## ECON-C4100 - Capstone: Econometrics I

Lectures 10&11: Causal parameters part II - Instrumental variables regression

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#### Learning outcomes

- At the end of these lectures, you understand
- 1 what simultaneous causality means
- 2 what is meant by an **endogeneity** problem
- 3 why it causes bias in the parameters
- 4 what an instrumental variable is and why it solves the endogeneity problem
- 5 what characteristics are required of an instrumental variable
- 6 what one should pay attention to when using an instrumental variable

#### Learning outcomes

- At the end of these lectures, you understand
- 7 what a **reduced form** equation/parameter is
- 8 what a **structural** equation/parameter is
- 9 how to "manually" estimate a model with simultaneous causality
- 10 what **2SLS** estimation means, how you do it and why it is used

#### Overview

- Demand experiment, market data analysis.
- Simultaneous causality.
- IV regression and 2SLS.
- NOTE: Instrumental variables are used in a large variety of contexts.
- We are exploring it in a particular but historically and practically very important setting.
- In Applied Microeconometrics I and II you will learn more about IV, its use and the interpretation of results.

#### Simultaneous causality

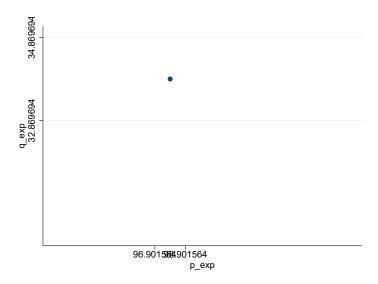
- Your task is to estimate the demand function for a homogeneous good sold at unit price.
- How would you do this in an experiment?

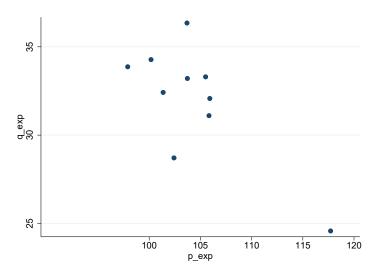
## Simultaneous causality

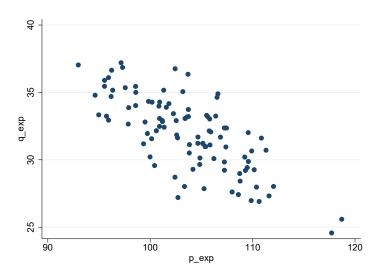
- Your task is to estimate the demand function for a homogeneous good sold at unit price.
- How would you do this in an experiment?
- By changing the price yourself ("at random") and observing how many units are sold at each price.

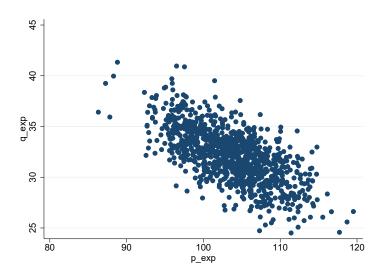
- What does "choosing prices" at random mean?
  - 1 We offer different randomized prices to individual consumers.
  - 2 We offer different randomized prices each to a group of consumers.
  - 3 Think either of geographically separate markets, or a given market over time.

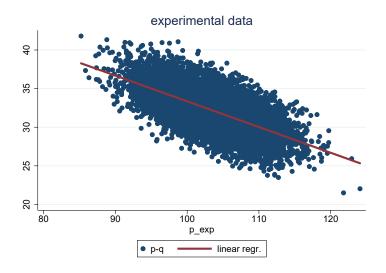
- We record quantity sold at different prices.
- We study the outcomes.
- For illustration, I have conducted such an experiment in my computer.
- We will get to the details of how I do it later, but now just imagine I have conducted the experiment in a real market.











• Question: why does sold quantity vary between two experiments where the prices are identical?

- Question: why does sold quantity vary between two experiments where the prices are identical?
- Answer: Demand is stochastic from the viewpoint of the econometrician.
- Let's study a simple set-up (the one I used in the experiment) in more detail.

