ECON-C5100 Digital Markets

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Lecture 4: Auction theory

Outline

- Why auctions (price discovery, competition)
- Auction methods
- Private vs. common value auctions

Example: Ascending price auction

- Seller asks for bids from potential buyers.
- Several rounds of bidding are possible.
- Price starts low. Price is increased until only one bidder is left.
- Remaining bidder pays her bid.
- Common applications
  - Traditional auction houses selling all sorts of fancy stuff.
  - eBay, huuto.net (Finnish eBay) and the like.

How should you bid?

- Your private value for the item, i.e. the maximum price you are willing to pay or the minimum price you are willing to sell the item
  - Take the last two digits of your student number
  - As an example, I'd have 96 (IIRC)
- You can bid above the current price or exit.
- Auction ends when just one bidder remains.
- Final bidder wins, and pays her/his bid.

Use the poll in Presemo presemo.aalto.fi/digimar to place bids.

Auctions online

- Online advertising is the key application
  - E.g. advertising made up 80% of Google's annual revenue in: 147 billion U.S. dollars in 2020.
  - Early enthusiasm of online auctions for other stuff (e.g. eBay) has quieted somewhat
- Data created through individual tracking and e.g. unique search term create thousands of ad sales opportunities *per second*.
- How to choose price for thousands of ads sold to billions of consumers?

