



Aalto University  
School of Electrical  
Engineering

# ELEC-E8126: Robotic Manipulation Control in contact

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14.2.2022

# Learning goals

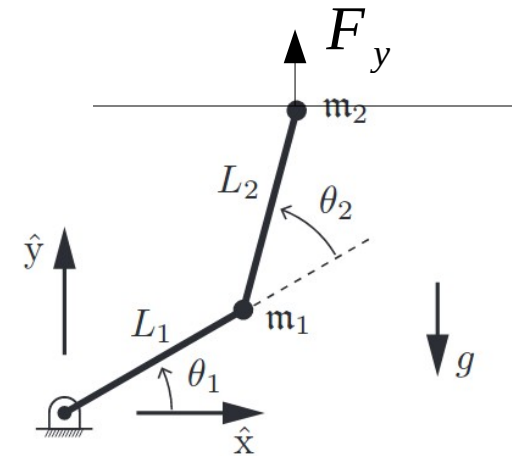
- Understand basic approaches of force and impedance control.
- Understand how control can be partitioned with multiple objectives.

# Contact

- Contact requires control of interaction forces.
- Approaches:
  - Control interaction forces to desired values → force control
  - Control interaction behavior → impedance control

# Force control

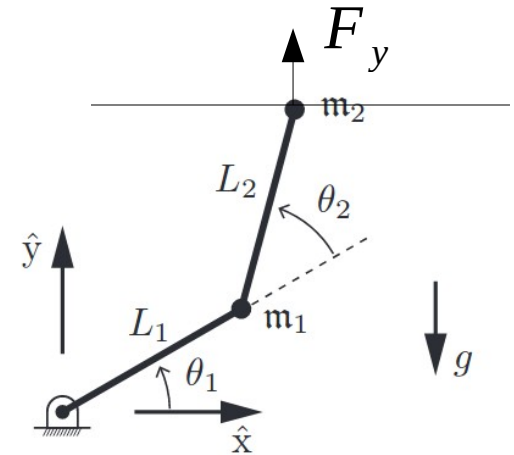
- Try to achieve vertical contact force  $F_{yd}$



- Propose a feedforward controller!

# Force control

- Try to achieve vertical contact force  $F_{yd}$



- Propose a feedforward controller!

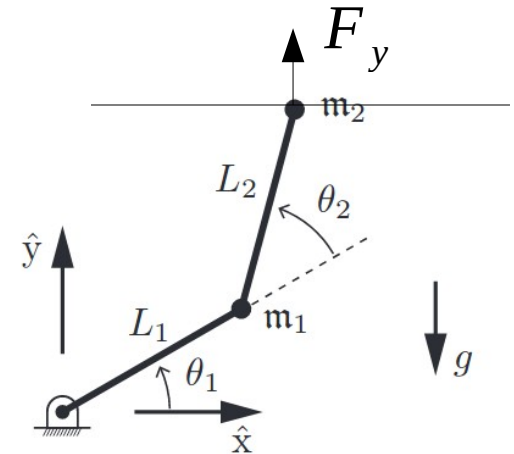
$$\boldsymbol{\tau} = \mathbf{J}_y^T(\boldsymbol{\theta}) F_{yd} + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

What's this?  
What is its meaning?

Dynamics (gravity)  
compensation

# Force control

- Try to achieve vertical contact force  $F_{yd}$



- Propose a feedforward controller!

