ELEC-C8201 Control and Automation
Exercise 2

1. Prove that the second-order differential equation and state presentation below describe the same system.

$$\ddot{y}(t)+4\dot{y}(t)+3y(t)=2u(t) \Leftrightarrow \begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases}$$

2. Obtain the state space representation of the following differential equation. Use the p-technique.

$$\ddot{y}(t)-3\dot{y}(t)+2y(t)=2\dot{u}(t)$$

3. Create a differential equation system corresponding to the electrical circuit in the figure, when the input voltage is $V_0(t)$ and output voltage is $V_2(t)$. Also, create a state-space representation. Tip: The quantities of energy that are stored in physical circuits are usually natural state space variable candidates. Examples of electronic circuits include the voltages across capacitors and the currents through coils.