Reminder: Preferences online

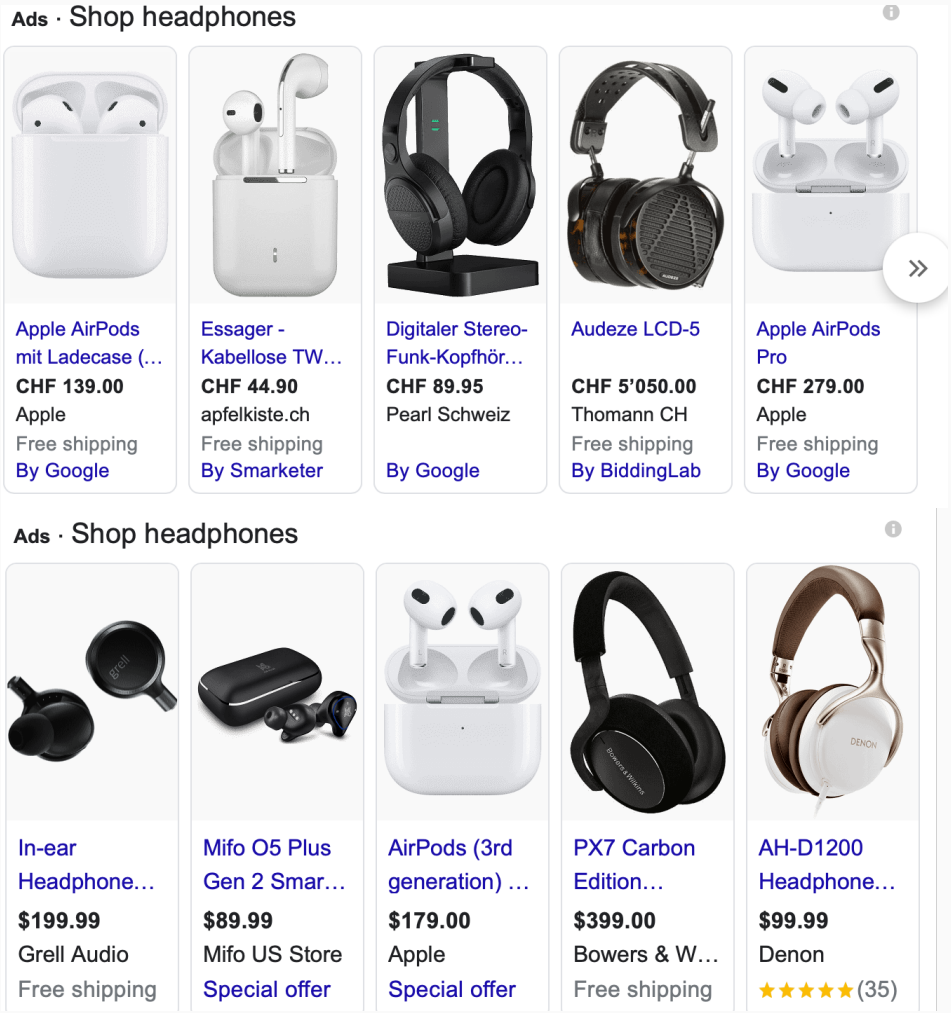
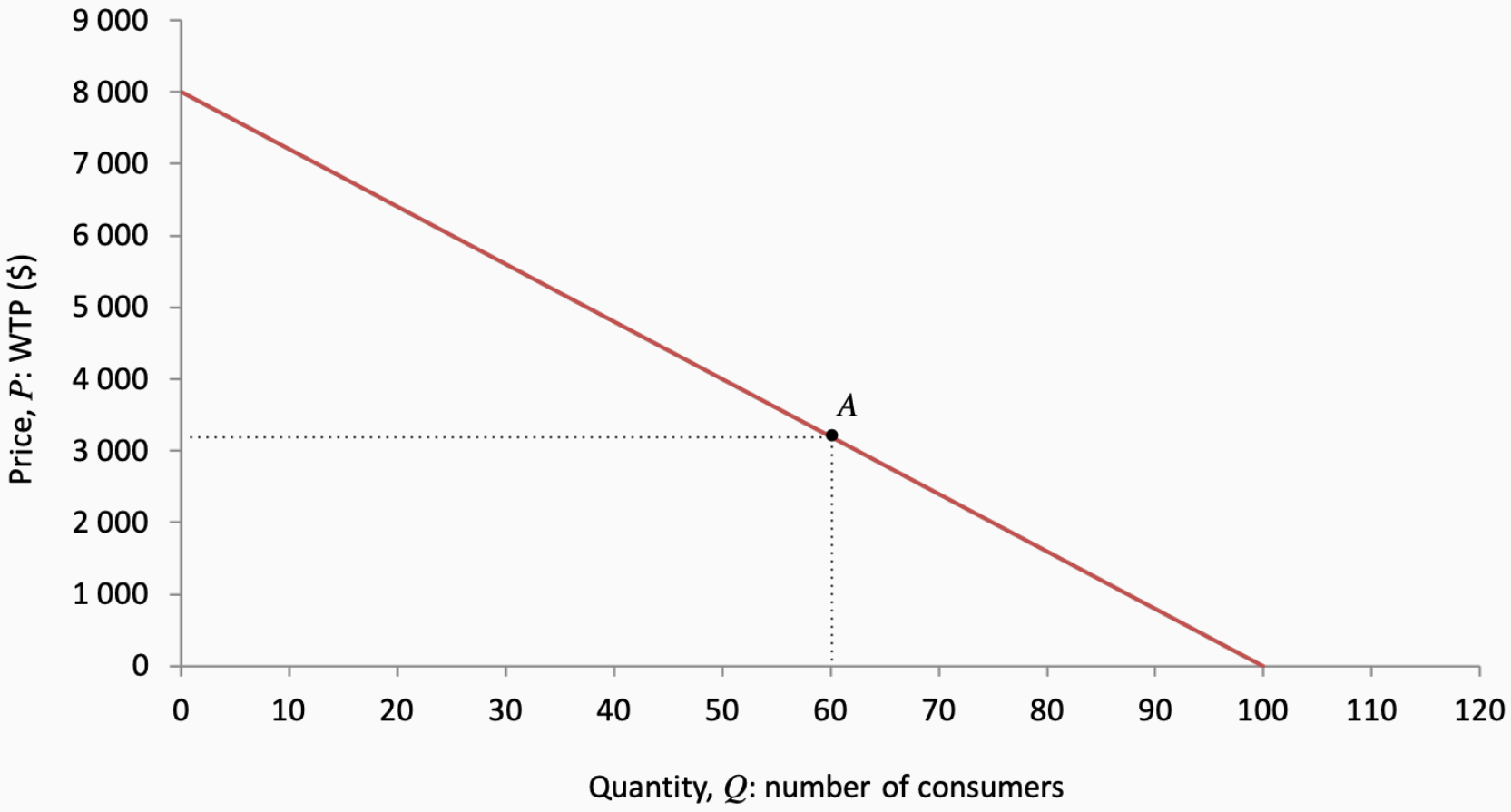