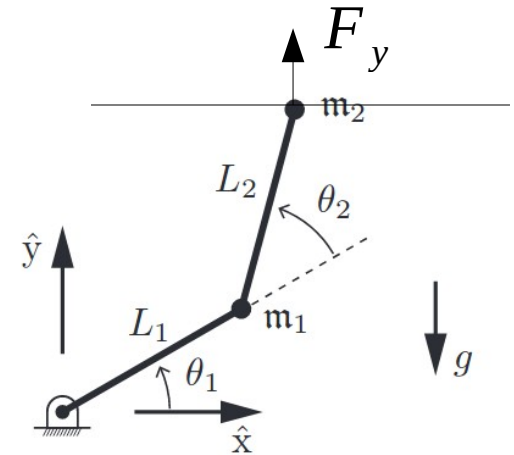
$$\boldsymbol{\tau} = \mathbf{J}_y^T(\boldsymbol{\theta}) F_{yd} + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

What's this?  
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# Force control

- Try to achieve vertical contact force  $F_{yd}$
- Now assume force can be measured, giving error  $F_{ye} = F_y - F_{yd}$



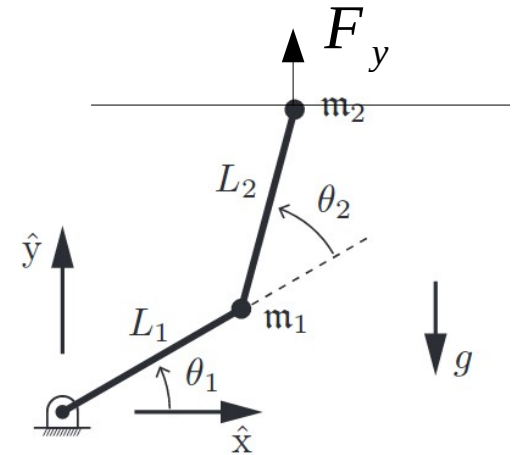
- PI-controller with feedforward can then be constructed:

$$\boldsymbol{\tau} = J_y^T(\boldsymbol{\theta}) \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Dynamics  
compensation

# Recap: Position control

- Propose a position controller to remain in horizontal position  $x_d$

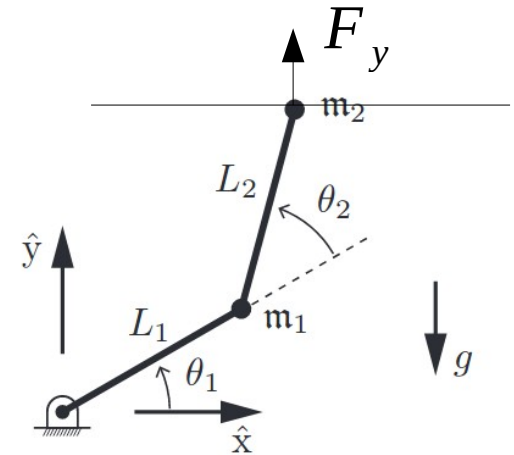




# Recap: Position control

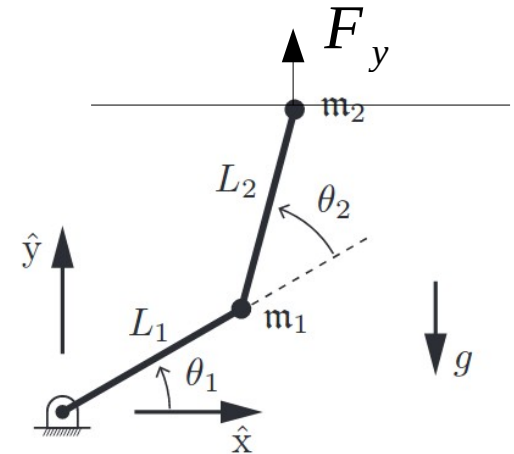
- Propose a position controller to remain in horizontal position  $x_d$
- Computed torque controller:

$$\boldsymbol{\tau} = J_{\hat{x}}^T(\boldsymbol{\theta}) \left( M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$



# Recap: Position control

- Propose a position controller to remain in horizontal position  $x_d$
- Computed torque controller:



$$\boldsymbol{\tau} = J_{\hat{x}}^T(\boldsymbol{\theta}) \left( M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

How to combine with force controller?

$$\boldsymbol{\tau} = J_y^T(\boldsymbol{\theta}) \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