#### Linear demand

Demand function

$$Q_i = a - bP_i + \epsilon_i$$

- a = average intercept.
- *b* = slope.
- $\epsilon_i$  = market specific deviation from the average intercept.
- i = a particular market realisation.
- Question: Where does this demand function come from?
- Answer: From consumers making utility-maximizing choices.
- Exercise: What does the utility function look like that produces a linear demand function?

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#### Linear demand

Inverse demand function

$$P_{i} = \frac{a}{b} - \frac{1}{b}Q_{i} + \frac{1}{b}\epsilon_{i} = \alpha + \beta Q_{i} + \tilde{\epsilon}_{i}$$

## Regression analysis

#### . regr q\_exp p\_exp

Source	SS	df	MS	Number of obs F(1, 9998)	=	10,000 6935.10
Model Residual	27500.3163 39645.8875	1 9,998	27500.3163 3.96538183	Prob > F R-squared	=	0.0000 0.4096
Total	67146.2038	9,999	6.71529191	Adj R-squared Root MSE	=	0.4095 1.9913

q_exp	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
p_exp _cons	3329958 66.65341		-83.28 160.79	0.000	3408339 65.84081	3251577 67.466

## Robustness analysis

```
. gen p exp2
                         = p exp^2
. regr q exp p exp*
                                                    Number of obs
                     SS
                                   df
                                            MS
                                                                          10,000
     Source
                                                     F(2, 9997)
                                                                         3468.48
                27506.3195
      Model
                                      13753.1598
                                                     Prob > F
                                                                          0.0000
                39639.8843
   Residual
                                9,997
                                       3.96517798
                                                    R-squared
                                                                          0.4096
                                                    Adj R-squared
                                                                          0.4095
      Total
                67146.2038
                                9,999 6.71529191
                                                     Root MSE
                                                                          1.9913
      q exp
                    Coef.
                             Std. Err.
                                            t
                                                  P>|t|
                                                            [95% Conf. Interval]
                -.4806486
                             .1200668
                                         -4.00
                                                  0.000
                                                           -.7160038
                                                                       -.2452935
      p exp
     p exp2
                 .0007134
                             .0005798
                                          1.23
                                                  0.219
                                                           -.0004231
                                                                         .0018498
      _cons
                 74.27605
                             6.208917
                                         11.96
                                                  0.000
                                                            62.10532
                                                                        86.44677
```

#### Parameters for inverse demand function

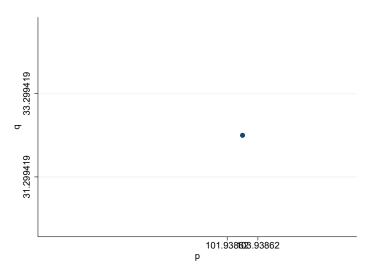
```
. scalar alpha_exp = - _b[_cons] / _b[p_exp]
. scalar beta_exp = -1 / _b[p_exp]
. scalar list alpha_exp beta_exp
alpha_exp = 200.16291
beta_exp = 3.003041
```

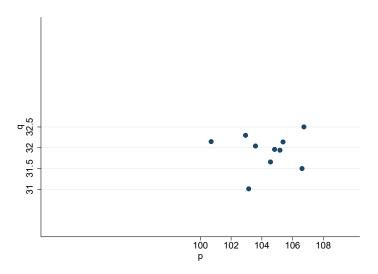
#### Market outcomes

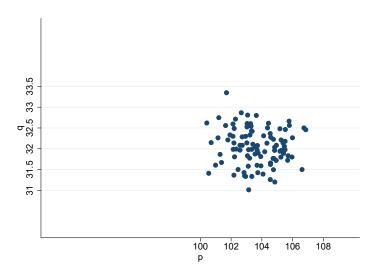
- Assume you are an outside observer of a market say a prospective buyer of a firm or the competition authority.
  - $\rightarrow$  you cannot run experiments.
- You would still want to know demand (to calculate e.g. price cost margins, consumer surplus).