Figure. Two identical Google searches, one done from Zurich, Switzerland (top) and another from Mexico City, Mexico (bottom)

How to set prices?



Source: CORE, The Economy.

Auctions as a tool for selling stuff

- We start by considering a situation where a seller has a single item for sale and there are a number of potential buyers.
- What is the right price to ask?
  - It will depend on how much buyers are willing to pay.
  - Buyers are not going to tell you their true value.
  - Auction is a mechanism for price discovery.
- Auctions also create a competition between buyers.

Simple model

- Seller looking to sell one item.
- There are n buyers
  - Buyers have their own valuations for the item $v_1, v_2, \ldots, v_n$.
  - These valuations are private information.
- We assume that the valuations are drawn from an uniform distribution $[0, 100]$.
  - In practice, the distribution of such valuations could be estimated from past decisions, or modeled based on some assumptions.
- Seller sets the rules for the auction.

Ascending auction

- Prices start at zero, and rises slowly.
- Buyers can bid at the current price or exit.
- Auction ends when just one bidder remains.
- Final bidder wins, and pays the price at which the second remaining bidder dropped out.

Ascending auction

- In this auction it is optimal for you to bid until the price is higher than your private value:
  - If you continue bidding above your value and win, you need to pay a price that is higher than your private value.
  - If you stop bidding below your value, you lose the potential gain from buying the item below your value.
- Bidder with the highest valuation will win and pay the second highest value.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - First exits at 25, second at 33 and the auction ends.

Expected revenue from the ascending auction

- Assume two bidders with random values.
- Expected revenue from the auction is equal to the expected value of the second highest bid.
- Expected values for the bids
  - Highest value is 66 ⅔.
  - Second highest value is 33 ⅓.
- So the expected revenue for the seller is 33 ⅓.

Role of competition

More competition increases the bids and the expected value for the seller:

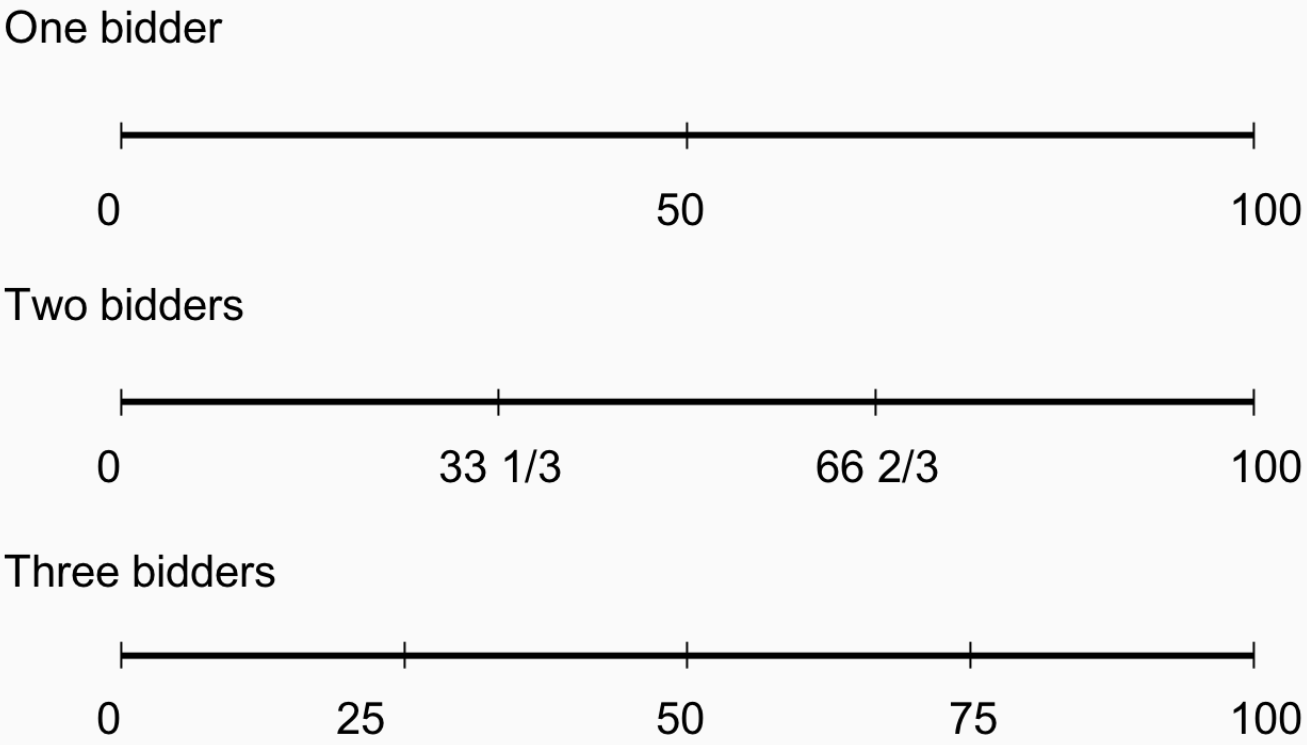


Figure. Expected bid values drawn from uniform $[0, 100]$ distribution as the number of bidders increases.

Why auctions have been popular online

- Arranging an auction is a trade-off between the benefit of price discovery vs. the cost of arranging an auction.
- An auction can be costly for both the seller and the buyers.
  - Transaction costs, think of a traditional auction selling arts.
  - But also indirect costs for both sides (delay, hassle).
- Online both the advantage and disadvantage change:
  - Many more participants possible online vs. physically.
  - Lower costs of arranging.
  - Emergence of popular sites has made it easier for people to know where to find the auction action.