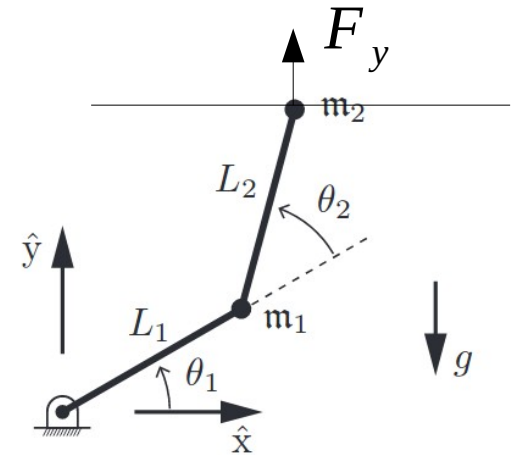
# Hybrid control (example)

- Hybrid just combination of the two

$$\boldsymbol{\tau} = J_x^T(\boldsymbol{\theta}) \left( M_C(\boldsymbol{\theta}) \left( \ddot{\mathbf{x}}_d + K_p \mathbf{x}_e + K_i \int \mathbf{x}_e + K_d \dot{\mathbf{x}}_e \right) \right) + J_y^T(\boldsymbol{\theta}) \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

- Note:  $J = \begin{bmatrix} J_x \\ J_y \end{bmatrix}$  (ignoring  $J_\theta$ )

- Can we write this now somehow in form  $\boldsymbol{\tau} = J^T(\dots)$ ?



# Hybrid control (example)

$$\begin{aligned} \boldsymbol{\tau} = & J_x^T(\boldsymbol{\theta}) \left( M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) \right) \\ & + J_y^T(\boldsymbol{\theta}) \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\tau} = & J^T(\boldsymbol{\theta}) \\ & \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) \right) \\ & + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \end{aligned}$$

$$\boldsymbol{\tau} = \mathbf{J}^T(\boldsymbol{\theta})$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \left( F_{yd} + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Orthogonal directions where controllers act

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} M_C(\boldsymbol{\theta}) \left( \ddot{x}_d + K_p x_e + K_i \int x_e + K_d \dot{x}_e \right) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \left( F_d + K_{fp} F_{ye} + K_{fi} \int F_{ye} \right) + \hat{\mathbf{h}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

# Hybrid control

- General formulation

$$\begin{aligned}
 \tau = J_b^T(\theta) & \left( \underbrace{P(\theta) \left( \tilde{\Lambda}(\theta) \left( \frac{d}{dt}([\text{Ad}_{X^{-1}X_d}]V_d) + K_p X_e + K_i \int X_e(t)dt + K_d V_e \right) \right)}_{\text{motion control}} \right. \\
 & + \underbrace{(I - P(\theta)) \left( \mathcal{F}_d + K_{fp} \mathcal{F}_e + K_{fi} \int \mathcal{F}_e(t)dt \right)}_{\text{force control}} \\
 & \left. + \underbrace{\tilde{\eta}(\theta, V_b)}_{\text{Coriolis and gravity}} \right). \tag{11.61}
 \end{aligned}$$

# Simultaneous tasks

## Some alternative formulations

- Torque control, two Cartesian space controllers

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta}) \left( P C_1(\boldsymbol{\theta}) + (I - P) C_2(\boldsymbol{\theta}) \right) \quad (+dyn)$$

- Velocity control

$$\dot{\boldsymbol{\theta}} = J_1^+ \dot{\mathbf{x}}_1 + N_1 \left( J_2^+ \dot{\mathbf{x}}_2 + N_2(\dots) \right)$$