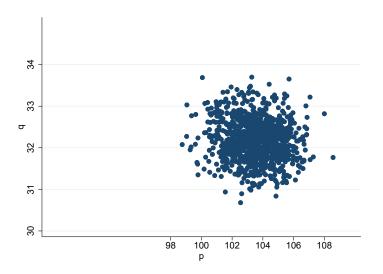
#### Market outcomes

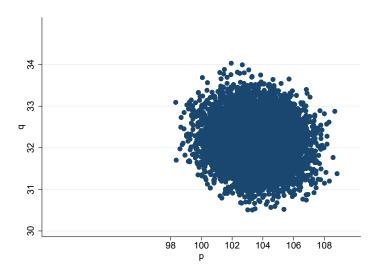
- We collect data from the market.
- We observe pairs  $(P_i, Q_i)$ , i = market.
- Let's think how such pairs are determined, using a simple monopoly model.
- Let's allow a monopolist to choose prices instead in the same market.

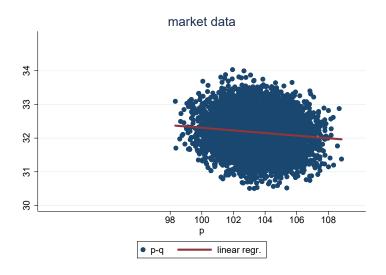


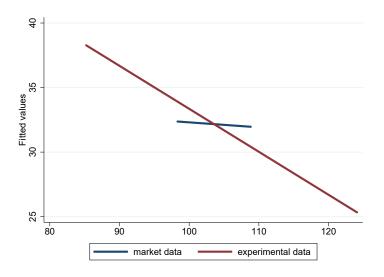












# Regression using market data

#### . regr q p

Source	SS	df	MS		er of obs	=	10,000 135.06
Model Residual	33.3509286 2468.76463	1 9,998	33.3509286	Froh	> F puared	=	0.0000 0.0133
Total	2502.11556	9,999	.250236579		R-squared MSE	=	0.0132 .49692
q	Coef.	Std. Err.	t	P> t	[95% Co	nf.	Interval]
p _cons	0386712 36.17252	.0033275	-11.62 105.01	0.000	045193 35.4972		0321487 36.84776

# Parameter comparison for demand and inverse demand functions

```
. scalar alpha ols
                             = - b[ cons] / b[p]
. scalar beta ols
                         = -1 / b[p]
. scalar a ols = b[ cons]
. scalar b ols = - b[p]
. scalar list a exp b exp a ols b ols
    a exp = 66.653405
    b \exp = .33299579
    a ols = 36.172515
    b ols = .03867121
. scalar list alpha exp beta exp alpha ols beta ols
alpha exp = 200.16291
beta exp = 3.003041
alpha ols = 935.38611
 beta ols = 25.859029
```

## Challenge with market data

- Price quantity pairs are a leading example of simultaneous causality.
- This generalizes to more complicated markets with:
  - 1 differentiated goods
  - 2 multiproduct firms
  - 3 endogenous entry and exit
  - 4 dynamic considerations (e.g. collusion, durable goods, ...)
  - 5 advertising
  - **6** ...

## Challenge with market data

- Need to address simultaneous causality.
- ullet ightarrow need to understand and exploit determinants of price and quantity.
- How did the experiment solve the problem?
- By having the researcher shift (=change) prices instead of the firm.

## Linear monopoly

Demand function

$$Q_i = a - bP_i + \epsilon_i$$

- a = average intercept.
- b = slope.
- $\epsilon_i$  = market specific deviation from the average intercept.
- NOTE: We assume the firm observes all these parameters.
- Question: What if the firm did not observe our "unobservable", i.e.,  $\epsilon_i$ ?

#### Linear monopoly

Inverse demand function

$$P_{i} = \frac{a}{b} - \frac{1}{b}Q_{i} + \frac{1}{b}\epsilon_{i} = \alpha + \beta Q_{i} + \tilde{\epsilon}_{i}$$

## How to get the supply function?

- Inverse demand function.
- Need to specify costs of production: constant marginal cost

$$c_i = c_0 + c_1 z_i + \eta_i$$

 Note: Here one should have an understanding of the production technology.