Illustration: Trade-off between auctions and posting prices

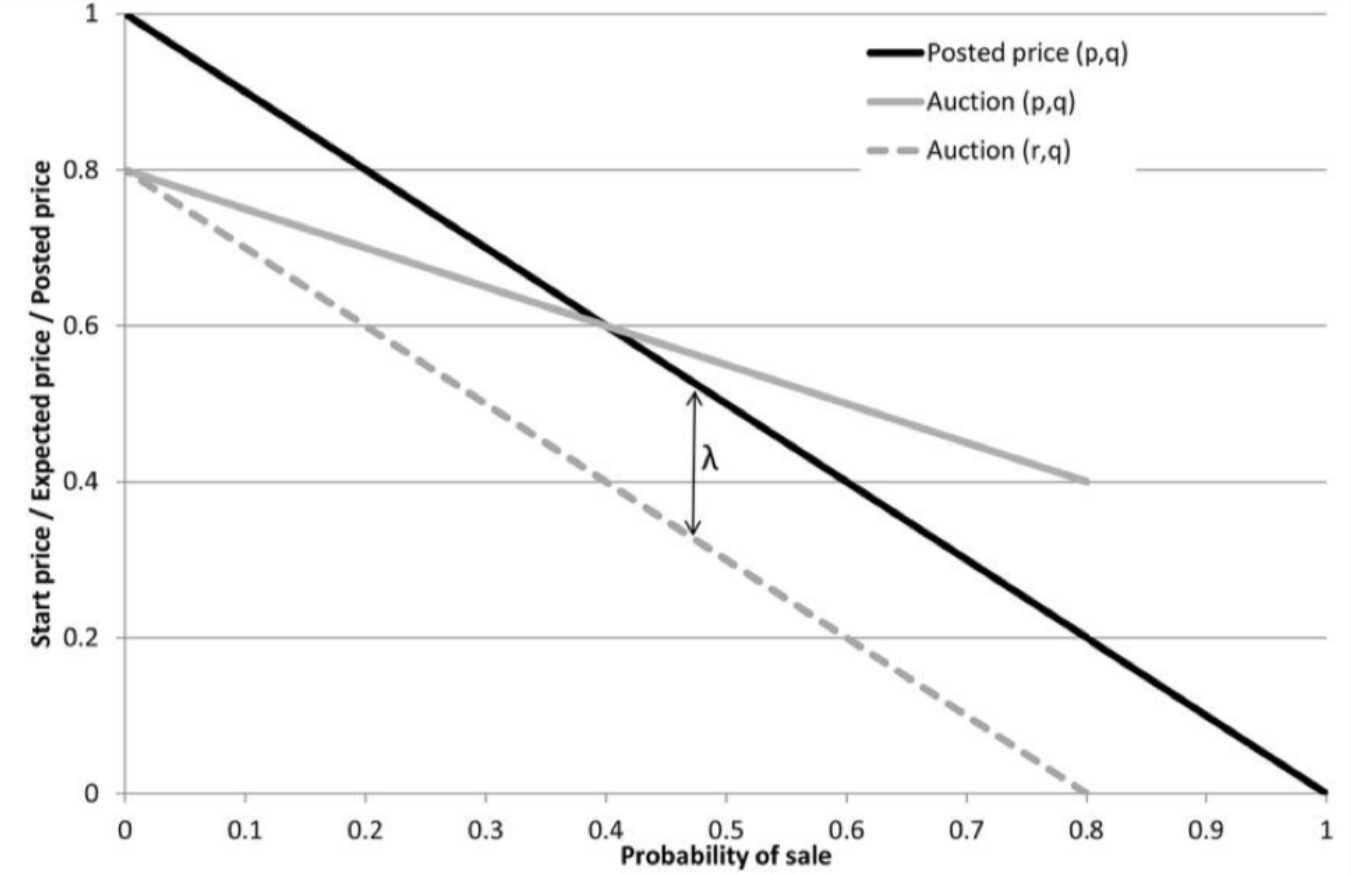


Figure. Trade-off between auctions and posting prices.

Figure: Einav et al. 2018.

Sealed bidding

- Seller asks for bids from potential buyers.
- There is only one round of bidding.
- Buyers deliver their bids so that the other buyers do not observe their bids.
- Highest bidder wins and pays their bid.
- Or a variation: Winning bidder pays the second highest bid.
- Common applications
  - Procurement contracts for commodities and services.
  - Private M&A ownership transactions between firms.

Sealed bidding – Second price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays the second highest bid.

How should you bid?

Second price auction – Optimal bidding

- Again, it is optimal for you to bid your private value.
- In equilibrium, everyone bids their value.
- Bidder with the highest valuation will win and pay the second highest value, exactly as in the ascending auction.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - Winner has value 75, pays 33, as in ascending auction.

Sealed bidding – First price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays her/his bid.

How does the optimal bidding change?

First price auction – Optimal bidding

- Optimal bid is less than your true value.
- A higher bid (but still below your value)
  - Increases your chances to win.
  - Decreases your profit if you do win.
- Optimal bid depends on what you think the others will bid.
- Variation: Descending price auction
  - Price starts high, is lowered until someone buys.
  - Highest bidder wins and pays her bid.
- We need to consider an equilibrium analysis.

First price auction – Equilibrium

- Recall the concept of Nash equilibrium: the strategic interaction now takes place through bidding strategies.
- A set of bidding strategies is a Nash equilibrium if each bidder's strategy maximizes his expected payoff given the strategies of the others.
- Bidders do not know their opponent's values, i.e. we are in incomplete information setting.
- In equilibrium, all bidding strategies maximize the expected payoff of the bidder taking in to account the uncertainty about opponent values.

