

## Game Theory Week 2: Monday Exercises

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1. Two firms are competing to develop a new technology faster. Firm  $i$  chooses to invest  $x_i \in [0, \infty)$  resources in the technology, and receives payoff

$$\frac{x_i}{x_1 + x_2} - \frac{1}{4}x_i$$

where the first term represents the likelihood that firm  $i$  succeeds in developing the technology first. Suppose that firm 1 publicly chooses how much to invest before firm 2. If  $x_1 = x_2 = 0$ , the first term is 0.

- (a) Define strategies in this game.
- (b) Solve for the unique subgame perfect equilibrium of this game.
- (c) Suppose firm 1 and 2 moved simultaneously. The unique Nash equilibrium of this game is symmetric. Find it (*Hint: It may make things a bit easier if you use symmetry here*). How does it compare to the equilibrium in (b)?
- (d) Show that in the game where player 1 moves first, it's a Nash equilibrium for player 1 to choose  $x_1 = 4$  and player 2 chooses  $x_2 = 0$  if  $x_1 = 4$ , otherwise player 2 chooses  $x_2 = 4$ . Why does this not contradict the uniqueness of the equilibrium in (b)?
- (e) Now suppose that there is a common pool of 2 units of resources, so  $x_1 + x_2 \leq 2$ . What is the subgame perfect equilibrium in the game where player 1 moves first?