

Game Theory Week 4: Monday Exercises

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1. There is a single seller selling an item to two buyers. The seller values the item at 0 and both buyers value the item at 1. The game proceeds as follows:
 - First, the seller chooses a buyer and then makes an offer p to the buyer.
 - That buyer chooses to accept or reject, if the buyer accepts they receive $(1 - p)$ and the seller receives p .
 - If the buyer rejects, the rejecting buyer makes an offer p' to the seller, and the seller chooses to accept or reject. The seller receives $\delta p'$ if they accept, and the buyer receives $\delta(1 - p')$, where $\delta < 1$.

If no one accepts all players get 0.

- (a) Draw the extensive form (note we aren't repeating anything yet).
- (b) What are the pure strategy SPEs of this game?
- (c) Now suppose the above bargaining protocol is repeated until an offer is accepted (with payoffs discounted in the standard way). This game has a stationary SPE where whenever it is the seller's turn to make an offer, the seller chooses a buyer uniformly at random and makes the same offer regardless of the identity of the buyer. Find such an equilibrium.¹
- (d) Suppose we modify the setting from (c) so that now the buyer makes offers before the seller (i.e. the seller chooses a buyer, that buyer makes them an offer, if rejected, the seller makes a counteroffer, repeat). Find a stationary SPE where the seller chooses a buyer uniformly at random. Compare it to the equilibrium you found in (c).²

¹Can you construct another SPE? If you want even more practice, you should be able to prove that the seller receives the same payoff in any SPE.

²Another practice problem, I think this game has a (non-stationary) SPE where the seller receives the same payoff as in the alternating offer bargaining game we saw in class. It even has an SPE where the seller receives δ . Can you construct these?