First price auction – Equilibrium

- In the above first price sealed bid auction example, there is an equilibrium in which both bidders bid half their value.
- Bidder with the highest value wins.
- In expectation, highest value is 66 ⅔, so the expected revenue is 33 ⅓. Which is the same as in the second price auction.
- Turns out this is a quite general result.

Auction efficiency

- Auction outcome is efficient if the high value bidder wins.
- The Nash equilibrium outcome is the same in several auction mechanisms:
  - The high value bidder wins.
  - The expected auction price equals the expected value of the second highest bidder.
- First discovered by Vickrey (1961), extended by Myerson (1981) and Riley and Samuelson (1981).

Revenue Equivalence Theorem

The Revenue Equivalence Theorem:

- Take any auction that allocates the goods efficiently and offers no profit to a zero value bidder.
- Each such auction has the same expected profits for every bidder valuation and the same expected revenue for the seller.

Revenue Equivalence Theorem

- Intuition: Auctions are a method to extract the valuations of bidders.
  - For each bidder it is 1) the probability of winning and 2) the expected price in the case of winning that matter.
  - In an equilibrium of the bidding game, a bidder correctly perceive how her bids map to the other bidders chances of winning and prices.
- It's assumed that bidders are subjective expected utility maximizers and can quantify the uncertainty over other bidders' private information.

Learnings today

- Auctions are an efficient way to organize markets.
  - They offer a method for price discovery and to induce competition.
  - Cost of arranging an auction low online.
- Several auction methods, but in theory often same expected revenues when an efficient method is employed.
- Details of the auction design matter in practice.

Materials for this week

Online resources (for Lecture 3):

- **Perfect competition**. www.core-econ.org 8.5 and 8.8.
- **Monopolies**. Here MRU section on Monopolies mru.org: Monopoly is better suited for the course than www.core-econ.org 7.5.1.
- **Oligopolies**. MRU on Cournot mru.org: Cournot.

Reading assignment 2 (for Lecture 4):

- Einav, Linar, Chiara Farronato, Jonathan Levin and Neel Sundaresan (2018) "Auctions versus Poster Prices in Online Markets". Very selective reading expected: Introduction, with the exception of the literature review in the end, and Section II.
- Easley, David and Jon Kleinberg (2010), Chapter 9 from "Networks, Crowds, and Markets: Reasoning about a Highly Connected World". Advanced material (9.7) not obligatory.

Next time

Market design

- Auction design
- Ad auctions
- Market design

Appendix

Auctions are loved by economists!

- Three Nobel prizes
- Including the 2020 prize to Paul Milgrom and Robert Wilson:
  - Common value auctions
  - Implementation of auctions
- Others main contributions
  - William Vickrey (1996): formal auction theory
  - Roger Myerson (2007): revelation principle

Additional topics in auctions: Common values

- In this course, we mostly assume that the buyers have some *private* valuation for the item.
- Reverse is also possible, it may be that the value of the item once acquired is the same for all buyers, but the value during the auction is uncertain.
- These are named *common* value auctions.
- Here optimal bidding strategies change: you want to bid cautiously to avoid *Winner's curse* (see Appendix).

Additional topics in auctions: Winner's curse

- In a common value auction the bidder with the highest valuation on the value of the item, i.e. the most optimistic bidder, wins.
- A bidder who fails to take this into account pays, on average, more than the item is worth.

