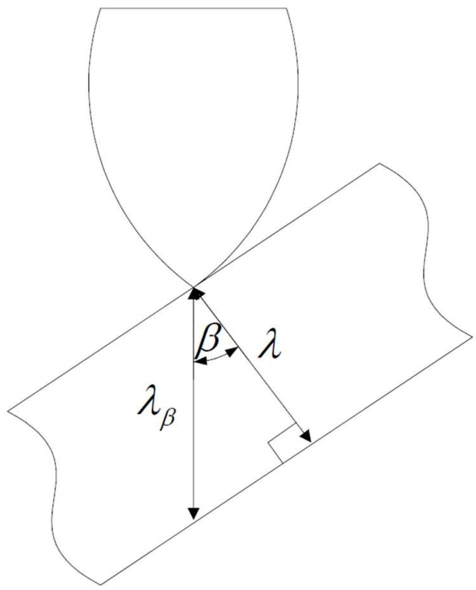

Figure 0-5 λ_β is the wave length experienced by the ship.

7. Introduction to maneuvering theory

In this section we will go much more in-depth on the theory behind maneuvering, including the equations of motion and the hydrodynamic derivatives. We will discuss how to determine these derivatives experimentally and discuss how a ship turns in more detail. Finally, we will cover some rudder design considerations.

1) Course keeping (or Steering) - The maintenance of a steady mean course or heading. Interest centers on the ease with which the ship can be held to the course.

2) Maneuvering - The controlled change in the direction of motion (turning or course changing). Interest centers on the ease with which change can be accomplished and the radius and distance required to accomplish the change.
3) Speed Changing - The controlled change in speed including stopping and backing. Interest centers on the ease, rapidity and distance covered in accomplishing changes.

Performance varies with water depth, channel restrictions, and hydrodynamic interference from nearby vessels or obstacles. Coursekeeping and maneuvering characteristics are particularly sensitive to ship trim. For conventional ships, the two qualities of coursekeeping and maneuvering may tend to work against each other: an easy turning ship may be difficult to keep on course whereas a ship which maintains course well may be hard to turn. Fortunately a practical compromise is nearly always possible.

Since controllability is so important, it is an essential consideration in the design of any floating structure. Controllability is, however, but one of many considerations facing the naval architect and involves compromises with other important characteristics. Some solutions are obtained through comparison with the characteristics of earlier successful designs. In other cases, experimental techniques, theoretical analyses, and rational design practices must all come into play to assure adequacy.

Three tasks are generally involved in producing a ship with good controllability:

1) Establishing realistic specifications and criteria for coursekeeping, maneuvering, and speed changing.
2) Designing the hull control surfaces, appendages, steering gear, and control systems to meet these requirements and predicting the resultant performance.
3) Conducting full-scale trials to measure performance for comparison with required criteria and predictions.

For the axis fixed with respect to the Earth, the equations of motion for maneuvering are

$$\begin{aligned} X_0 &= m_\Delta \ddot{x}_{0G} \text{ Surge} \\ Y_0 &= m_\Delta \ddot{y}_{0G} \text{ Sway} \\ N &= I_z \ddot{\psi} \quad\ \text{Yaw} \end{aligned} \tag{0-25}$$

However, for convenience we want to discuss the ship forces and motions from the ship-fixed reference frame. To do that, we need to express the variables in the previous equations from the ship-fixed coordinate system rather than in the Earth reference frame.

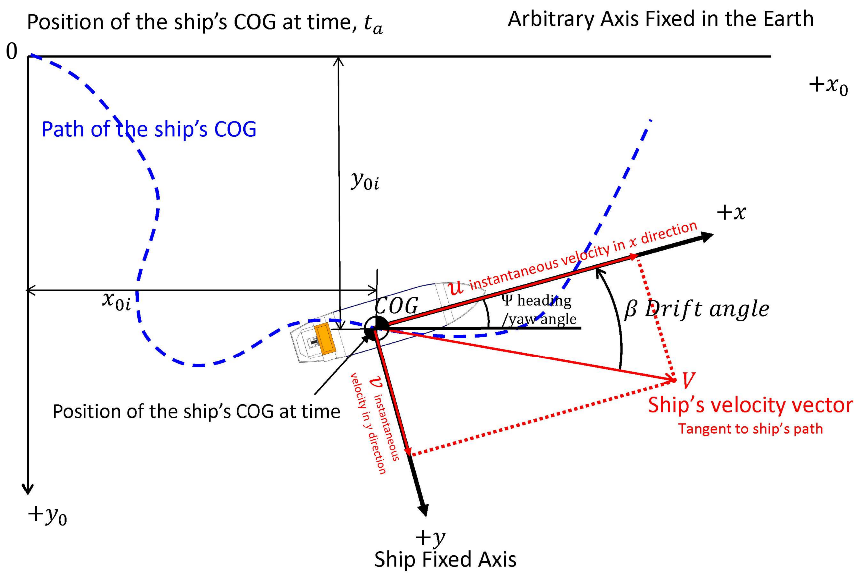

Figure 0-6 Coordinate System for Maneuvering.

Consider the velocities:

$$\begin{aligned} \dot{x}_{0G} &= u\cos\psi + v\sin\psi \\ \dot{y}_{0G} &= -u\sin\psi + v\cos\psi \end{aligned} \tag{0-26}$$

To get accelerations we need to take the derivative of the velocities:

$$\begin{aligned} \ddot{x}_{0G} &= \dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi} \\ \ddot{y}_{0G} &= -\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi} \end{aligned} \tag{0-27}$$

Plugging these into the equations of motion (Note: the forces are still in the Earth reference frame):

$$\begin{aligned} X_0 &= m_\Delta(\dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi}) \\ Y_0 &= m_\Delta(-\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi}) \end{aligned} \tag{0-28}$$

Now consider the forces in the ship-fixed reference frame (same transformation as for the velocities):

$$\begin{aligned} X_0 &= X\cos\psi + Y\sin\psi \\ Y_0 &= -X\sin\psi + Y\cos\psi \end{aligned} \tag{0-29}$$

Plugging into the previous equations and simplifying gives the equations of motion in the forces, velocities, and accelerations measured in the ship-fixed reference frame:

$$\begin{aligned} X &= m_\Delta(\dot{u} + v\dot{\psi}) \\ Y &= m_\Delta(\dot{v} - u\dot{\psi}) \end{aligned} \tag{0-30}$$

The angular equation is unchanged by the shift in coordinate system. Since the other variables (u, v) are velocities, let's replace the angular velocity $\dot{\psi}$ with r (now velocities have no dot and accelerations are all represented with one dot). Now, the equations of motion are:

$$X = m_\Delta(\dot{u} + vr)$$
$$Y = m_\Delta(\dot{v} - ur) \qquad (0\text{-}31)$$
$$N = I_z\dot{r}$$

The forces and moments (left hand side) of the equations of motion consist of four types of forces that act on a ship during a maneuver:

1) Hydrodynamic forces acting on the hull and appendages due to ship velocity and acceleration, rudder deflection, and propeller rotation
    a. Due to relative velocity and acceleration of the surrounding fluid
    b. Due to rudder deflection
    c. Due to propeller action
2) Inertial reaction forces caused by ship acceleration
    a. Rigid body forces acting on a moving body - due to body accelerations
3) Environmental forces due to wind, waves and currents
4) External forces such as tugs, thrusters, mooring lines, etc.

We will only deal with the top two!

The force components X, Y and moment component $N$are assumed to be composed of several parts, some of which are functions of the velocities and accelerations of the ship. For now, we will assume that the forces are composed only of the forces and moments arising from motions of the ship which, in turn, have been excited by disturbances whose details we need not be concerned with here.

$$X = F_x(u, \dot{u}, v, \dot{v}, r, \dot{r})$$
$$Y = F_y(u, \dot{u}, v, \dot{v}, r, \dot{r}) \qquad (0\text{-}32)$$
$$N = F_\psi(u, \dot{u}, v, \dot{v}, r, \dot{r})$$

The forces are comprised of velocity dependent forces arising from hull drag through the water (in surge, sway and yaw) and acceleration dependent forces arising from the mass of the ship and the added mass of the fluid being accelerated in surge, sway, and yaw.

