ECON-C4200 - Econometrics II

Lecture 10: Time series III

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Learning outcomes

- At the end of lecture 10, you know

1 what dynamic causal effects are

2 what the assumptions behind such a model are

3 how to estimate a dynamic causal model

4 how to interpet the results of a dynamic causal model

5 what **H**eteroskedasticity and **A**utoregression **C**onsistent (HAC) standard errors are

6 what a **V**ector **A**utoregressive **R**egression (VAR) model is

7 how to estimate a VAR

8 how to estimate a VAR how to and interpret the results of a VAR model

Dynamic causal effect

- Think of macroeconomic data and questions.

- Example: The effect of a change in the ECB interest rate on inflation next month, in 6 months, in 12, in 24,... months.

- Often, we only have one "subject", say the country affected by ECB decisions.

- Difficult to think of an RCT.

Dynamic causal effect

- Substitute thought experiment: different treatments to the same (subject / observation unit) at different points in time.

- Treatment $=$ e.g. level of interest rate t periods ago.

- If the different times are drawn from the same distribution (i.e., Y_y, X_t is stationary), then we can estimate the dynamic causal effects from an OLS regression of Y_t on X_t and its lags.

- Such an estimator is called a **distributed lag** estimator.

The distributed lag model

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1}... + \beta_{t-p+1} X_{t-p} + u_t$$

- Interpretation of β_1: The **impact effect of change in** X_t

- Notice that we are keeping the **past** constant!

- β_2: The **1-period dynamic multiplier**. The effect of a change in X_{t-1} on Y_t, keeping X_t, $X_{t-2},...$ X_{t-p} constant.

- β_3: The **2-period dynamic multiplier**. The effect of a change in X_{t-2} on Y_t, keeping X_t, X_{t-1}, $X_{t-3}...$ X_{t-p} constant.

- **j-period cumulative dynamic multiplier** $= \sum_{k=1}^{j-1} \beta_k$

- Example: 3-period cumulative dynamic multiplier $= \beta_1 + \beta_2 + \beta_3 + \beta_4$

  ...that is, the effect of all X_j for $j = t, t-1, t-2, t-3$.

The distributed lag model: assumptions

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1}... + \beta_{t-p+1} X_{t-p} + u_t$$

- **Exogeneity**: $\mathbb{E}[u_t|X_t, X_{t-1}, ...] = 0$
- **Strict exogeneity**: $\mathbb{E}[u_t|..., X_{t+1}, X_t, X_{t-1}, ...] = 0$
- Notice that if X is strictly exogenous, then it is also exogenous.
- We will proceed with assuming exogeneity.

The distributed lag model: assumptions

A1 $\mathbb{E}[u_t | X_t, X_{t-1}, ...] = 0$

A2 i) Y, X are stationary and ii) (Y_t, X_t) and (Y_{t-j}, X_{t-j}) become independent as j grows large.

A3 (Y_t, X_t) have 8 nonzero finite moments; large outliers are unlikely.

A4 There is no perfect multicollinearity.

The distributed lag model: assumptions

- Assumptions 1 and 4 are familiar.

- Assumption 2 is different. The difference is due to us studying time series.

- Assumption 3 is familiar otherwise, but now need 8 finite moments. This is to ensure "good" (=HAC) standard errors.

The distributed lag model: assumptions

- Assumption 2.i yields
  1. internal validity
  2. external validity
  3. This is the time series equivalent of the "identically distributed" of the cross-section i.i.d assumption

- Assumption 2.ii
  1. enables the use of a version of CLT
  2. This is the time series equivalent of the "independently distributed" of the cross-section i.i.d assumption

- The intuition is that when the time periods are sufficiently separated, we have independent experiments.

Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors

- Under the above assumptions a distributed lag model, estimated via OLS, yields consistent estimates of the coefficients β_k.

- What about the standard errors? The variance of the sampling distribution is different as X_t and u_t may be autocorrelated (think of what the equivalent would mean in a cross-section context).

- $\rightarrow$ need standard errors that are robust not only to heteroskedasticity but also to autocorrelation.

Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors

- In (well-behaving) cross-section data, the variance of β_1 in a univariate regression is

$$var(\hat{\beta}_1) = \frac{\sigma_u^2}{T(\sigma_X^2)^2}$$

- Let's consider a time-series with $T = 2$, with $cov(u_t, u_{t-1}) \neq 0$:

$$
\begin{aligned}
var\left(\frac{1}{T}\sum_{t=1}^{T} u_t\right) &= var[0.5(u_1 + u_2)] \\
&= 0.25[var(u_1) + var(u_2) + 2cov(u_1, u_2)] \\
&= 0.5(\sigma_u^2 + \rho)
\end{aligned}
$$

- Autocorrelation means that the usual variance formula for the error term does not apply.

