

# ELEC-C1310 Automaatio- ja systeemitekniikan laboratoriotyöt

## Control of an inverted pendulum

### Instructor:

- Hoang Nguyen Khac ([hoang.kh.nguyen@aalto.fi](mailto:hoang.kh.nguyen@aalto.fi))

### NOTICE:

The workbook “**Rotary Pendulum Workbook (Student).pdf**” (1) is used for this lab.

Numerical values of parameters can be found in:

- “**Rotary Servo Base Unit – User Manual.pdf**”
- “**Rotary Inverted Pendulum – User Manual.pdf**”

For questions 6-8, please read the “**IP\_Lab\_Appendix.pdf**” carefully before starting.

This lab will cover chapters 1-4.

**IMPORTANT:** Solution must be submitted to Mycourses prior to the lab session. One copy is enough from each group.

Carefully read the workbook (1) and answer these following questions.

1. The nonlinear equations of motion for the SRV02 rotary inverted pendulum are given in equation **2.2** and equation **2.3**. Linearize these equations by the classical linearization method.

The initial conditions for all variables are set to zero.

#### Hints:

- let variable  $z$  be  $z = [\theta, \alpha, \dot{\theta}, \dot{\alpha}, \ddot{\theta}, \ddot{\alpha}]^T$  and  $z_0 = [0, 0, 0, 0, 0, 0]^T$
  - consider the left-hand side of equation 2.2 and 2.3 as functions of  $z$ ,  $f_1(z)$ ,  $f_2(z)$ . Then linearize  $f_1(z)$  and  $f_2(z)$ .
2. Find the linear state-space representation of the system when the input  $u$  is the torque applied at the servo load gear. (matrices C and D have been given)
  3. What is the characteristic equation of this system?
  4. What are the open-loop poles of the system? What can be said of the system based on the poles you just found?
  5. Recall that the input of the state-space model you just found is the torque applied at the servo load gear. However, in the lab session, we control the servo input voltage instead.  
By using the voltage-torque relationship in Equation **2.4**, derive a new state-space model with the voltage to be the input. **Notice:** we assume that the gear's and the motor's efficiencies ( $\eta_g, \eta_m$ ) are 1.

In Matlab, write a script to find the open-loop poles of the new state-space model.

**Hints:** make sure your final state-space model is similar with the following one, otherwise your later results will be meaningless.

State-Space Matrix	A	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 80.3 & -45.8 & -0.930 \\ 0 & 122 & -44.1 & -1.40 \end{bmatrix}$
State-Space Matrix	B	$\begin{bmatrix} 0 \\ 0 \\ 83.4 \\ 80.3 \end{bmatrix}$
State-Space Matrix	C	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$
State-Space Matrix	D	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

6. From this point, **only the state-space model in terms of servo input voltage** is considered.
  - What is the characteristic polynomial of A?
  - Compute the corresponding companion matrices ( $\tilde{A}, \tilde{B}$ ) of A and B.
7. In Figure 3.1, this system has four closed-loop poles and two of them ( $p_1$  and  $p_2$ ) are complex conjugate dominant poles. Find the location of these two dominant poles so that they satisfy the specifications given in **Section 3.1**. Assume that  $p_3 = -30$  and  $p_4 = -40$ , find the desired characteristic equation.
8. Calculate companion gain  $\tilde{K}$  that assigns the poles to the locations found in question 8. Convert the companion gain  $\tilde{K}$  into controller gain  $K$ . (Hint: read **Section 3.2.4**)
 

**Notice:**

  - Typo in equation 3.1:  $A^n \rightarrow A^{n-1}$
  - Typo in the formula of  $\tilde{T}$ :  $\tilde{A}^n \rightarrow \tilde{A}^{n-1}$

### In-lab experiment instructions:

- The first chapter is for the validation of the state-space model calculated from the pre-lab questions.
- In the second chapter, controller gain K from question 8 will be tested with the balance control of the pendulum. Some practices in LabView will also be included.
- The swing-up control will be briefly explained, and the demo will be shown where the pendulum is swung up and kept balanced up right.

**Should you have any problems or questions, please ask the instructor!**