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$$c_i = c_0 + c_1 z_i + \eta_i$$

- $c_0$  = average of marginal cost when  $z_i = 0$ .
- z<sub>i</sub> = a component of marginal cost that varies across markets (cost of raw materials / unit of output, cost of labor / unit of output, ..., with c<sub>1</sub> being the number of units of the input needed to produce one unit of output).
- $\eta_i$  = "shock" to average marginal cost / deviation from the avg. This is observed by the firm but not by the econometrician.
- If the firm did not observe  $\eta_i$ , how could it take it into account in its decision?

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The monopolist's problem:

$$max_{P_i}\pi_i = (P_i - c_i) \times Q_i$$

Equilibrium price:

$$P_i = \frac{a}{2b} + \frac{1}{2}c_i + \frac{1}{2b}\epsilon_i$$

$$= \frac{a}{2b} + \frac{1}{2}(c_0 + c_1z_i + \eta_i) + \frac{1}{2b}\epsilon_i$$

#### Equilibrium quantity

$$Q_i = \frac{a}{2} - \frac{b}{2}(c_0 + c_1 z_i + \eta_i) + \frac{1}{2}\epsilon_i$$

Equilibrium price and equilibrium quantity are functions of:

(fixed) demand parameters a and b

- $\bigcirc$  (fixed) demand parameters a and b
- 2 (fixed) supply side parameters  $c_0$  and  $c_1$

- (fixed) demand parameters a and b
- $oldsymbol{2}$  (fixed) supply side parameters  $c_0$  and  $c_1$
- 3 variable cost determinant z<sub>i</sub>

- (fixed) demand parameters a and b
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- **4** cost shock  $\eta_i$

- (fixed) demand parameters a and b
- $oldsymbol{2}$  (fixed) supply side parameters  $c_0$  and  $c_1$
- 3 variable cost determinant zi
- **4** cost shock  $\eta_i$
- **5** demand shock  $\epsilon_i$

### Market data challenge

- Both eq. price and eq. quantity are functions of:
- $\mathbf{0}$  demand shock  $\epsilon_i$
- 2 supply (cost) shock  $\eta_i$ 
  - → simultaneous causality.
  - $\rightarrow$  price is an endogenous variable ( $\epsilon_i$  and  $\eta_i$  are the "omitted" variables that affect both price and quantity).

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### Market data solution

- We want to learn the demand curve.
- Could we mimic the experimental approach with market data?
- Needed: Something that shifts firm's (supply) decision at "random".
- Random = without being affected by demand shock  $\epsilon_i$ .

- Imagine the firm still sets the price,
- but we choose (=randomize)  $z_i = z_i^{exp}$ .
- Recall that  $c_i = c_0 + c_1 z_i^{\text{exp}} + \eta_i$ .
  - $\rightarrow$  we "shift" firm's marginal cost.

• Now the firm sets each time the price

$$P_{i} = \frac{a}{2b} + \frac{1}{2}(c_{0} + c_{1}z_{i}^{exp} + \eta_{i}) + \frac{1}{2b}\epsilon_{i}$$

- $\rightarrow$  equivalent to running an experiment.
- Change in price due to (known) change in  $z_i^{exp}$ .

- Imagine we raise  $z_i^{exp}$  by 1 unit.
- By how much does
  - **1** marginal cost  $c_i = c_0 + c_1 z_i + \eta_i$  change? Answer:  $c_1$ .
  - 2 price change? Answer:  $\frac{1}{2}c_1$  (by the equilibrium price equation).
  - **3** demand change? Answer:  $-b\frac{1}{2}c_1$  (by the equilibrium quantity equation).