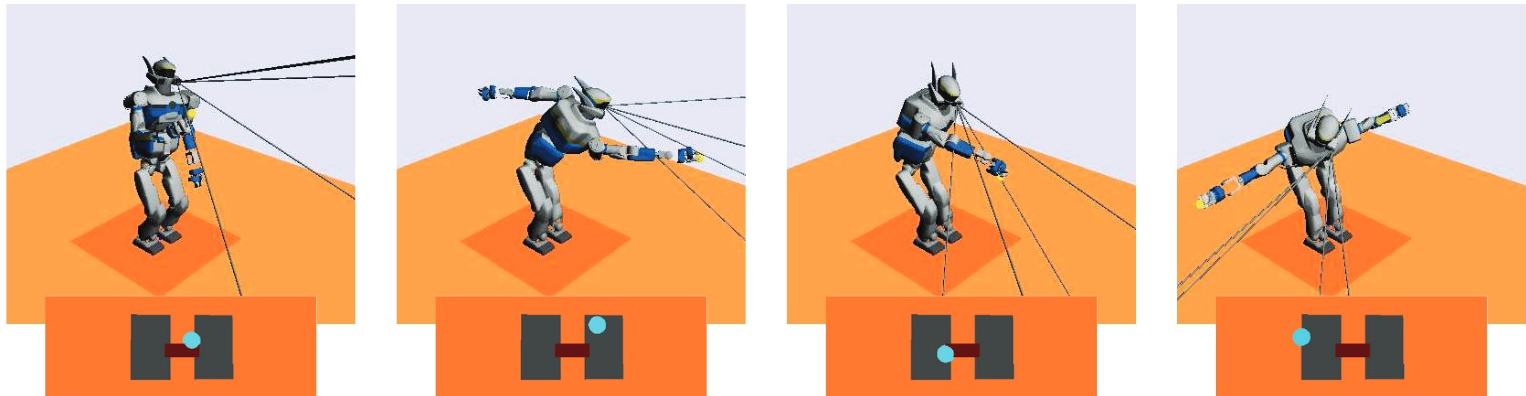
Jacobian of 1<sup>st</sup> task

Null space of 1<sup>st</sup> task

$$N_i = I - J_i^+ J_i$$

# Task formalism for control

- Stack of tasks (e.g. Mansard 2009) provides hierarchical approach of execution of multiple simultaneous tasks.
- Tasks prioritized.
- Example: simultaneous balancing, reaching, field of view

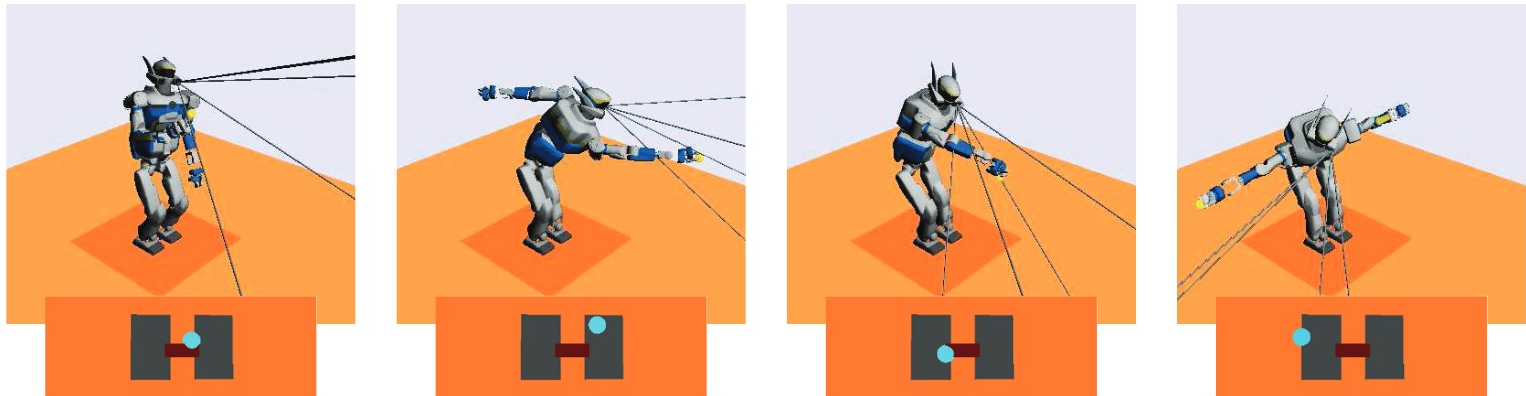


Escande et al., IJRR 2014



# Task formalism for control

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Back to in-contact control!

