

MEC-E8001 Finite Element Analysis; Mathematica

“Structure is a collection of elements connected by nodes. Geometry, displacement, temperature etc. of the structure are defined by the nodal values of coordinates, translation, rotation, temperature etc. of which some are known and some unknown.”

STRUCTURE

$prb = \{ele, fun\}$ where

$ele = \{prt_1, prt_2, \dots\}$ elements

$fun = \{val_1, val_2, \dots\}$ nodes

Element

$prt = \{typ, pro, geo\}$ where

$typ = \text{BAR} | \text{TORSION} | \text{BEAM} | \text{RIGID} | \dots$ model

$pro = \{p_1, p_2, \dots, p_n\}$ properties

$geo = \text{Point}[\{n_1\}] | \text{Line}[\{n_1, n_2\}] | \text{Polygon}[\{n_1, n_2, n_3\}] | \dots$ geometry

Nodes

$val = \{crd, trn, rot\} | \{crd, trn, rot, tmp\}$ where

$crd = \{X, Y, Z\}$ structural coordinates

$trn = \{u_X, u_Y, u_Z\}$ translation components

$rot = \{\theta_X, \theta_Y, \theta_Z\}$ rotation components

$tmp = \mathcal{G}$ temperature

DISPLACEMENT ANALYSIS

Constraint

- {JOINT,{{},{u_X,u_Y,u_Z}},Point[{n₁}}]displacement constraint
- {JOINT,{{},Line[{n₁,n₂}}]displacement constraint
- {RIGID,{{},{u_X,u_Y,u_Z},{θ_X,θ_Y,θ_Z}},Point[{n₁}}]displacement/rotation constraint
- {RIGID,{{},Line[{n₁,n₂}}]rigid constraint
- {SLIDER,{n_X,n_Y,n_Z},Point[{n₁}}] slider constraint

Force

- {FORCE,{F_X,F_Y,F_Z},Point[{n₁}}] point force
- {FORCE,{F_X,F_Y,F_Z,M_X,M_Y,M_Z},Point[{n₁}}]point load
- {FORCE,{f_X,f_Y,f_Z},Line[{n₁,n₂}}]distributed force
- {FORCE,{f_X,f_Y,f_Z},Polygon[{n₁,n₂,n₃}}]distributed force

Beam model

- {BAR,{{E},{A},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}]bar mode
- {TORSION,{{G},{J},{m_X,m_Y,m_Z}},Line[{n₁,n₂}}] torsion mode
- {BEAM,{{E,G},{A,I_{yy},I_{zz}},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}]beam
- {BEAM,{{E,G},{A,I_{yy},I_{zz}},{j_X,j_Y,j_Z},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}]beam

Plate model

- {PLANE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}] thin slab mode
- {PLANE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃,n₄}}] thin slab mode
- {PLATE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}] bending mode
- {SHELL,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}]plate

Solid model

- {SOLID,{{E,ν},{f_X,f_Y,f_Z}},Tetrahedron[{n₁,n₂,n₃,n₄}}]solid
- {SOLID,{{E,ν},{f_X,f_Y,f_Z}},Hexahedron[{n₁,n₂,n₃,n₄,n₅,n₆,n₇,n₈}}]solid
- {SOLID,{{E,ν},{f_X,f_Y,f_Z,m_X,m_Y,m_Z}},Tetrahedron[{n₁,n₂,n₃,n₄}}]solid

OPERATIONS

- prb* = REFINE[*prb*] refine structure representation
- Out = FORMATTED[*prb*]display problem definition

Out = STANDARDFORM[*prb*] display virtual work expression

sol = SOLVE[{DISP}, *prb*] | SOLVE[*prb*] solve the unknowns

VIBRATION ANALYSIS

Constraint

- {JOINT, { } | { { u_X, u_Y, u_Z } }, Point[{ n_1 }]}displacement constraint
- {JOINT, { }, Line[{ n_1, n_2 }]}displacement constraint
- {RIGID, { } | { { u_X, u_Y, u_Z }, { $\theta_X, \theta_Y, \theta_Z$ } }, Point[{ n_1 }]}displacement/rotation constraint
- {RIGID, { }, Line[{ n_1, n_2 }]}rigid constraint
- {SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }]} slider constraint