See Paul Milgrom's treatment on the topic in "Auctions and Bidding: A Primer." Journal of Economic Perspectives. https://www.aeaweb.org/articles?id=10.1257/jep.3.3.3

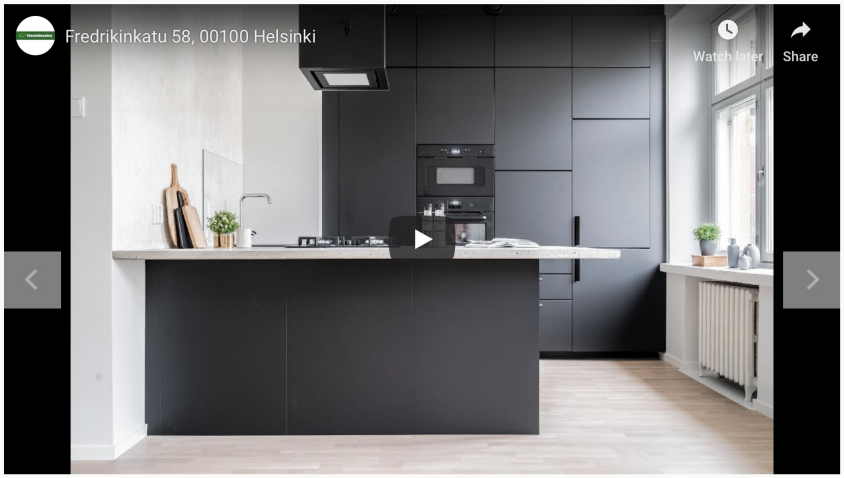
Additional topics in auctions: Other auction methods

- Auctions can be generalized to auctions when many identical items are sold simultaneously.
  - Uniform price auctions where all successful bidders get the same price.
  - Pay-as-bid auctions where each winning bidder pays her own price.
- All-pay auctions where bidders submit bids, highest bidder wins, and everyone has to pay their own bid.
  - Winner pays less in expectation, because everyone pays.
  - Not often seen in monetary auctions, but can be used to model e.g. R&D competition or lobbying

Example: Online auction for an apartment

Fredrikinkatu 58, Etu-Töölö, Helsinki

40 m² | KT, 1h, avok, kph, vh



Annettu	Asiakasnumero	Tyyppi	Summa
01.01.2019	**1493587**	**Kirjallinen tarjous**	**337 000,00 €**
30.12.2018	1500570	Kirjallinen tarjous	336 000,00 €
29.12.2018	1493587	Kirjallinen tarjous	335 000,00 €
27.12.2018	1500570	Kirjallinen tarjous	332 000,00 €
25.12.2018	1493587	Kirjallinen tarjous	331 000,00 €
25.12.2018	1500570	Kirjallinen tarjous	329 000,00 €
23.12.2018	1493587	Kirjallinen tarjous	328 000,00 €
22.12.2018	1500570	Kirjallinen tarjous	326 000,00 €
21.12.2018	1493587	Kirjallinen tarjous	324 000,00 €
20.12.2018	1499837	Kirjallinen tarjous	319 000,00 €
20.12.2018	1493587	Kirjallinen tarjous	318 000,00 €
19.12.2018	1500570	Kirjallinen tarjous	315 000,00 €
19.12.2018	1500082	Kirjallinen tarjous	307 000,00 €
19.12.2018	1493587	Kirjallinen tarjous	306 000,00 €

Source: Kiinteistömaailma.

Example: Online auction for an apartment

Date	Bidder id	Bid
27.01.2019	1	347 000 €
25.01.2019	2	346 000 €
24.01.2019	1	345 000 €
23.01.2019	2	343 000 €
22.01.2019	1	342 000 €
20.01.2019	2	340 000 €
01.01.2019	3	337 000 €
30.12.2018	4	336 000 €
29.12.2018	3	335 000 €
27.12.2018	4	332 000 €
25.12.2018	3	331 000 €
25.12.2018	4	329 000 €
23.12.2018	3	328 000 €
22.12.2018	4	326 000 €
21.12.2018	3	324 000 €
20.12.2018	5	319 000 €
20.12.2018	3	318 000 €
19.12.2018	4	315 000 €
19.12.2018	6	307 000 €
19.12.2018	3	306 000 €