# Making robot compliant to external forces

- Instead of particular contact force, make robot mimic desired impedance characteristics (mass, spring, damper) when responding to external forces.
  - Robot acts as a virtual tool, e.g. with interacting human.
- Two approaches:
  - Sense robot (endpoint) motion and command torques  
→ impedance control
  - Sense interaction forces and command positions  
→ admittance control

# Impedance control

Measure position difference!

- Desired behavior: mass-spring-damper with respect to a reference trajectory

$$F_{ext} = M \ddot{x}_e + B \dot{x}_e + K x_e$$

diff. to desired

- Ideal control law

$$\tau = J^T(\theta) \left( \underbrace{M_C(\mathbf{x}) \ddot{\mathbf{x}} + \mathbf{h}_C(\mathbf{x}, \dot{\mathbf{x}})}_{\text{dynamics compensation}} - \underbrace{(M \ddot{x}_e + B \dot{x}_e + K x_e)}_{\text{desired behavior}} \right)$$

Negate true dynamics

Replace with desired dynamics

# Impedance control in practice

- Typical control law

$$\boldsymbol{\tau} = \mathbf{J}^T(\boldsymbol{\theta}) \left( -\mathbf{K}(\mathbf{x} - \mathbf{x}_d) - \mathbf{B}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) \right) + \mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

May contain only gravity

What's the inertia that's felt?



Compare to ideal

$$\boldsymbol{\tau} = \mathbf{J}^T(\boldsymbol{\theta}) \left( \mathbf{M}_C(\mathbf{x}) \ddot{\mathbf{x}} + \mathbf{h}_C(\mathbf{x}, \dot{\mathbf{x}}) - \left( \mathbf{M} \ddot{\mathbf{x}}_e + \mathbf{B} \dot{\mathbf{x}}_e + \mathbf{K} \mathbf{x}_e \right) \right)$$

difficult to  
measure

When force measurement and position (not torque) input are available

# Admittance control

- Measure external force  $F_{ext}$ , respond according to desired impedance behavior

$$F_{ext} = M \ddot{x} + B \dot{x} + K x$$

- Desired acceleration then

$$\ddot{x}_d = M^{-1} (F_{ext} - K x - B \dot{x})$$

- Desired accelerations in joint space

$$\ddot{\theta}_d = J^+ (\theta) (\ddot{x}_d - \dot{J} (\theta) \dot{\theta})$$

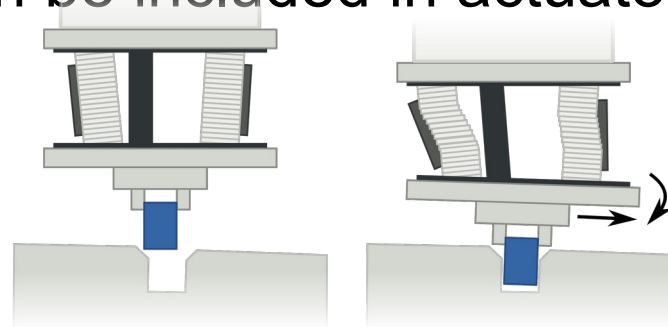
# Actuator effects

- Actuators do not produce torque exactly and may have significant internal dynamics.
  - Gearing introduces backlash.
  - Strain gauges may be used for torque measurement to close loop in torque.

Also variable impedance possible.

- Passive compliance can be included in actuator.

Torsional spring of series elastic actuator



Remote center of compliance device

# Summary

- Force control is used when desired forces can be specified.
- Impedance control used for designed contact interaction characteristics, typical for physically interacting with humans.

# Next time: Grasping and statics

- Readings:
  - Lynch & Park, Chapter 12.-12.1.3