For stability analyses, we need to consider a vessel moving in equilibrium that experiences a disturbance. To consider the effect of a disturbance on the forces acting on the vessel, we can use the Taylor Series expansion technique. "If the function of a variable, x, and all its derivatives are continuous at a particular value of x, say x_1, then the value of the function at the value of x not far removed from x_1 can be expressed as follows":

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx} + \frac{(x - x_1)^2}{2!}\frac{d^2f(x_1)}{dx^2} + \frac{(x - x_1)^3}{3!}\frac{d^3f(x_1)}{dx^3} + \cdots \qquad (0\text{-}33)$$

If the change in the variable $(x - x_1)$is kept small, the higher order terms (HOT) can be neglected, leaving

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx} \qquad (0\text{-}34)$$

For multivariable functions,

$$f(x,y) = f(x_1, y_1) + (x - x_1)\frac{\partial f(x_1, y_1)}{\partial x} + (y - y_1)\frac{\partial f(x_1, y_1)}{\partial y} \tag{0-35}$$

So, if we write the linearized sway force we get

$$Y = F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) + (u - u_1)\frac{\partial Y}{\partial u} + (v - v_1)\frac{\partial Y}{\partial v} + \cdots + (\dot{r} - \dot{r}_1)\frac{\partial Y}{\partial \dot{r}} \tag{0-36}$$

For Straight Line Stability, many of the velocities and accelerations are zero. For example, for a vessel moving at constant forward speed, there are no acceleration terms, no sway or yaw velocities and no Y force before a disturbance. The forward velocity is equal to the ship speed, U.

$$\begin{gathered} u_1 = U \\ v_1 = r_1 = 0 \\ \dot{u}_1 = \dot{v}_1 = \dot{r}_1 = 0 \\ F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) = 0 \end{gathered} \tag{0-37}$$

Because of symmetry, there can be no Y force due to forward velocity or acceleration, so

$$\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial \dot{u}} = 0 \tag{0-38}$$

The Sway Force Equation now becomes,

$$Y = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r} \tag{0-39}$$

We can perform the same technique to get the linearized Surge and Yaw equations:

$$\begin{gathered} X = \frac{\partial X}{\partial u}(u - U) + \frac{\partial X}{\partial \dot{u}}\dot{u} \\ N = \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r} \end{gathered} \tag{0-40}$$

Now we have the forces for the basic equations of motion, we can combine (and move everything over to the right hand side) and get

$$\begin{aligned} 0 &= m_\Delta \dot{u} + m_\Delta vr - \frac{\partial X}{\partial u}(u - U) - \frac{\partial X}{\partial \dot{u}}\dot{u} && \text{Surge} \\ 0 &= m_\Delta \dot{v} - m_\Delta Ur - \frac{\partial Y}{\partial v}v - \frac{\partial Y}{\partial \dot{v}}\dot{v} - \frac{\partial Y}{\partial r}r - \frac{\partial Y}{\partial \dot{r}}\dot{r} && \text{Sway} \\ 0 &= I_z\dot{r} - \frac{\partial N}{\partial v}v - \frac{\partial N}{\partial \dot{v}}\dot{v} - \frac{\partial N}{\partial r}r - \frac{\partial N}{\partial \dot{r}}\dot{r} && \text{Heave} \end{aligned} \tag{0-41}$$

The partial derivatives are called the *Hydrodynamic Derivatives* and we need to find them to solve the equations of motion! To define a standard notation for maneuvering (rather than writing out the partial derivatives every time), SNAME (1952) specified the following rule:

- Replace the partial derivative with the letter for force (or moment) and the subscript with the motion. For example,

$$\frac{\partial Y}{\partial v} \equiv Y_v$$
$$\frac{\partial N}{\partial \dot{r}} \equiv N_{\dot{r}} \qquad (0\text{-}42)$$

Rewriting the equations of motion using this notation gives the official Linearized Maneuvering Equations of Motion:

$$\begin{gathered} -X_u(u-U) + (m_\Delta - X_{\dot{u}})\dot{u} + m_\Delta vr = 0 \\ -Y_v v + (m_\Delta - Y_{\dot{v}})\dot{v} - (Y_r + m_\Delta U)r - Y_{\dot{r}}\dot{r} = 0 \\ -N_v v - N_{\dot{v}}\dot{v} - N_r r + (I_z - N_{\dot{r}})\dot{r} = 0 \end{gathered} \qquad (0\text{-}43)$$

For convenience in analysis, we will non-dimensionalize the equations. For maneuvering the main effects are on sway and yaw - we can neglect surge since changes in forward velocity will be small relative to the mean forward velocity, U.

$$\begin{gathered} -Y_v^{'}v^{'} + \left(m_\Delta^{'} - Y_{\dot{v}}^{'}\right)\dot{v}^{'} - \left(Y_r^{'} + m_\Delta^{'}\right)r^{'} - Y_{\dot{r}}^{'}\dot{r}^{'} = 0 \\ -N_v^{'}v^{'} - N_{\dot{v}}^{'}\dot{v}^{'} - N_r^{'}r^{'} + \left(I_z^{'} - N_{\dot{r}}^{'}\right)\dot{r}^{'} = 0 \end{gathered} \qquad (0\text{-}44)$$

(The U disappeared in the sway equation since the velocities are non-dimensionalized by U, so $U' = 1$)

It is important to note that all the terms in the previous equations must include the effect of the ship's rudder held at zero degrees (on the centerline). On the other hand, if we want to consider the path of a ship with controls working, we must include terms expressing the control forces and moments created by rudder deflection (and any other control devices) as functions of time. The linearized y-component of the force created by rudder deflection is $Y_\delta \delta_R$. The linearized component of the moment created by rudder deflection about the z-axis of the ship is $N_\delta \delta_R$.

δ_R = rudder-deflection angle, measured from xz -plane of the ship to plane of the rudder; positive deflection corresponds to a turn to port for rudder(s) located at the stern.

$Y_\delta N_\delta$ = linearized derivatives of Y and N with respect to rudder-deflection angle δ_R

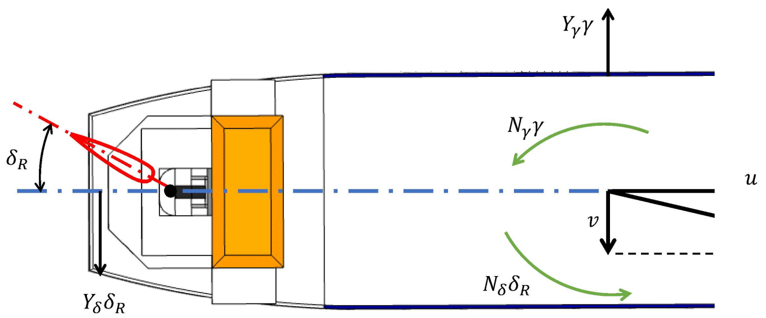

Figure 0-7 Rudder Induced Turning moments

For small rudder deflections (due to small disturbances, for example) and for usual ship configurations,

$$Y_r' \approx 0$$
$$N_{\dot{v}}' \approx 0 \tag{0-45}$$

Applying these assumptions and including the rudder force and moment, the equations of motion become:

$$(I_z' - N_{\dot{r}})\dot{r}' - N_v' v' - N_r' r' = N_\delta' \delta_R \qquad \text{Yaw Moment}$$
$$(m_\Delta' - Y_{\dot{v}}')\dot{v}' - Y_v' v' - (Y_r' + m_\Delta')r' = Y_\delta' \delta_R \qquad \text{Sway Force} \tag{0-46}$$

For conventional ship configurations, we can simplify the mass and inertial terms as follows:

$$(m_\Delta' - Y_{\dot{v}}') \cong 2m_\Delta'$$
$$(I_z' - N_{\dot{r}}') \cong 2I_z' \tag{0-47}$$

We can evaluate the hydrodynamic derivatives for the effect of the rudder on the hull, where δ_R is the rudder angle in radians (positive to **port**):

$$N_\delta' = \frac{\partial N}{\partial \delta_R}$$
$$Y_\delta' = \frac{\partial Y}{\partial \delta_R} \tag{0-48}$$

To make numerical predictions it is necessary to obtain values for some or all of the coefficients or derivatives involved. This is primarily done by means of captive model tests.

8. Captive Model Tests (PMM)

Consider a ship experiencing transverse acceleration, $\dot{v}$ (see Figure 0-8). If the acceleration is to starboard (positive), there will be a reaction force $Y_{\dot{v}}$to port due to the resistance of the water. For a transverse acceleration the force will always resist the direction of acceleration. This is shown in Figure 0-8 with the sway force versus sway acceleration showing a negative slope.

