ELEC-C8201: Control Theory and Automation

Exercise 6

The problems marked with an asterisk ($\star$) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Plot the frequency response of $G(j\omega)$

$$G(s) = \frac{5000}{(s+70)(s+500)}$$

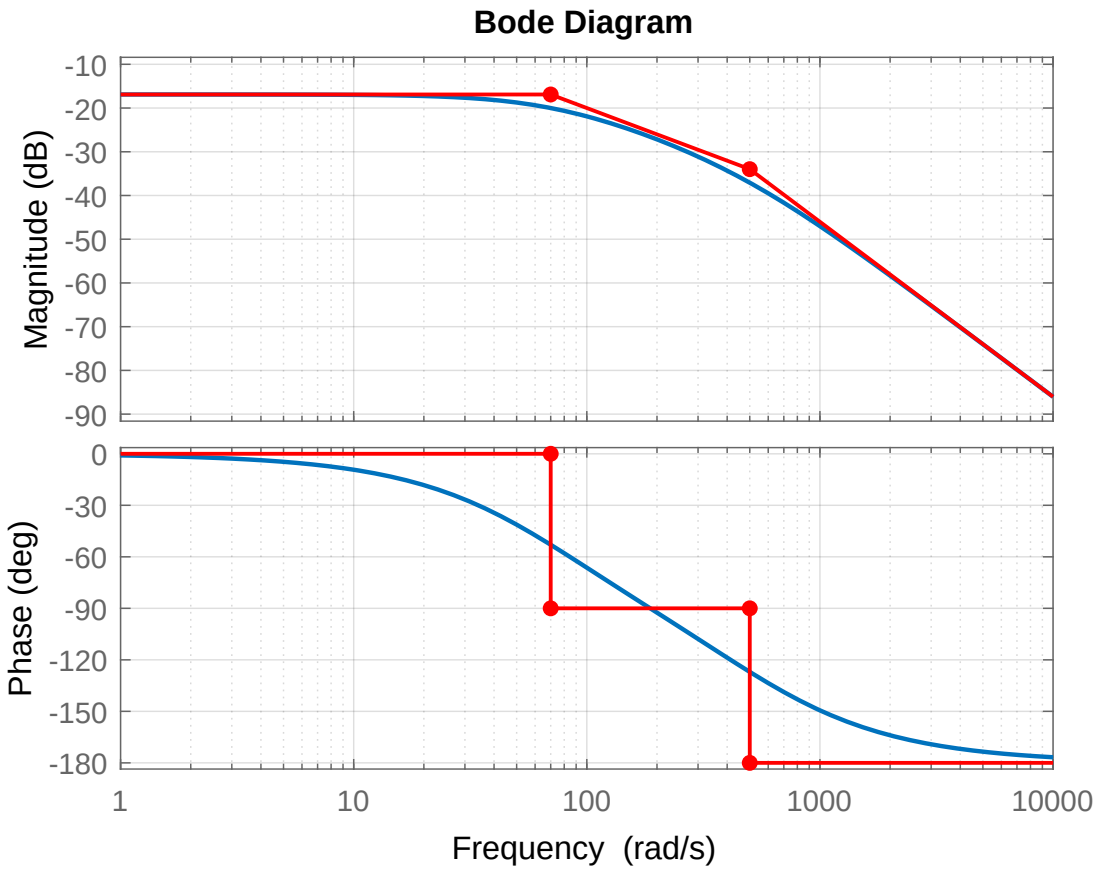
   a) Find the magnitude of $G(j\omega)$ at $\omega = 10$

   b) Find the magnitude of $G(j\omega)$ at $\omega = 500$

   c) Find the phase of $G(j\omega)$ at $\omega = 700$

   d) Is the system stable (by using the Bode plot)?

Solution. The Bode plot:



a)

$$|G(j10)| = \frac{5000}{\sqrt{10^2+70^2}\sqrt{10^2+500^2}} = -16.99dB$$

b)

$$|G(j500)| = \frac{5000}{\sqrt{500^2+70^2}\sqrt{500^2+500^2}} = -37.07dB$$

c)