Force

- {FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }]} point force
- {FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }]}point load
- {FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }]}distributed force
- {FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }]}distributed force
- {FORCE, { { m } }, Point[{ n_1 }]}inertia effect
- {FORCE, { { m, J } }, Point[{ n_1 }]}inertia effect
- {FORCE, { { m, J }, { { i_X, i_Y, i_Z }, { j_X, j_Y, j_Z } } }, Point[{ n_1 }]}inertia effect

Beam model

- {BAR, { { E, ρ }, { A }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }]}bar mode
- {TORSION, { { G, ρ }, { J }, { m_X, m_Y, m_Z } }, Line[{ n_1, n_2 }]} torsion mode
- {BEAM, { { E, G, ρ }, { A, I_{yy}, I_{zz} }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }]} beam
- {BEAM, { { E, G, ρ }, { A, I_{yy}, I_{zz} }, { j_X, j_Y, j_Z } }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }]}beam

Plate model

- {PLANE, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }]} thin slab mode
- {PLANE, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }]} thin slab mode
- {PLATE, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }]} bending mode
- {SHELL, { { E, ν, ρ }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }]}plate

Solid model

- {SOLID, { { E, ν, ρ }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }]}solid
- {SOLID, { { E, ν, ρ }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }]}solid
- {SOLID, { { E, ν, ρ }, { $f_X, f_Y, f_Z, m_X, m_Y, m_Z$ } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }]}solid

OPERATIONS

$prb = \text{REFINE}[prb]$ refine structure representation
Out = $\text{FORMATTED}[prb]$ display problem definition
Out = $\text{STANDARDFORM}\{\text{VIBR}\}, prb$ display virtual work expression
sol = $\text{SOLVE}\{\text{VIBR}\}, prb$ solve the eigenfrequencies and modes
sol = $\text{SOLVE}\{\text{VIBR,ini}\}, prb$ solve the unknowns (does not work with DAEs)

STABILITY ANALYSIS

Constraint

{JOINT,{{},{u_X,u_Y,u_Z}},Point[{n₁}}]displacement constraint
{JOINT,{{},Line[{n₁,n₂}}]displacement constraint
{RIGID,{{},{u_X,u_Y,u_Z},{θ_X,θ_Y,θ_Z}},Point[{n₁}}]displacement/rotation constraint
{RIGID,{{},Line[{n₁,n₂}}]rigid constraint
{SLIDER,{n_X,n_Y,n_Z},Point[{n₁}}] slider constraint

Force

{FORCE,{F_X,F_Y,F_Z},Point[{n₁}}] point force
{FORCE,{F_X,F_Y,F_Z,M_X,M_Y,M_Z},Point[{n₁}}]point load
{FORCE,{f_X,f_Y,f_Z},Line[{n₁,n₂}}]distributed force
{FORCE,{f_X,f_Y,f_Z},Polygon[{n₁,n₂,n₃}}]distributed force

Beam model

{BAR,{{E},{A},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}]bar mode
{TORSION,{{G},{J},{m_X,m_Y,m_Z}},Line[{n₁,n₂}}] torsion mode
{BENDING,{{E},{I_y,I_z},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}] bending mode
{BEAM,{{E,G},{A,I_{yy},I_{zz}},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}]beam
{BEAM,{{E,G},{A,I_{yy},I_{zz}},{j_X,j_Y,j_Z},{f_X,f_Y,f_Z}},Line[{n₁,n₂}}] beam

Plate model

{PLANE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}] thin slab mode
{PLANE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃,n₄}}] thin slab mode
{PLATE,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}] bending mode
{SHELL,{{E,ν},{t},{f_X,f_Y,f_Z}},Polygon[{n₁,n₂,n₃}}]plate

Solid model

{SOLID,{{E,ν},{f_X,f_Y,f_Z}},Tetrahedron[{n₁,n₂,n₃,n₄}}]solid
{SOLID,{{E,ν},{f_X,f_Y,f_Z}},Hexahedron[{n₁,n₂,n₃,n₄,n₅,n₆,n₇,n₈}}]solid
{SOLID,{{E,ν},{f_X,f_Y,f_Z,m_X,m_Y,m_Z}},Tetrahedron[{n₁,n₂,n₃,n₄}}]solid