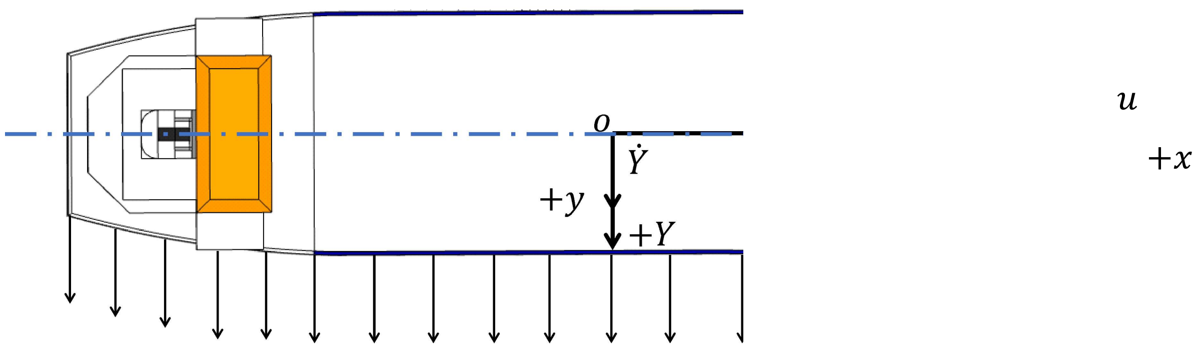

Figure 0-8 Ship Experiencing Transverse Acceleration

Consider a ship experiencing angular acceleration, r (see Figure 0-9). If the acceleration is positive (bow to starboard), there will be a reaction moment $N_{\dot{r}}$ in the negative direction due to the resistance of the water. For an angular acceleration the moment will always resist the direction of acceleration. Therefore, a plot of yaw moment versus yaw acceleration will always have a negative slope and will look like Figure 0-9. Figure 0-11 shows the forces on a body with a sway velocity, v, added to a forward velocity, u. Both the bow and the stern experience a lift force oppositely directed to v. Therefore, Y_v is always negative (see Figure 0-12). However, the bow contribution is usually larger than that of the stern, so there is a negative moment contribution N_v. Yet the addition of a rudder at the stern will increase the magnitude of the stern force and so decrease the negative magnitude of N_v. If the rudder force were sufficiently large, it might even cause N_v to be positive (not usually the case). Figure 0-12 shows the possible relationships between $N_v$and v. Figure 0-14 shows the effect of an angular velocity, r, in addition to forward velocity, u, on $Y$and N. Due to the angular velocity, point B near the bow has a positive transverse velocity, v_B, producing a negative $Y - force$ and a negative $N - moment$. Point S near the stern has a negative transverse velocity, v_B, producing a positive $Y - force$ and a negative $N - moment$. So the moments can combine to produce a large negative moment, but the sway forces oppose each other resulting in a small positive or negative $Y - force$. Figure 0-14 shows the relationship between $Y$and N and r.