- How could we get the slope of the demand function from these changes?
- Yes, by dividing the change in quantity by the change in demand:

$$-\frac{b\frac{1}{2}c_1}{\frac{1}{2}c_1} = -b$$

- How could we get those numbers?
- 1. Regress  $P_i$  on  $z_i^{exp}$  to get  $\frac{1}{2}c_1$ .

$$P_{i} = \frac{a}{2b} + \frac{1}{2}(c_{0} + c_{1}z_{i}^{exp} + \eta_{i}) + \frac{1}{2b}\epsilon_{i}$$

$$= (\frac{a}{2b} + \frac{1}{2}c_{0}) + \frac{1}{2}c_{1}z_{i}^{exp} + (\frac{1}{2}\eta_{i} + \frac{1}{2b}\epsilon_{i})$$

$$= \gamma_{0} + \gamma_{1}z_{i} + e_{i}$$

$$\gamma_{1} = \frac{1}{2}c_{1}, \gamma_{0} = (\frac{a}{2b} + \frac{1}{2}c_{0})$$

2. Regress  $Q_i$  on  $z_i^{exp}$  to get  $-b\frac{1}{2}c_1$ .

$$Q_i = \mu_0 + \mu_1 z_i + w_i$$

$$\mu_1 = -b\frac{1}{2}c_1$$

- One can link the  $Q_i$  equation to the equilibrium  $Q_i$ -expression just like was done for  $P_i$  on the previous slide.
- The P<sub>i</sub> and Q<sub>i</sub> regression equations on this and previous slide are called reduced form equations.

• The regression equations we estimated, i.e.,

$$P_i = \gamma_0 + \gamma_1 z_i + e_i$$

$$Q_i = \mu_0 + \mu_1 z_i + w_i$$

• are called **reduced form** equations.

### What are reduced form equations?

- Proper definition: A reduced form equation is an equation whose parameters are functions of the structural parameters.
- In our model, structural parameters are a, b,  $c_0$  and  $c_1$ .
  - 1 They are building blocks of the theory model
  - 2 They are determined outside our model
  - 3 They are not functions of any other parameters (or variables) of the model
- The parameters  $(\gamma_0, \gamma_1, \mu_0, \mu_1)$  of the two regressions  $(P_i \text{ on } z_i \text{ and } Q_i \text{ on } z_i)$  we ran are functions of the structural parameters (= those in the theoretical model, i.e., a, b,  $c_0$ ,  $c_1$ ).

### What are reduced form equations?

- Commonly used meaning: A **reduced form equation** is an equation that is not derived from a theoretical model.
- Examples: The regressions in the papers we have studied, i.e.,
  - Bronnenberg et al., 2015.
  - Kleven et al., 2011.

- When would this work?
  - 1  $z_i^{exp}$  has to have an impact on the decision of the firm, i.e., have an effect on  $c_i$ .
    - $\rightarrow$   $c_1$  cannot be (insignificantly different from) zero.
  - 2  $z_i^{exp}$  may not have an effect on  $Q_i$  directly, but only via  $c_i$ .

### Let's regress P on z.

. regr p z

	Source	SS	df	MS		r of obs	=	10,000
_	Model Residual	9783.4964 12517.9151	1 9,998	9783.4964 1.25204192	2 R-squ	> F	=	7814.03 0.0000 0.4387 0.4386
	Total	22301.4115	9,999	2.23036418		-	=	1.1189
	р	Coef.	Std. Err.	t	P> t	[95% Co	nf.	Interval]
	z _cons	.4986242 101.0138	.0056407	88.40 3322.11	0.000	.487567 100.954		.5096812 101.0734

.  $scalar red_1 = b[z]$ 

### Let's regress Q on z.

. regr q z

	Source	SS	df	MS	Numb	er of obs	3 =	10,000
-					<ul><li>F(1,</li></ul>	9998)	=	8117.89
	Model	1121.21997	1	1121.2199	7 Prob	> F	=	0.0000
	Residual	1380.89558	9,998	.13811718	2 R-sq	uared	=	0.4481
-					- Adj	R-squared	1 =	0.4481
	Total	2502.11556	9,999	.25023657	9 Root	MSE	=	.37164
	P	Coef.	Std. Err.	t	P> t	[95% C	Conf.	Interval]
	z	1687997 33.01561	.0018735	-90.10 3269.17	0.000	17247		1651273 33.0354

