

1. Proof of Exercise 5 Demo

Let x be a p -variate continuous random variable. Show that $\text{Cov}[x]$ is positive semidefinite.

Recall the definition for positive semi-definiteness:

Definition 1.1. A symmetric and real-valued $p \times p$ matrix A is said to be positive semidefinite if the scalar $a^\top A a$ is non-negative for every real-valued column vector $a \in \mathbb{R}^p$.

Now,

$$\begin{aligned} a^\top \text{cov}[x] a &= a^\top \mathbb{E} \left[(x - \mathbb{E}[x]) (x - \mathbb{E}[x])^\top \right] a \\ &= \mathbb{E} \left[a^\top (x - \mathbb{E}[x]) (x - \mathbb{E}[x])^\top a \right] && | \quad y = a^\top (x - \mathbb{E}[x]) \in \mathbb{R} \\ &= \mathbb{E} [y y^\top] = \mathbb{E} [y^2] \geq 0. \end{aligned}$$