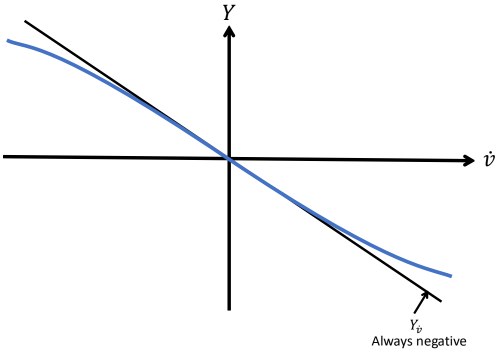

Figure 0-9 Sway Force versus Sway Acceleration

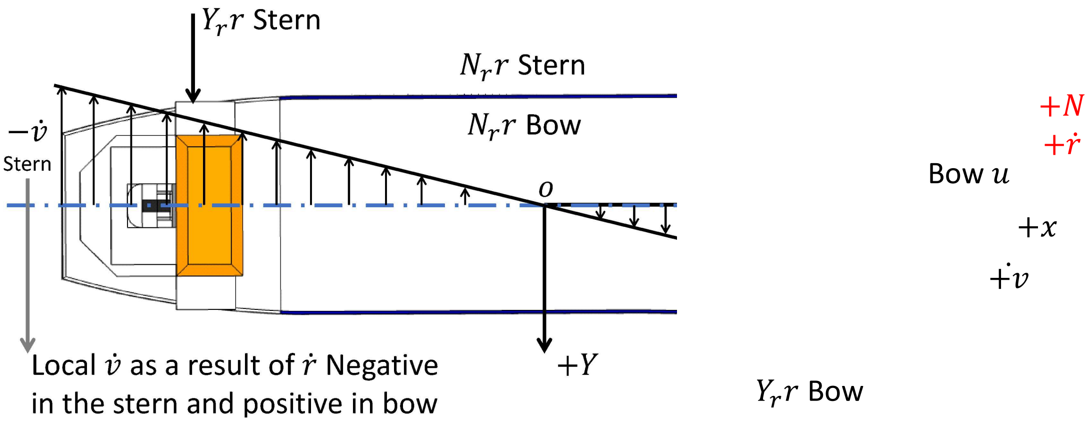

Figure 0-10 Ship Experiencing Angular Acceleration

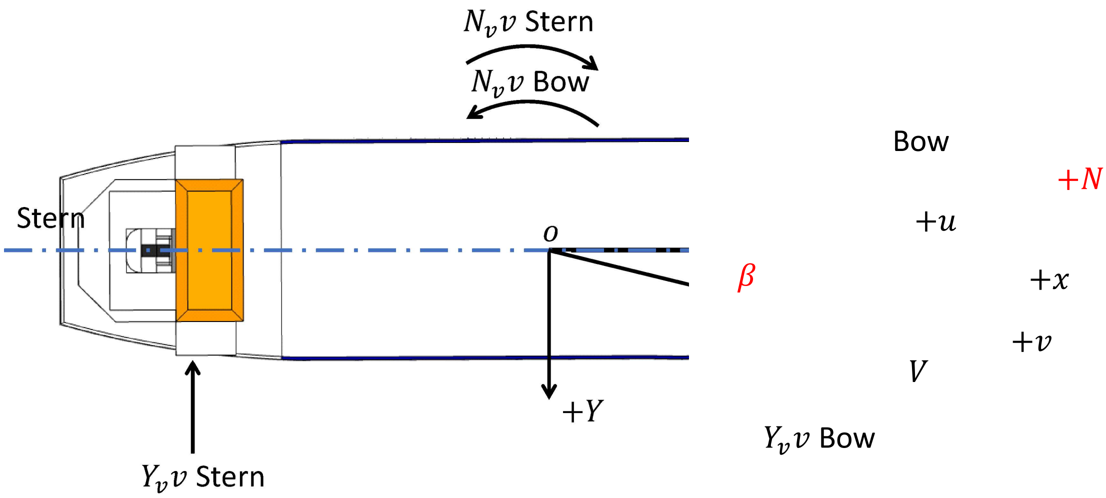

Figure 0-11 Ship Experiencing Forward Velocity and Transverse Velocity

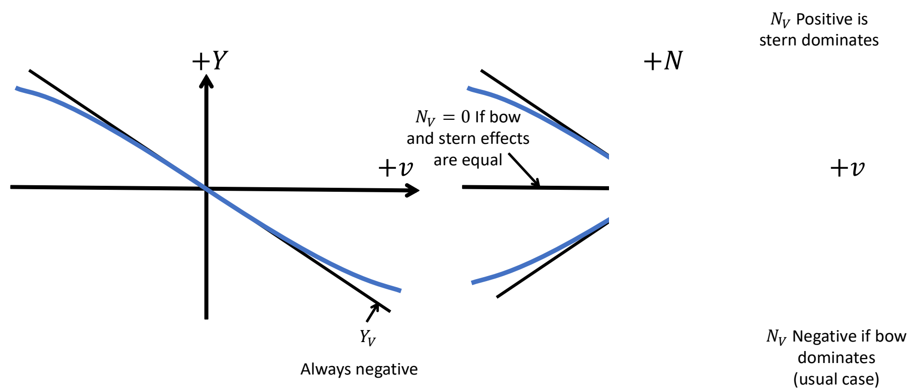

Figure 0-12 Sway Force and Yaw Moment versus Transverse Velocity

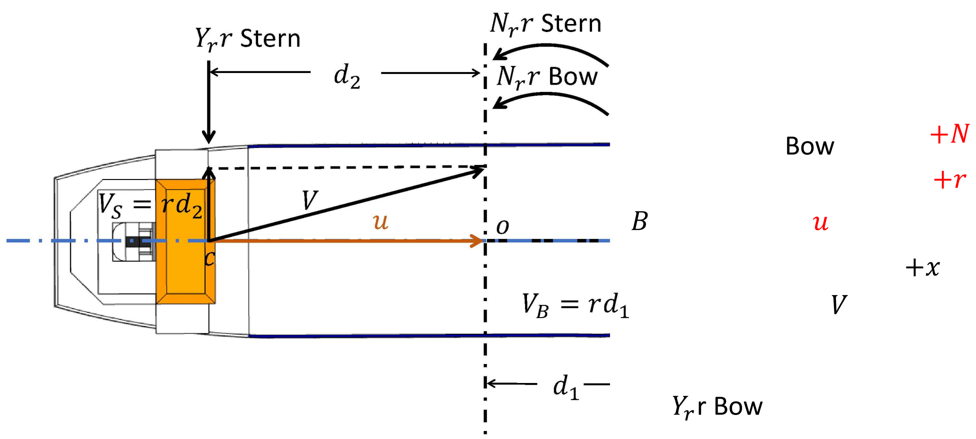

Figure 0-13 Ship Experiencing Forward Velocity and Angular Velocity

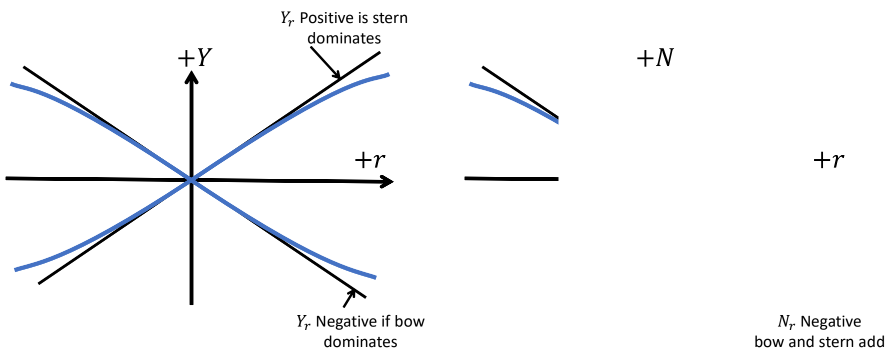

Figure 0-14 Sway Force and Yaw Moment versus Angular Velocity

9. Straight-Line Tests in a Towing Tank

The velocity-dependent derivatives Y_v and $N_v$of a ship at any draft and trim can be determined from measurements on a model of the ship, ballasted to a geometrically similar draft and trim, towed in a conventional towing tank at a constant velocity, U, corresponding to a given ship Froude number, at various angles of attack, β, to the model path. The figure below (Figure 0-15) shows the orientation of the model with respect to the tow tank. From the figure you can see that the transverse velocity component (from the vessel coordinate system) is produced along the y-axis such that

$$v = -U \sin\beta \qquad (0\text{-}49)$$

where the negative sign is due to the sign convention (see Figure 0-6). The $Y - force$ and $N - moment$are measured on the model for each value of β tested. The force or moment versus sway velocity is then plotted and the hydrodynamic coefficient is the slope of the curve near $v = 0$. Figure 0-12 shows an example of sway force (Y) and yaw moment (N) versus sway velocity (v). The slope of the straight line through the curve at $v = 0$ is the hydrodynamic coefficient. So, for the plot Yversus v, you can find the coefficient Y_v and for the plot Nversus v, you can find the coefficient N_v, Let's review:

1) Test a model fixed in yaw (specified drift angle, β) at a constant forward velocity,U.
2) The sway velocity felt by the model is equal to $-U\ sin(\beta)$
3) The sway force and yaw moment are measured on the model
4) For a given U and β you have one point on the Y versus v plot and one point on the N versus v plot. To get additional points, run the test at various drift angles.

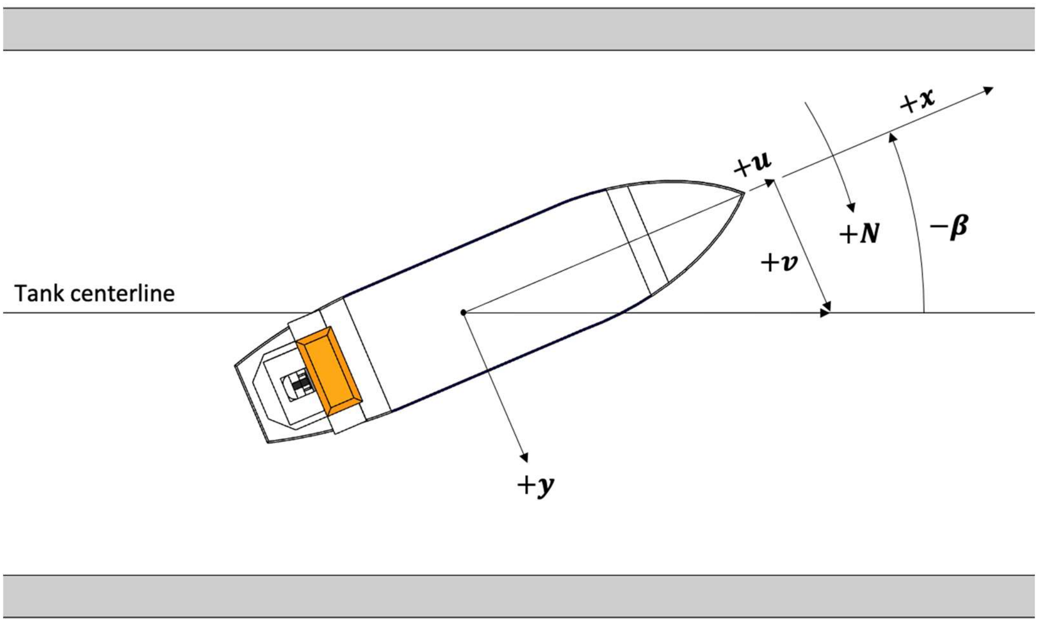

Figure 0-15 Straight Line Tow Testing

The propeller will usually exert an important influence on the hydrodynamic derivatives. Therefore, the model tests to determine these derivatives should be conducted with the propeller operating, preferably at the ship propulsion point. Also, since the undeflected rudder contributes significantly to the derivatives the model tests should also include the rudder in the amidships position.

The technique described above can also be used to determine the control derivatives $Y_\delta$and N_δ . If the model is oriented with zero angle of attack ($\beta = 0$), but the model were towed down the tank at various values of rudder angle, δ_R, the force and moment measurements would determine the force $Y$and moment N as a function of rudder angle. Plots of these against rudder angle will indicate the values of the derivatives Y_δ and N_δ.

Straight-line tests can also be used to determine the cross-coupling effect of v on Y_δ and N_δ and of δ_R on Y_v and N_v.

To measure the rotating derivatives $Y_r$and $N_r$on a model a special type of towing tank and apparatus called a rotating-arm facility is occasionally employed.

- An angular velocity is imposed on the model by fixing it to the end of a radial arm and rotating the arm about a vertical axis fixed in the tank.
- The model revolves about the tank axis, rotates at rate r while its transverse velocity component v is zero at all times (yaw angle of attack or drift angle−$\beta = 0$).
- The model is rotated at a constant linear speed at various radii R and the sway force $Y$and yaw moment N are recorded.
- The angular velocity is given by $r = U / R$, so the only way to vary r at constant U is to vary R.
- The plots of Y and N versus r provide the hydrodynamic derivatives $Y_r$and N_r.

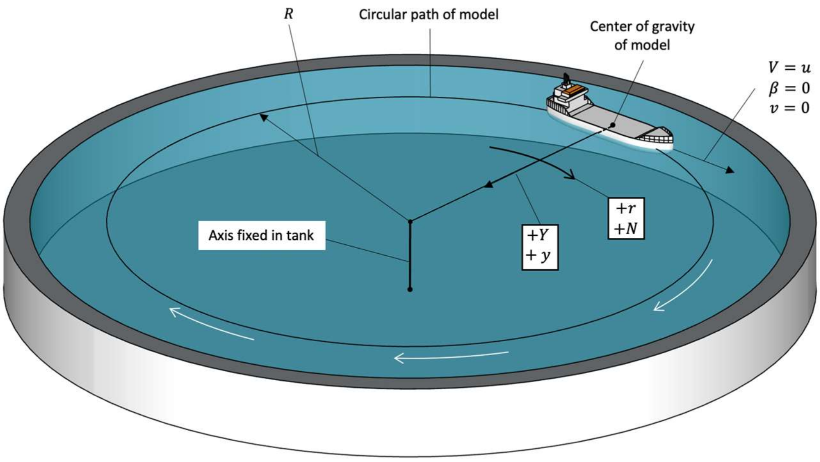

Figure 0-16 Model in Rotating-Arm Facility

Some disadvantages of rotating-arm tests:

1) Require a specialized facility of substantial size. (There are only a few rotating-arm facilities in the world. One is at the David Taylor Research Center in Carderock, MD. Another was at the Davidson Laboratory at Stevens Institute of Technology.)
2) The model must be accelerated and data obtained within a single revolution. Otherwise the model will be running in its own wake and its velocity with respect to the fluid will not be accurately known.
3) In order to obtain values of the derivatives Y_r, N_r , Y_v , and N_v at $r = 0$, data at small values of r are necessary. This means that the ratio of the radius of turn, R to the model length L must be large.

10. Planar Motion Mechanism (PMM) Technique

To avoid the large expense of a rotating-arm facility, a device known as a Planar Motion Mechanism (PMM) was developed for use in the conventional long and narrow towing tank to measure the velocity-dependent and acceleration derivatives.

