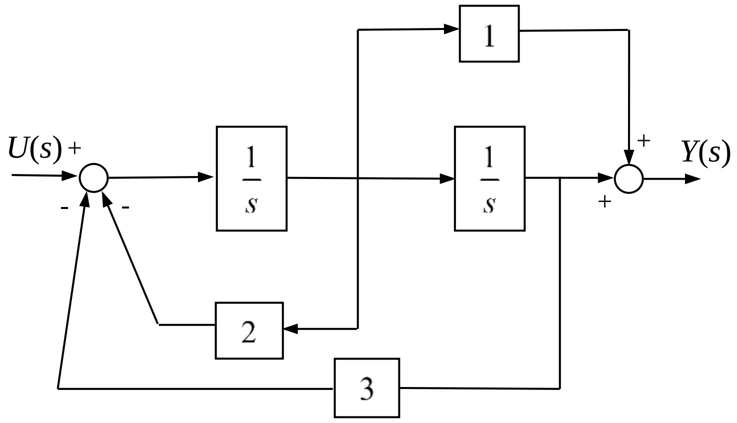
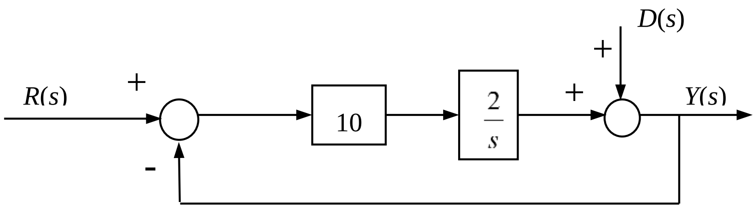
ELEC-C8201 Control and Automation

Exercise 4

1. Specify the total transfer function for the system given below. (The numerator and denominator of the transfer function are polynomials in *s*)



2. For the system model given below, calculate output $y(t)$, when the reference is $r(t) = 5.0u_s(t)$ and the disturbance is $d(t) = 5.0(\cos(t))u_s(t)$. Note: $u_s(t-t_0)$ means a step function entering at time t_0.



3. Calculate the transfer function corresponding to the following state space model:

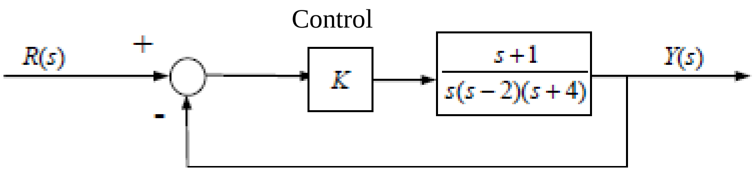
$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(t) \end{cases}$$

4. Examine how many roots do the following polynomials have on the right half plane:

a. $s^4 + 6s^3 + 13s^2 + 12s + 4$ **b.** $2s^5 + s^4 + 3s^2 + s + 2$

c. $s^4 + 2s^3 + 4s^2 + 8s + 10$

5. The process of the system below is unstable. Does this mean that the closed loop system is unstable?



Tips:

Matrix A inverse:

First we obtain the components of the adjoint matrix (adjA) from the following expression:

$$a_{ij} = (-1)^{i+j} \det A_{ij} \ ,$$

where $\boldsymbol{A}_{ij}$ *is the submatrix of A obtained by removing line i and column j (Note in particular, the order of indexes).*

Inverse of a matrix is obtained from its adjoint matrix by the following relation:

$$A^{-1} = \frac{adjA}{\det A} \ .$$

Laplace transform expressions:

Definition: $F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$	
Laplace transformation	Time domain function
$F(s)$	$f(t)$
$C_1F_2(s) + C_2F_2(s)$	$C_1f_2(t) + C_2f_2(t)$
$F(s+a)$	$e^{-at}f(t)$
$e^{-as}F(s)$	$\begin{cases} 0, & t \le a \\ f(t-a), & t > a \end{cases}$
$\frac{1}{a}F\left(\frac{s}{a}\right)$	$f(at)$
$F_1(s)F_2(s)$	$\int_0^t f_1(\tau) f_2(t-\tau)\,d\tau$
$sF(s) - f(0)$	$f'(t)$
$s^nF(s) - \left[s^{n-1}f(0) + \ldots + f^{(n-1)}(0)\right]$	$f^{(n)}(t)$
Initial and final value theorems: If the limits of $f(t)$ and $F(s)$ exist and are finite, then: $\lim_{s\to 0}\{sF(s)\} = \lim_{t\to\infty}\{f(t)\}$ $\quad \lim_{s\to\infty}\{sF(s)\} = \lim_{t\to 0}\{f(t)\}$	

Laplace transformations and Time domain responses

Laplace transformation	Time domain function
1	$\delta(t)$
$1 / s$	1
$1 / s^2$	t
$1 / s^{n+1}$	$t^n / n!$
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^{n+1}}$	$\frac{t^n e^{-at}}{n!}$
$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{1}{ab(b-a)}\left(ae^{-bt}-be^{-at}\right)$
$\frac{a}{s^2+a^2}$	$\sin(at)$
$\frac{s}{s^2+a^2}$	$\cos(at)$
$\frac{a}{(s+b)^2+a^2}$	$e^{-bt}\sin(at)$
$\frac{s+b}{(s+b)^2+a^2}$	$e^{-bt}\cos(at)$
$\frac{s+a}{s+b}$	$\delta(t)+(a-b)e^{-bt}$