Example: A (sort-of) Phillips curve

$$infl_t = \beta_0 + \beta_1 \delta UE_t + \beta_2 \delta UE_{t-1} + u_t$$

- Let's estimate the model with regular and HAC s.e.

Sort-of Phillips curve

```
. regr dlnpind_euro d.lnue_euro d2.lnue_euro, robust

Linear regression                               Number of obs   =      187
                                                F(2, 184)       =     0.23
                                                Prob > F        =   0.7956
                                                R-squared       =   0.0030
                                                Root MSE        =    .0041

                          Robust
dlnpind_euro    Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]

   lnue_euro
        D1.   -.0234946   .0368771   -0.64   0.525   -.0962509    .0492617
        D2.    .0059077   .0289012    0.20   0.838   -.0511127     .062928

       _cons    .001453   .0002997    4.85   0.000    .0008616    .0020444


. newey dlnpind_euro d.lnue_euro d2.lnue_euro, lag(1)

Regression with Newey-West standard errors      Number of obs   =      187
maximum lag: 1                                  F(  2,     184) =     0.22
                                                Prob > F        =   0.8056

                        Newey-West
dlnpind_euro    Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]

   lnue_euro
        D1.   -.0234946    .037917   -0.62   0.536   -.0983027    .0513134
        D2.    .0059077   .0276402    0.21   0.831   -.0486248    .0604402

       _cons    .001453   .0003067    4.74   0.000    .0008479    .0020581
```

Estimation of the distributed lag model

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1}... + \beta_{t-p+1} X_{t-p} + u_t$$

- You can estimate the model with OLS.

- You directly get the impact effect of X and the k-period dynamic multipliers & their standard errors

- You do not get the cumulative multipliers directly, nor their standard errors.

Estimating the cumulative dynamic multipliers

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1}... + \beta_{t-p+1} X_{t-p} + u_t$$

- Option 1: you estimate the above specification and then use post-estimation commands to calculate the cumulative dynamic multipliers and their standard errors.

Cumulative dynamic multiplier

```
. newey dlnpind_euro d.lnue_euro d2.lnue_euro, lag(3)

Regression with Newey-West standard errors          Number of obs    =       187
maximum lag: 3                                       F(  2,     184) =      0.23
                                                     Prob > F         =    0.7916
```

dlnpind_euro	Coef.	Newey-West Std. Err.	t	P>|t|	[95% Conf. Interval]
lnue_euro					
D1.	-.0234946	.0356168	-0.66	0.510	-.0937645 .0467753
D2.	.0059077	.0245427	0.24	0.810	-.0425137 .054329
_cons	.001453	.0002694	5.39	0.000	.0009215 .0019845

```
. scalar cdm = _b[d.lnue_euro] + _b[d2.lnue_euro]

. scalar list cdm
        cdm = -.01758694

. testnl _b[d.lnue_euro] + _b[d2.lnue_euro] = 0

  (1)  _b[d.lnue_euro] + _b[d2.lnue_euro] = 0

              chi2(1) =        0.37
          Prob > chi2 =      0.5450
```

Estimating the cumulative dynamic multipliers

- Option 2: Transform the model to yield direct estimates of the cumulative dynamic multipliers.

- Example: A 1-lag distributed lag model:

$$
\begin{aligned}
Y_t =& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
=& \beta_0 + \beta_1 X_t - \beta_1 X_{t-1} + \beta_2 X_{t-1} + \beta_1 X_{t-1} + u_t \\
=& \beta_0 + \beta_1 (X_t - X_{t-1}) + (\beta_2 + \beta_1) X_{t-1} + u_t \\
=& \beta_0 + \beta_1 \Delta X_t + (\beta_2 + \beta_1) X_{t-1} + u_t
\end{aligned}
$$

- Same trick works for higher order distributed lag models.