Basically the PMM consists of two oscillators, one of which produces a transverse oscillation at the bow and the other produces a transverse oscillation at the stern while the model moves down the towing tank at a constant forward velocity, U_0 (measured along the centerline of the tank). Figure 0-17 shows a sample model in a PMM. The forces required from each oscillator are recorded along with the transverse position of the model at each oscillator. The point B near the bow is oscillated transversely with a small amplitude, a_0, and at frequency ω. Point S near the stern is oscillated transversely with the same amplitude, a_0, and the same frequency, ω. The phase difference between the oscillations allows the model to experience yaw. If $\epsilon = 0$, the model experiences pure sway with zero yaw, as shown in Figure 0-18. For a pure sway test, the model is moving transversely in a sinusoidal motion. The sway velocity and acceleration can be found by taking the time derivatives of the position.

$$\begin{aligned} y &= a_0 \, sin \, \omega \, t \\ \frac{dy}{dt} &= v = \omega a_0 \, cos \, \omega \, t \\ \frac{d^2y}{dt^2} &= \dot{v} = -\omega^2 a_0 \, sin \, \omega \, t \end{aligned} \tag{0-50}$$

Therefore, the magnitude of the velocity and acceleration is given by

$$\begin{aligned} v &= a_0 \omega \\ \dot{v} &= \omega^2 a_0 \end{aligned} \tag{0-51}$$

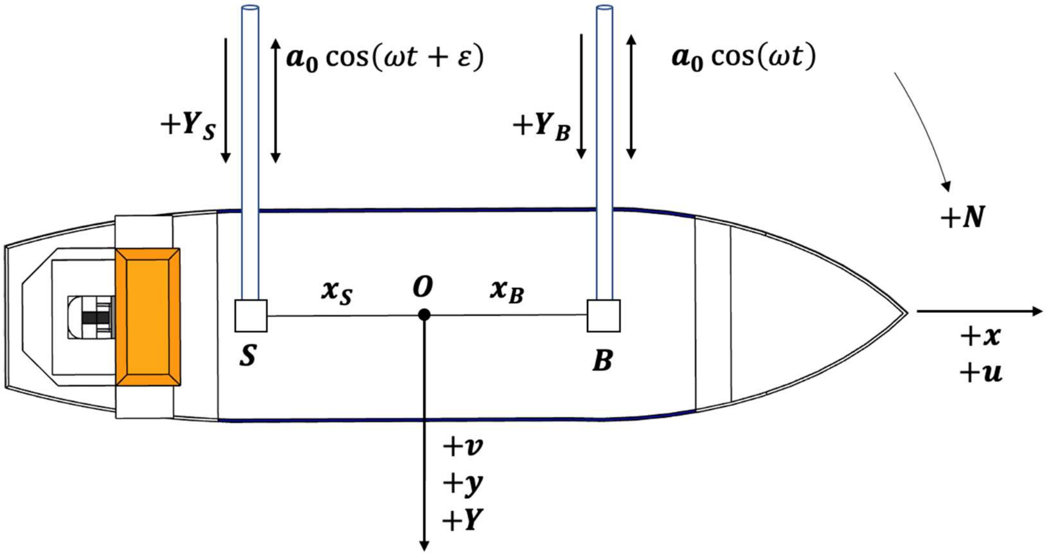

Figure 0-17 Model setup for planar motion tests

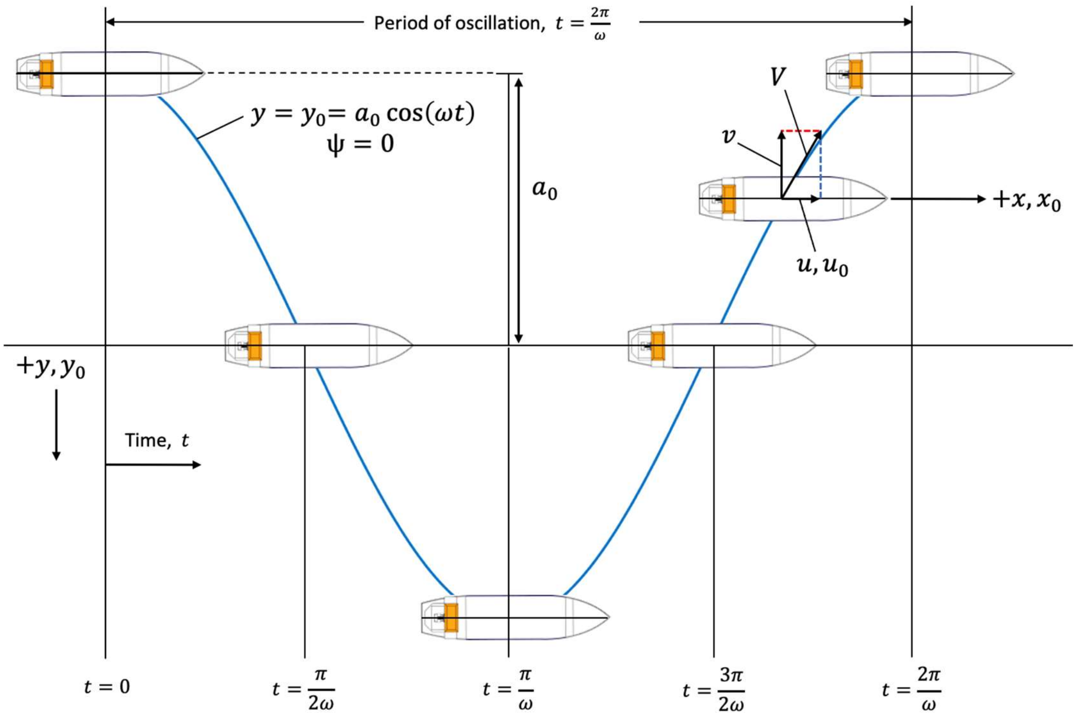

Figure 0-18 Path and orientation of model for pure sway motion

Each oscillator measures the Y-forces experienced by the model as a result of the swaying motion (Y_B and Y_S). To find the Y_v derivative, we need to consider the $Y - force$ in-phase with the velocity (or 90° out-of-phase with the position). To get the magnitude of the $Y - force$ in-phase with the velocity we need to do a FFT of the signal.

This time, however, we will find the sine and cosine components of the signal, rather than the total magnitude. Once we have the components in-phase with the velocity (the cosine components) we can find the derivative Y_v by plotting the Y_{vel} term versus the sway velocity.

$$Y_{vel} = Y_{B_{\cos}} + Y_{S_{\cos}} \tag{0-52}$$

For the yaw moment derivative, a similar procedure can be applied. In this case, the sway force at each oscillator must be multiplied by a distance to get the moment. The distance, X_s, is typically chosen as measured from amidship: (and each point B and S must be equidistant from amidship). This means the hydrodynamic derivative N_v can be determined from plotting the cosine component of the yaw moment versus the sway velocity.

$$N_{vel} = \left(Y_{B_{\cos}} - Y_{S_{\cos}}\right)x_s \tag{0-53}$$

The components of the sway force and yaw moment that are in-phase with the acceleration are the sine components. Therefore, the derivatives Y_v and N_v are found by plotting the $Y_{acc}$and N_{acc} versus the sway acceleration $\dot{v}$.

$$\begin{aligned} Y_{acc} &= Y_{B_{\sin}} + Y_{S_{\sin}} \\ N_{acc} &= \left(Y_{B_{\sin}} - Y_{S_{\sin}}\right)x_s \end{aligned} \tag{0-54}$$

To obtain the angular derivatives Y_r and N_r from planar motion tests, the measurements must be made when $r = 0, v = 0$ and $\dot{v} = 0$. Similarly, for $Y_{\dot{r}}$ and $N_{\dot{r}}$ the measurements need to be taken when $r = 0, v = 0$, and $\dot{v} = 0$. To impose an angular velocity and an angular acceleration on a body with v and $\dot{v}$ equal to zero, the model must be towed down the tank with the centerline of the model always tangent to its path, see Figure 0-19. This means the sway velocity (relative to the model) is always zero. To obtain pure yaw motion using the two oscillators in the PMM, the phase angle, ϵ, must be equal to

$$tan \ \ \epsilon/2 = \frac{\omega x_s}{U} \tag{0-55}$$

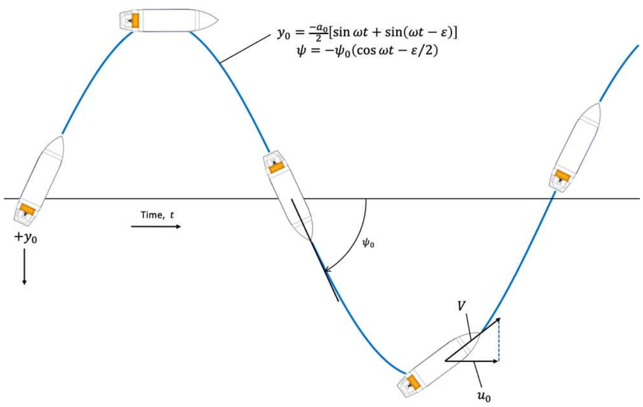

Figure 0-19 Path and orientation of model for pure yaw motion

The yaw oscillation is a sinusoidal motion and of the form

$$\psi = -\psi_0 \, sin(\omega t - \epsilon/2)$$
$$r = \dot{\psi} = -\omega\psi_0 \, cos(\omega t - \epsilon/2) \tag{0-56}$$
$$\dot{r} = \ddot{\psi} = \omega^2\psi_0 \, sin(\omega t - \epsilon/2)$$

The yaw velocity, r is out-of-phase with the angle ψ and the angular acceleration r is in-phase with the angle ψ. Therefore, the amplitudes of Y_B and Y_S measured 90° out-of-phase with ψ will determine the force and moment due to rotation r and the amplitudes of Y_B and Y_S in-phase with the ψ will determine the forces and moment due to angular acceleration r .

$$Y_{\text{angvel}} = Y_{B_{\cos}} + Y_{S_{\cos}}$$
$$N_{\text{angvel}} = \left(Y_{B_{\cos}} - Y_{S_{\cos}}\right)x_s$$
$$Y_{\text{angacc}} = Y_{B_{\sin}} + Y_{S_{\sin}} \tag{0-57}$$
$$N_{\text{angacc}} = \left(Y_{B_{\sin}} - Y_{S_{\sin}}\right)x_s$$

Plotting these forces versus velocity and acceleration can provide the yaw derivatives. The slope of Y_{angvel} versus r gives$(Y_r + m_\Delta U)$, the slope of N_{angvel} versus r gives N_r, the slope of Y_{angacc}versus $\dot{r}$ gives $Y_{\dot{r}}$, and the slope of N_{angvel} versus r gives$(N_{\dot{r}} - I_z)$.