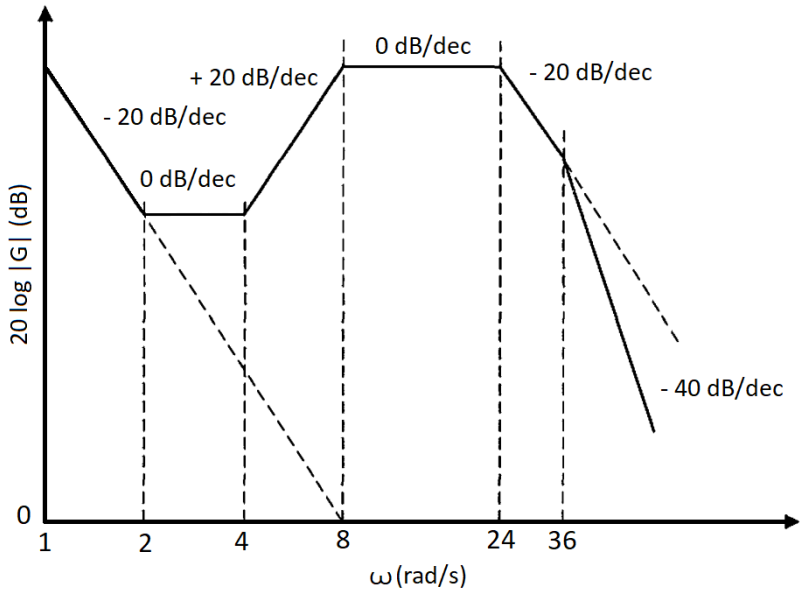
$$\angle G(j700) = -\tan^{-1}(\frac{700}{70}) - \tan^{-1}(\frac{700}{500}) = -138.75^\circ \Big(Matlab :>> atand() \Big)$$

d) The magnitude at the frequency with the phase of -180° satisfies $20\log|G| < 0$. Therefore, system is stable!

2. The magnitude plot of a transfer function

$$G(s) = \frac{K(1+0.5s)(1+as)}{s(1+s/8)(1+bs)(1+s/36)}$$

is shown in the following figure.



Determine K, a, and b from the plot.

Solution.

- before $\omega < 2$, the magnitude is

$$20\log|G| = 20\log\frac{K}{\omega}$$

This line crosses the horizontal axis at $\omega = 8$. So,

$$20\log\frac{K}{8} = 0$$

Then, $K = 8$.

- The slope is increased at $\omega = 2$ and $\omega = 4$, which means we have two zeros at these points. So, $a = \frac{1}{4}$.
- The slope is decreased at $\omega = 8$, $\omega = 24$ and $\omega = 36$, which means we have three poles at these points. So, $b = \frac{1}{24}$.

3. Sketch the Bode plot of the frequency response for the following transfer functions:

a)

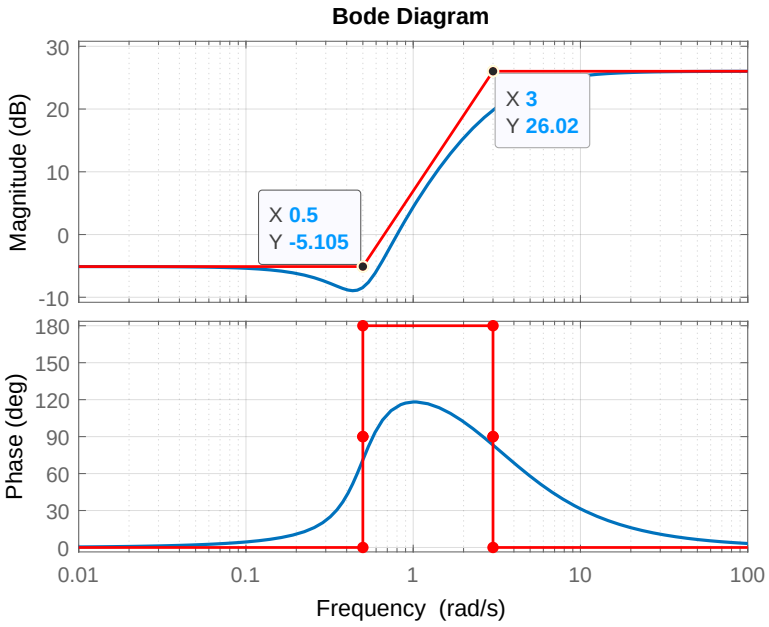
$$G = \frac{5(4s^2 + 1.4s + 1)}{(s+3)^2}$$

b)

$$G = \frac{4(s+0.5)(s^2+3s+2)}{(4s+1)(9s^2+4s+1)(s+3)}$$

Solution.

a) Break frequencies are at $\omega = 0.5$ and $\omega = 3$.



b) Break frequencies are at $\omega = 0.5$, $\omega = 1$, $\omega = 2$, $\omega = 0.25$, $\omega = 0.33$ and $\omega = 3$.