.  $scalar red_2 = b[z]$ 

### Let's calculate b.

```
. scalar b_red = red_2 / red_1
. scalar list b_red
    b_red = -.33853094
```

#### Instrumental variable

- Instrumental variable = a variable that causes variation in price (explanatory variable X) but does not affect demand (dependent variable Y) directly.
- If the variable cost component z<sub>i</sub> varies "at random", i.e., without
  affecting demand directly,
  - ightarrow market data allows us to use the "experimental approach" indirectly.

# Approach #2

- Could we proceed differently?
  - **1** Regress  $P_i$  on  $z_i$ . Calculate predicted price  $\hat{P}_i = \hat{\gamma}_0 + \hat{\gamma}_1 z_i$ .
  - **2** Regress  $Q_i$  on  $\hat{P}_i$  to get b (and a).
- Equation  $Q_i = a bP_i + \epsilon_i$  is a **structural** equation. Why?

# Approach #2

- Could we proceed differently?
  - **1** Regress  $P_i$  on  $z_i$ . Calculate predicted price  $\hat{P}_i = \hat{\gamma}_0 + \hat{\gamma}_1 z_i$ .
  - 2 Regress  $Q_i$  on  $\hat{P}_i$  to get b (and a).
- Equation  $Q_i = a bP_i + \epsilon_i$  is a **structural** equation. Why?
- Because it is a function of structural parameters only  $(+ P_i)$  which is determined within the model).

### Approach #2

- The parameters of a structural equation are part of the model primitives, i.e.,
  - they are not determined within the model
  - they are not functions of other parameters of the model
- Reduced form parameters are functions of structural parameters.

### Regress P on z, create predicted values

. regr p z

Source	SS	df	MS	Number of obs F(1, 9998)	=	10,000 7814.03
Model Residual	9783.4964 12517.9151	1 9,998	9783.4964 1.25204192	Prob > F R-squared	=	0.0000 0.4387
Total	22301.4115	9,999	2.23036418	Adj R-squared Root MSE	=	0.4386 1.1189

р	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
z _cons	.4986242 101.0138	.0056407			.4875673 100.9542	.5096812 101.0734

<sup>.</sup> predict p hat

(option xb assumed; fitted values)

# Regress Q on $\hat{P}$

#### . regr q p\_hat

Source	SS	df	MS		r of obs	=	10,000 8117.89
Model Residual	1121.21997 1380.89558	1 9,998	1121.2199 .13811718	7 Prob 2 R-squ	> F	=	0.0000 0.4481 0.4481
Total	2502.11556	9,999	.25023657		-	=	.37164
P	Coef.	Std. Err.	t	P> t	[95% Co	nf.	Interval]
p_hat _cons	3385309 67.21192	.0037573	-90.10 172.80	0.000	34589 66.449		3311659 67.97433

# Why / how do these approaches work?

- **1** Regress  $P_i$  on  $z_i$  to get  $\frac{1}{2}c_1 = \frac{cov(P_i, z_i)}{var(z_i)}$ .
- 2 Regress  $Q_i$  on  $z_i$  to get  $-b\frac{1}{2}c_1 = \frac{cov(Q_i,z_i)}{var(z_i)}$ .

$$\rightarrow -b = \frac{cov(Q_i, z_i)}{cov(P_i, z_i)}$$
.