11 Principles of directional stability

Now that we have experimental values for our hydrodynamic derivatives, we can solve the linear sway and yaw equations of motion. Solutions to the linear sway and yaw equations provide linear transfer functions permitting review of the stability of motion. There are various kinds of motion stability associated with ships and they are classified by the attributes of their initial state of equilibrium that are retained in the final path of their centers of gravity. For example, consider Figure 0-20. In each of the cases, the ship is assumed to be traveling at a constant speed along a straight path.

1) For case I - Straight Line Stability: the final path after the disturbance is finished retains the straight-line attribute of the initial state of equilibrium, but not its direction.
2) For case II - Directional Stability: the final path after the disturbance is finished retains not only the straight-line attribute of the initial path, but also its direction.
3) For case III - Directional Stability: the result is the same as for Case II, but without the oscillations.
4) For case IV - Positional Motion Stability: the ship returns to the original path – not only does the final path have the same direction as the original path, but also its same transverse position relative to the surface of the earth.

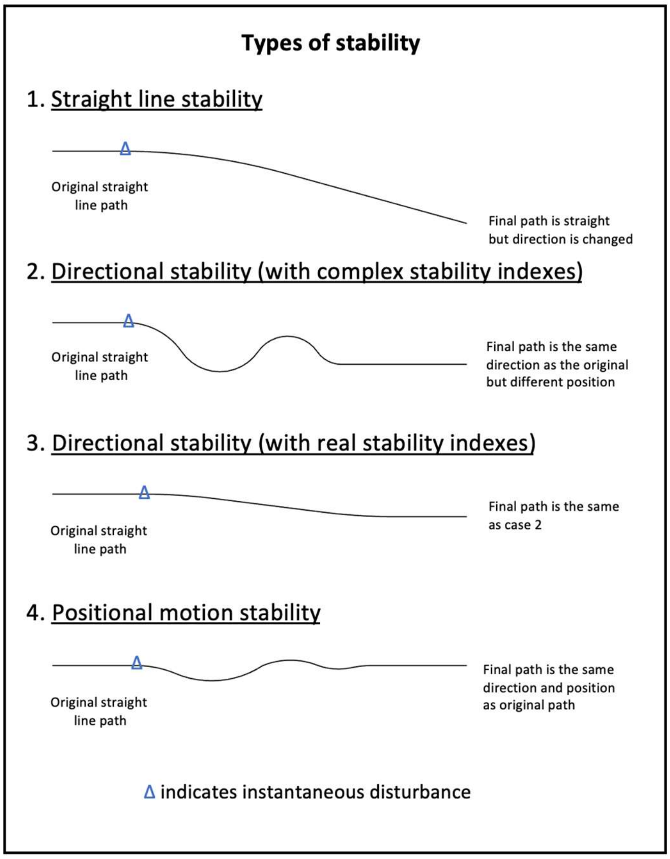

Figure 0-20 Various kinds of motion stability (PNA III, Arentzen 1960)

When operating with controls-fixed in the horizontal plane in the open ocean with stern propulsion, a surface ship does not have directional stability (i.e. if disturbed from its original course it will not return to that course by itself). However, the ship can have Straight-Line Stability (i.e. if disturbed from its original straight-line course, the ship will settle on a final path that is also a straight line).

When operating with controls working you can achieve directional stability. You want the ship to have directional stability with controls working, but also to have straight-line stability with controls fixed. This results in a compromise between rudder size and deadwood size.

We will start by using the linear equations of motion to evaluate the straight-line stability characteristics of a ship.

- We want to understand the effect of ship design features on maneuverability.
- With the rudder fixed on the centerline, we want the ship to have straight-line stability, but just barely.
- This will reduce the size of the rudder and steering gear needed for good maneuverability.

The simultaneous solution of the sway and yaw equations for the sway and yaw velocities yields a second-order differential equation. Working with non-dimensional variables, the solutions for v' and r' correspond to the standard solutions of second-order differential equations:

$$v' = V_1 e^{\sigma_1 t} + V_2 e^{\sigma_2 t}$$
$$r' = R_1 e^{\sigma_1 t} + R_2 e^{\sigma_2 t} \tag{0-58}$$

The variables V_1, V_2, R_1 and R_2 are constants of integration and $\sigma_1$and σ_2 are the stability indexes. If both values of a are negative, v' and r' will approach zero with increasing time which means that the path of the ship will eventually resume a new straight-line direction. If either σ_1 or σ_2 are positive, v' and r' will increase with increasing time and a straight-line path will never be resumed. We can relate these stability indexes, σ , to the hydrodynamic derivatives by substituting the solutions back into the equations of motion. If this is done, a quadratic equation in σ is obtained:

$$A\sigma^2 + B\sigma + C = 0 \tag{0-59}$$

A, B, and C are as follows:

$$A = \left(Y'_{\dot{v}} - m'_{\Delta}\right)\left(N'_{\dot{r}} - I'_z\right) - Y'_{\dot{r}} N'_{\dot{v}}$$
$$B = Y'_v\left(N'_{\dot{r}} - I'_z\right) + N'r\left(Y'_{\dot{v}} - m'_{\Delta}\right) - N'_{\dot{v}}\left(Y'_r + m'_{\Delta}\right) - Y'_{\dot{r}} N'_v \tag{0-60}$$
$$C = Y'_v N'_r - N'_v\left(Y'_r + m'_{\Delta}\right)$$

The two roots, both of which must be negative for *controls-fixed stability* are:

$$\sigma_{1,2} = \frac{-B/A \pm [(B/A)^2 - 4C/A]^{1/2}}{2} \tag{0-61}$$

For both stability roots to be negative (all changes with respect to time are decreasing), two conditions must be met:

$$\frac{B}{A} > 0$$
$$\frac{C}{A} > 0 \tag{0-62}$$

- For conventional ships A is large and positive.
- It can be shown that B is usually large and positive and on the same order of magnitude as A.
- This means that the determining factor will be C.

For both stability roots to be negative, $C > 0$! Therefore,

$$C = Y'_v N'_r - N'_v\left(Y'_r + m'_{\Delta}\right) > 0 \tag{0-63}$$

Rewriting we can say,

$$\frac{N'_r}{Y'_r + m'_{\Delta}} - \frac{N'_v}{Y'_v} > 0 \tag{0-64}$$

We can calculate the directional straight-line stability after having performed the PMM tests on a model, but what can we say generally about controls-fixed straight-line stability from what we know about the hydrodynamic derivatives?