OPERATIONS

prb = REFINE[*prb*] refine structure representation

Out = FORMATTED[*prb*]display problem definition
Out = STANDARDFORM[{STAB}, *prb*] display virtual work expression
sol = SOLVE[{STAB, *p*}, *prb*]find the critical values of *p* and the modes

NONLINEAR ANALYSIS

Constraint

- {JOINT, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ } }, Point[{ n_1 }] }displacement constraint
- {JOINT, { }, Line[{ n_1, n_2 }] }displacement constraint
- {RIGID, { } | { { $\underline{u}_X, \underline{u}_Y, \underline{u}_Z$ }, { $\underline{\theta}_X, \underline{\theta}_Y, \underline{\theta}_Z$ } }, Point[{ n_1 }] }displacement/rotation constraint
- {RIGID, { }, Line[{ n_1, n_2 }] }rigid constraint
- {SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

- {FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force
- {FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] }point load
- {FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] }distributed force
- {FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] }distributed force

Beam model

- {BAR, { { E }, { A }, { f_X, f_Y, f_Z } }, Line[{ n_1, n_2 }] }bar mode

Plate model

- {PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode
- {PLANE, { { E, ν }, { t }, { f_X, f_Y, f_Z } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode

Solid model

- {SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] }(nonlinear) solid
- {SOLID, { { E, ν }, { f_X, f_Y, f_Z } }, Hexahedron[{ $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ }] }(nonlinear) solid
- {SOLID, { { E, ν }, { $f_X, f_Y, f_Z, m_X, m_Y, m_Z$ }, } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] } ... (nonlinear) solid

OPERATIONS

- prb* = REFINE[*prb*] refine structure representation
- Out = FORMATTED[{NONL}, *prb*]display problem definition
- Out = STANDARDFORM[{NONL}, *prb*] display virtual work expression
- sol* = SOLVE[{NONL}, *prb*] find the likely numerical solution
- sol* = SOLVE[{NONL, ALL}, *prb*] find all solutions

THERMO-MECHANICAL ANALYSIS

Constraint

{JOINT, { } | { { u_X, u_Y, u_Z } }, Point[{ n_1 }] } displacement constraint

{JOINT, { }, Line[{ n_1, n_2 }] } displacement constraint

{RIGID, { } | { { u_X, u_Y, u_Z }, { $\theta_X, \theta_Y, \theta_Z$ } }, Point[{ n_1 }] } displacement/rotation constraint

{RIGID, { }, Line[{ n_1, n_2 }] } rigid constraint

{SLIDER, { n_X, n_Y, n_Z }, Point[{ n_1 }] } slider constraint

Force

{FORCE, { F_X, F_Y, F_Z }, Point[{ n_1 }] } point force

{FORCE, { $F_X, F_Y, F_Z, M_X, M_Y, M_Z$ }, Point[{ n_1 }] } point load

{FORCE, { f_X, f_Y, f_Z }, Line[{ n_1, n_2 }] } distributed force

{FORCE, { f_X, f_Y, f_Z }, Polygon[{ n_1, n_2, n_3 }] } distributed force

Beam model

{BAR, { { E, α, k }, { A }, { { f_X, f_Y, f_Z }, { s, \mathcal{G}_0 } } }, Line[{ n_1, n_2 }] } bar mode

Plate model

{PLANE, { { E, ν, α, k }, { t }, { { f_X, f_Y, f_Z }, { s, \mathcal{G}_0 } } }, Polygon[{ n_1, n_2, n_3 }] } thin slab mode

{PLANE, { { E, ν, α, k }, { t }, { { f_X, f_Y, f_Z }, s, \mathcal{G}_0 } } }, Polygon[{ n_1, n_2, n_3, n_4 }] } thin slab mode

Solid model

{SOLID, { { E, ν, α, k }, { { f_X, f_Y, f_Z }, { s, \mathcal{G}_0 } } }, Tetrahedron[{ n_1, n_2, n_3, n_4 }] } solid

Functions

prb = REFINE[*prb*] refine structure representation

Out = FORMATTED[*prb*] display problem definition

Out = STANDARDFORM[{TMEC}, *prb*] display virtual work expression

sol = SOLVE[{TMEC}, *prb*] solve the unknowns