# Regress Q on $\hat{P}$

$$b = \frac{cov(Q_i, \hat{P}_i)}{var(\hat{P}_i)} = \frac{cov(Q_i, \hat{\gamma}_0 + \hat{\gamma}_1 z_i)}{var(\hat{\gamma}_0 + \hat{\gamma}_1 z_i)}$$

$$\rightarrow b = \frac{\hat{\gamma}_1 cov(Q_i, z_i)}{\hat{\gamma}_1^2 var(z_i)} = \frac{1}{\hat{\gamma}_1} \frac{cov(Q_i, z_i)}{var(z_i)}$$

$$\text{because } \hat{\gamma}_1 = \frac{cov(P_i, z_i)}{var(z_i)} \rightarrow$$

$$b = \frac{var(z_i)}{cov(P_i, z_i)} \frac{cov(Q_i, z_i)}{var(z_i)} = \frac{cov(Q_i, z_i)}{cov(P_i, z_i)}$$

### 2SLS / instrumental variables regression

- In practice, want to use the so called Two Stage Least Squares (2SLS) or instrumental variables regression command. In Stata, ivregress or from SSC ivreg2.
- Manual and ivregress command(s) produce same point estimates, but the latter corrects the standard errors.
- This is important, as the manual approach yields too small standard errors: It ignores the uncertainty in the parameters  $\hat{\gamma}_0$  and  $\hat{\gamma}_1$  used to calculate  $\hat{P}_i$ .

### 2SLS estimation of demand

```
. ivregress 2sls q (p = z)
```

Instrumental variables (2SLS) regression Number of obs = 10,000 Wald chi2(1) = 2506.07 Prob > chi2 = 0.0000 R-squared = Root MSE = .66888

q	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
p _cons	3385309 67.21192				351785 65.83988	3252769 68.58395

Instrumented: p Instruments: z

### Requirements for an instrument

- Think of our normal regression  $Y = \beta_0 + \beta_1 X + u$ .
  - Instrument relevance: The instrument Z has to affect the (endogenous) explanatory variable X of the equation of interest ("2nd stage equation") in the equation

$$X = \alpha_0 + \alpha_1 Z + v.$$

2 Instrument exogeneity: The instrument Z may not be correlated with the error term of the equation of interest, i.e.,

$$cov(Z, u) = 0.$$

### Instrument relevance / Weak instrument

- Relevance = instrument Z needs to be "correlated enough" with the endogenous explanatory variable X.
- What happens when  $cov(Z, X) \rightarrow 0$ ?
- β<sub>1</sub> becomes undefined!

### Instrument relevance / Weak instrument

- $\rightarrow$  you want to check that your instrument is relevant.
- = you don't have a weak instrument.
- Rule of thumb: F-statistic of Z when you regress X on Z (and possible further controls) > 10.
- Note: With 1 instrument, F-test is the square of the t-test.
- Notice that the test for weak instruments is stricter than our usual 5% confidence level (t-stat 2).

### 2SLS estimation

#### . estat firststage

#### First-stage regression summary statistics

Variable	R-sq.	Adjusted R-sq.	Partial R-sq.	F(1,9998)	Prob > F
р	0.4387	0.4386	0.4387	7814.03	0.0000

#### Minimum eigenvalue statistic = 7814.03

Critical Values Ho: Instruments are weak	<pre># of endogenous regressors: 1 # of excluded instruments: 1</pre>
2SLS relative bias	5% 10% 20% 30% (not available)
2SLS Size of nominal 5% Wald test LIML Size of nominal 5% Wald test	10% 15% 20% 25% 16.38 8.96 6.66 5.53 16.38 8.96 6.66 5.53

### Instrument relevance / Weak instrument

- There are more sophisticated tests.
- There are ways of allowing for weak instruments.
- We leave all that for later courses.
- Good instruments are hard to find...

#### Instrument correlated with error

= "exogeneity" assumption or exclusion restriction:

$$\mathbb{E}[u|\mathbf{X}]=0$$

If this condition does not hold  $\rightarrow$  biased estimate of  $\beta_1$ .

Similar to omitted variable bias.

#### Instrument correlated with error

- What can be done?
  - 1 Strong story for why no correlation between instrument and error.
  - 2 With multiple instruments, may do tests.
  - 3 There are ways of allowing for (some) correlation to check robustness of your results (for later).