The terms $N_r^{'}$ and $Y_v^{'}$ are always negative, and generally large relative to $Y_r^{'}$ and $N_v^{'}$. If the bow is dominate (the usual condition), $Y_r^{'}$ and $N_v^{'}$ are negative. So, in a conventional craft, the ration $\frac{N_v^{'}}{Y_v^{'}}$ will be small and since $\frac{N_r^{'}}{Y_r^{'}+m_\Delta^{'}}$ is likely to be larger, the ship will have directional stability. For a conventional hull (where the bow dominates), directional stability can be increased by increasing the magnitude of $Y_v^{'}$ and $N_r^{'}$. Adding a larger rudder in the stern of the ship increases the directional stability of the ship by decreasing the magnitudes of $Y_r^{'}$ and $N_v^{'}$. The response of the ship to deflection of the rudder, and the resulting forces and moments produced by the rudder, can be divided into 2 portions:

1) An initial transient one in which significant surge, sway and yaw accelerations occur.
2) A steady turning portion in which rate of turn and forward speed are constant and the path of the ship is circular

Figure 0-21 shows the turning path of a ship. Generally, the turning path of a ship is characterized by four numerical measures: advance, transfer, tactical diameter, and steady turning diameter. All but the last are related to heading positions of the ship rather than tangents to the turning path. The advance is the distance from the origin at "execute" to the x-axis of the ship when that axis has turned 90°. The **transfer** is the distance from the original approach course to the origin of the ship when the x-axis has turned $90°$. The **tactical diameter** is the distance from the approach course to the x-axis of the ship when that axis has turned $180°$. These parameters of a ship's turning circle are useful for characterizing maneuvers in the open sea. There are three phases in turns defined as follows :

- **Phase I:** The first phase starts the instant the rudder begins to deflect and may be completed by the time the rudder reaches full deflection. The rudder force $(Y_\delta \delta_R)$ and the rudder moment $(N_\delta \delta_R)$ produce accelerations and are opposed solely by the inertial reaction of the ship (hydrodynamic responses have not yet materialized). For this phase the ship has not changed direction, so $\beta = v/U = r = 0$. The linearized, dimensional equations for the first phase of turning are

$$\begin{aligned}(m_\Delta - Y_{\dot{v}})\dot{v} - Y_{\dot{r}}\dot{r} = Y_\delta \delta_R \\ (I_z - N_{\dot{r}})\dot{r} - N_{\dot{v}}\dot{v} = N_\delta \delta_R\end{aligned} \qquad (0\text{-}65)$$

These accelerations ($\dot{v}$ and $\dot{r}$) exist only for a moment, for they quickly give rise to a drift angle, β, and a rotation, r, of the ship.

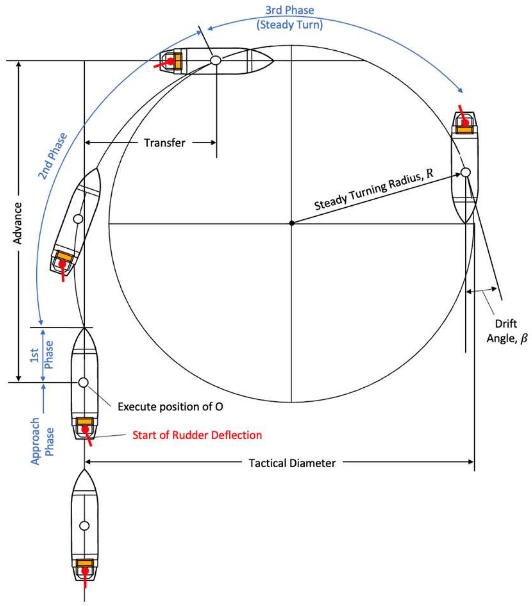

Figure 0-21 Turning Path of a Ship

- **Phase II:** The second phase starts with the introduction of the drift angle, β, and a rotation, r, of the ship. Here the accelerations of the ship coexist with the velocities and all the terms of the equations of motion along with the excitation terms $(Y_\delta \delta_R)$ and $(N_\delta \delta_R)$ are fully operative. The crucial event at the beginning of the second phase of the turn is the creation of a $Y_v v - force$ positively directed towards the center of the turn. This force is introduced due to the drift angle, β. The magnitude of this force soon becomes larger than the $Y_\delta \delta_R - force$ which is directed to the outside of the circle. The acceleration $\dot{v}$ ceases to grow to the outside of the circle and eventually becomes zero as the inwardly directed $Y_v v - force$ comes into balance with the outwardly directed force of the ship. In the second phase of the turn, the path of the center of gravity of the ship at first responds to the $Y_\delta \delta_R$-force and tends towards the outside of the circle before the $Y_v v - force$ grows large enough to enforce the inward turn.
- **Phase III:** Finally, after some oscillation (some of which is due to the settling down of the main propulsion machinery and is characteristic of the particular ty pe of machinery and its control system) the second phase of turning ends with the establishment of the final equilibrium of forces. When this equilibrium is reached, the ship settles down to a turn of constant radius.

  This is the third, or steady, phase of the turn. In this phase v and r have non-zero values, but $\dot{v}$ and $\dot{r}$ are zero. For this phase of the turn, the linearized equations of motion are:

$$\begin{gathered} -Y_v v - (Y_r + m_\Delta U) r = Y_\delta \delta_R \\ -N_v v - N_r r = N_\delta \delta_R \end{gathered} \tag{0-66}$$

  These two simultaneous equations can be solved for r and v provided that the stability derivatives $(Y_v, Y_r, N_v,$ and $N_r)$and the control derivatives $(Y_\delta$ and $N_\delta)$ are known. Note that

$$r' = \frac{rL}{U} \quad r = \frac{U}{R} \quad r' = \frac{L}{R} \tag{0-67}$$

Solving the non-dimensional version of the linearized equations of motion shown above, we can solve for the turning radius, R, and the sway velocity, v':

$$\frac{R}{L} = -\frac{1}{\delta_R}\left[\frac{Y_v' N_r' - N_v'\left(Y_r' + m_\Delta'\right)}{Y_v' N_\delta' - N_v' Y_\delta'}\right]$$
$$v' = -\beta = \delta_R\left[\frac{N_\delta'\left(Y_r' + m_\Delta'\right) - Y_\delta' N_r'}{Y_v' N_r' - N_v'\left(Y_r' + m_\Delta'\right)}\right] \tag{0-68}$$

A positive R denotes a starboard turn. The equation for the turn radius shows that :

- ✓ The steady turning radius is proportional to ship length and inversely proportional to rudder angle.
- ✓ Side velocity is equal to the drift angle and that is directly proportional to the rudder angle.
- ✓ Denominator in the equation for R introduces the effect of the rudder on the hull $\left(N_\delta' \text{ and } Y_\delta\right)$
  - o Sign of denominator is always positive
  - o If the numerator is negative (straight-line stability) and the rudder is at the stern, a negative δ_R will give a positive R.

To decrease the turning radius we can:

- ✓ Decrease Y_v'- could increase L/T ratio, but this is de-stabilizing
- ✓ Generally increase N_v' (if N_v' is negative) - this is a result of different bow and stern shapes. Changes could be made by cutting away skeg and deadwood aft or increasing forefoot forward.
- ✓ Increase (obvious choice) - the trick is to do it without increasing $1/\delta$ too much. Want to move the rudder as far aft as possible and make the rudder as efficient as possible.
- ✓ Increase Y_δ' (only if N_v' is negative) - can do this with a larger and/ or more efficient rudder.

12 Questions

1-What is the added resistance? Why is it a second-order problem? Discuss the role of added resistance in ship design.

2- when does the added resistance reach its peak? Why? When does the radiation/diffraction induced resistance dominate the wave added resistance? Use neat sketches to elaborate your answers.

3- Discuss briefly the methods used to calculate added resistance in waves.

4- Explain the course keeping, maneuvering and speed changing. Why are these controllability elements vital for ships? How can we design a ship with good controllability?

5- What are the types of forces acting on a ship during maneuvering? Why the control forces and moments are vital during maneuvering ?

6- Explain briefly with neat sketch the straight-line test, rotating arm test and Planar Motion Mechanism (PMM) Technique.

7- Explain the directional stability? What are the types of motion stability and how are they classified? Use neat sketches and brief explanations to elaborate your answers

8- What are the three phases of a turn? How can we decrease the turning radius? Use neat sketch and brief explanations to elaborate your answers

9- Use a neat sketch to describe the main parameters and hydrodynamic forces of a standard spade rudder.

10- What are the basic considerations in rudder design to reduce vibration and optimize its design?

13 References

Arribas, F. P. (2007). Some methods to obtain the added resistance of a ship advancing in waves. *Ocean Engineering* 34 (7):946-955.

Faltinsen, O.(1993). *Sea loads on ships and offshore structures*. Cambridge university press.

Gerritsma, J., and Beukelman, W. (1972). Analysis of the resistance increase in waves of a fast cargo ship. *International shipbuilding progress*, 19 (217):285-293.

Journée, J.M.J. (2001). *Theoretical manual of SEAWAY.* Delft University of Technology Ship hydromechanics Laboratory.

Salvesen, Nils (1978). Added resistance of ships in waves, *Journal of Hydronautics*, 12 (1):24-34.

Ström-Tejsen, J. (1973). *Added resistance in waves,* The Naval Ship R&D Center, Bethesda, Maryland, USA, Paper 3 of the Annual Meeting New York of The Society of Naval Architects and Marine Engineers, SNAME Transactions 1973, Paper: T1973-1

Appendix 1

Review of Probability and Statistics for Marine Applications.

We have shown that the irregular time histories of waves can be characterized in terms of energy spectra and various statistical quantities. Seakeeping studies, however, often demand a more intimate knowledge of waves. In particular, we need to be able to answer questions like "What is the likelihood of a particular wave height being exceeded?" We can use wave energy spectra and probability distributions to answer this type of question.

Probability Density Function (PDF)

The probability density function is defined such that the area enclosed by the PDF curve over a bin is equal to the probability of the measurement falling within that bin. So, the probability of the x-axis value falling between a and b is equal to the area under the curve from a to b. Figure 3.17 shows the area from a to b for a normal probability distribution curve. The probability the water elevation falls between these two limits is equal to the shaded area on the plot. The area under the entire probability density function equals one, since there is 100% probability that any measurement falls within the set of collected measurements. Water elevation typically follows a Gaussian or normal distribution. This is the typical "bell" curve, see Figure 3.18. The empirical rule states that there is about a 68% probability a measurement will fall between $\pm\sigma$ (one standard deviation), there is about a 95% probability a measurement will fall between $\pm 2\sigma$, and a 99% probability any measurement will fall between $\pm 3\sigma$ .While *water elevation* typically follows a Gaussian distribution, *wave heights* (and amplitudes) follow a Rayleigh distribution for narrow-banded wave spectra.

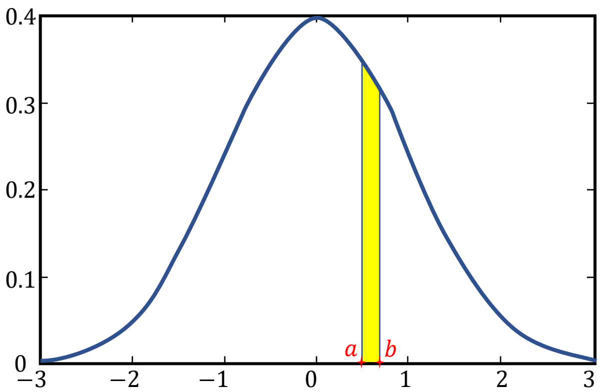

Figure A-0-1 The probability of the wave elevation falling between a and b equals the area under the pdf curve between these two values

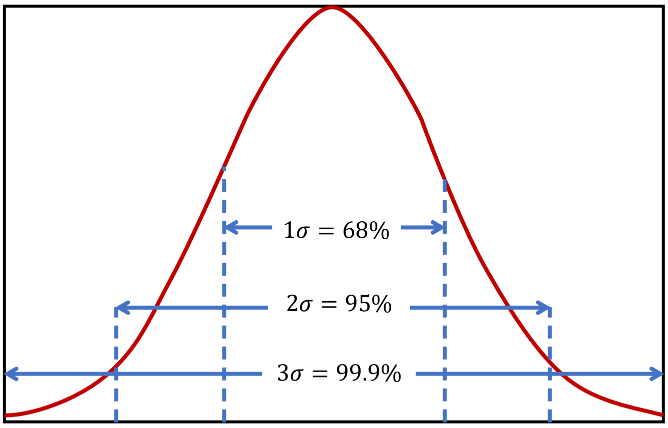

Figure A-0-2 Gaussian or Normal Probability Distribution

The probability for the wave amplitudes depends on the variance of the water elevation. Figure A-0-3 shows a typical Rayleigh distribution. The probability a wave amplitude would fall between two heights is equal to the area under the curve between those two points. The Rayleigh probability distribution equals

$$f = \frac{\zeta_a}{m_0} e^{-\frac{\zeta_a^2}{2m_0}} \tag{A-1}$$

where ζ_a is the wave amplitude and m_0 is the variance from the water elevation time history or area under the wave energy spectrum curve.

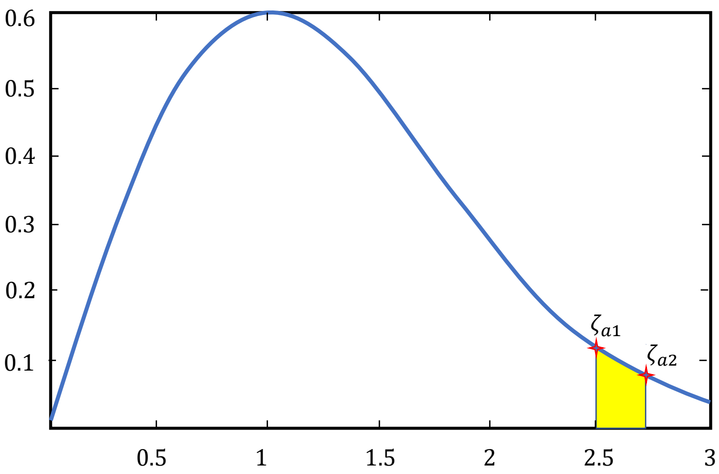

Figure A-0-3 Rayleigh Probability Distribution

Considering the wave amplitudes (Rayleigh probability distribution), the probability that an amplitude ζ_a will exceed a specific amplitude, is

$$P(\zeta_a > \zeta_{A1}) = e^{-\frac{\zeta_{A1}^2}{2m_0}} \tag{A-2}$$

The probability that the wave amplitude will fall *between* amplitudes and ζ_{A2} is

$$P(\zeta_{A1} < \zeta_a < \zeta_{A2}) = e^{-\frac{\zeta_{A2}^2}{2m_0}} - e^{-\frac{\zeta_{A1}^2}{2m_0}} \tag{A-3}$$

i.e. the probability of exceeding ζ_{A2} minus the probability of exceeding .

Significant Wave Height and Related Statistics

The significant wave height is the mean of the highest 1/3rd of the heights recorded in a wave time history. It closely correlates with the average wave height estimated visually by an experienced observer. It is expected that the experienced sailor's estimates of "average" wave heights might be similar to the significant wave height. The Rayleigh formula for the mean value of the highest $1/n$th of all observations is

$$\zeta_{\frac{1}{n}} = \sqrt{-2m_0 \, ln \frac{1}{n}} \tag{A-4}$$

So, for n = 1, the mean of all amplitudes, $\bar{\zeta}_a = 1.25\sigma_0$where σ_0 is the standard deviation from the water surface elevation$\left(\sigma_0 = \sqrt{m_0}\right)$. For the significant wave amplitude $\bar{\zeta}_{1/3} = 2.00\sigma_0$. Significant wave height, $\bar{H}_{1/3}$ = $4.00\sqrt{m_0}$. This is the same as saying that the significant wave height is equal to twice the significant wave amplitude. These results are widely assumed to apply to all wave records. However, this is only strictly true if the Rayleigh formula applies. Table below shows the values that can be multiplied by the water elevation standard deviation (σ_0) to determine the average of the $1/n^{th}$ highest amplitudes.

n	$\frac{\zeta_{1/n}}{\sigma_o}$	n	$\frac{\zeta_{1/n}}{\sigma_o}$
1	1.25	10	2.54
2	1.77	100	3.34
3	2.00	1000	3.72

Probability of Exceedance

So, what is the procedure for predicting the probability of the wave amplitude exceeding a particular value ζ_B in a specific sea state?

1. First we build an ITTC wave energy spectrum for our sea state (using the mean significant wave height and most probable modal period). We will need to convert the modal period, T_0, into modal frequency, $\omega_0 = 2\pi/T_0$.

$$S_\zeta(\omega) = \frac{1.25}{4}\left(\frac{\omega_0}{\omega}\right)^4 \frac{\bar{H}_{1/3}^2}{\omega} e^{-1.25\left(\frac{\omega_0}{\omega}\right)^4} \tag{A-5}$$

2. We can find the variance, m_0 from this wave energy spectrum.

$$m_0 = \int_0^\infty S_\zeta(\omega)d\omega \tag{A-6}$$

3. Then, we use the variance and the value of interest to calculate the probability of exceedance

$$P(\zeta_a > \zeta_B) = e^{-\frac{\zeta_B^2}{2m_0}} \tag{A-7}$$

<u>Example</u>

Consider the ocean spectrum for a Brettschneider sea state 6. For the time history recorded (a total of 23.5 $minutes$), the variance of the water elevation was $16.81f^2$. Find the significant wave height and the probability of a wave height exceeding 25 ft. Find the probability of exceeding the significant wave height. How did we get m_0 (variance of the water elevation)? It was found either by taking the variance of the time history (as in this problem) or by finding the area under the wave energy spectrum (as explained in the procedure above). Once we have it, we can find the significant wave height directly

$$\bar{H}_{1/3} = 4.00\sqrt{m_0} = 4.00\sqrt{16.81} = 16.4\text{ft} \tag{A-8}$$

To find the probability of exceedance we plug this into the equation 3.12. This equation requires us to use the wave amplitude. Since we want the probability of exceedance for a wave height of 25 ft, the corresponding wave amplitude is $25/2 \ = \ 12.5 \ ft$.

$$P(\zeta_a > 12.5) = e^{-\frac{12.5^2}{2(16.81)}} = 0.0096 = 0.96\% \tag{A-9}$$

So, there is a 0.96% probability that we will encounter a wave height over 25 ft. The probability we will exceed the significant wave height of 16.4 ft (amplitude of 8.2 ft) is

$$P(\zeta_a > 8.2) = e^{-\frac{8.2^2}{2(16.81)}} = 0.1353 = 13.53\% \tag{A-10}$$