Cumulative dynamic multiplier

```
. gen diff = d.lnue_euro - d2.lnue_euro
(6 missing values generated)

. newey dlnpind_euro diff d2.lnue_euro, lag(3)

Regression with Newey-West standard errors          Number of obs    =        187
maximum lag: 3                                      F(  2,     184) =       0.23
                                                    Prob > F         =     0.7916
```

dlnpind_euro	Coef.	Newey-West Std. Err.	t	P>|t|	[95% Conf. Interval]	
diff	-.0234946	.0356168	-0.66	0.510	-.0937645	.0467753
lnue_euro D2.	-.0175869	.0290564	-0.61	0.546	-.0749134	.0397396
_cons	.001453	.0002694	5.39	0.000	.0009215	.0019845

Distributed lag model with autocorrelation

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1}... + \beta_{t-p+1} X_{t-p} + u_t$$

- Now assume that

$$u_t = \tilde{u}_t + \phi u_{t-1}$$

  in other words, u_t is autocorrelated (AR(1)).

- Besides using HAC standard errors, we could consider transforming the regression equation so that the error term in the transformed equation is i.i.d.

- As an example, let's work with the following distributed lag model:

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t$$

Estimating the cumulative dynamic multipliers

- We can proceed as follows:

$$
\begin{aligned}
Y_t =& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
Y_t - \phi Y_{t-1} =& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t - \phi Y_{t-1} \\
=& \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t \\
& - \phi \underbrace{(\beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + u_{t-1})}_{=Y_{t-1}} \\
=& (\beta_0 - \phi\beta_0) + \beta_1 X_t + (\beta_2 - \phi\beta_1)X_{t-1} - \phi\beta_2 X_{t-2} + \tilde{u}_t \\
& + \phi u_{t-1} - \phi u_{t-1} \\
Y_t =& \delta_0 + \phi Y_{t-1} + \delta_1 X_t + \delta_2 X_{t-1} + \delta_3 X_{t-2} + \tilde{u}_t
\end{aligned}
$$

- The outcome is an ADL(1,2) model with i.i.d error term.

Vector autoregressive (VAR) models

- What if you want to model two or more series simultaneously?

- What if they interact, i.e., the past values of Y affect current value of X and vice versa?

- VAR models are designed for these situations. Below an example:

$$Y_t = \beta_{10} + \beta_{11} Y_{t-1} + \beta_{12} Y_{t-2} + \gamma_{11} X_{t-1} + \gamma_{12} X_{t-2} + u_{1t}$$
$$X_t = \beta_{20} + \beta_{21} X_{t-1} + \beta_{22} X_{t-2} + \gamma_{21} Y_{t-1} + \gamma_{22} Y_{t-2} + u_{2t}$$

Vector autoregressive (VAR) models

- Without any constraints on the coefficients, one could estimate the VAR equation by equation.

- Often, theory suggests some constraints on the coefficients, e.g., in macroeconomic models.

- Example of VAR: The Euro-area Phillips curve.

- We use data from 2000m1 - 2015m12.

Plotting the Phillips curve

Stata code

```
1  use "mytimeseries.dta", clear
2  tsset time_ind
3  label var dlnpind_euro "inflation"
4  label var ue_euro "UE rate"
5  scatter dlnpind_euro ue_euro , ///
6      title("Euro area Phillips curve") ///
7      subtitle("2000m1 − 2015m12") ///
8      graphregion(color(white)) bgcolor(white)
9
10   graph export "L_II_10_4_phillips.pdf", replace
```

Plotting the Phillips curve

Euro area Phillips curve
2000m1 - 2015m12
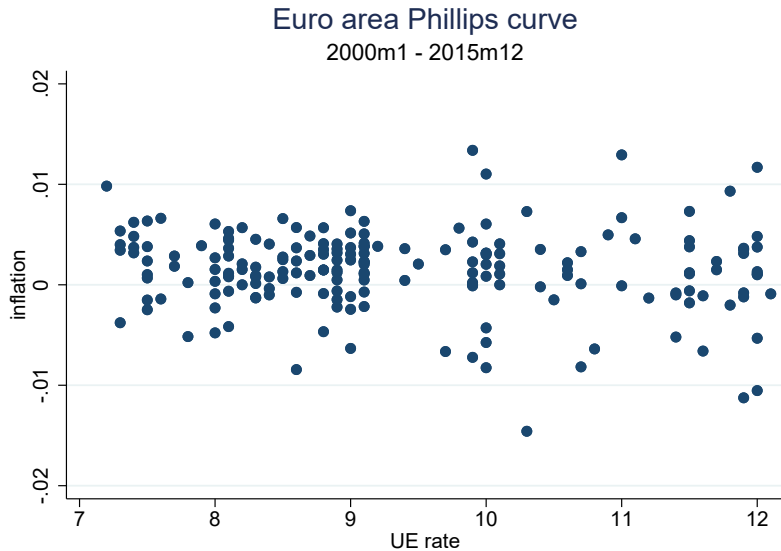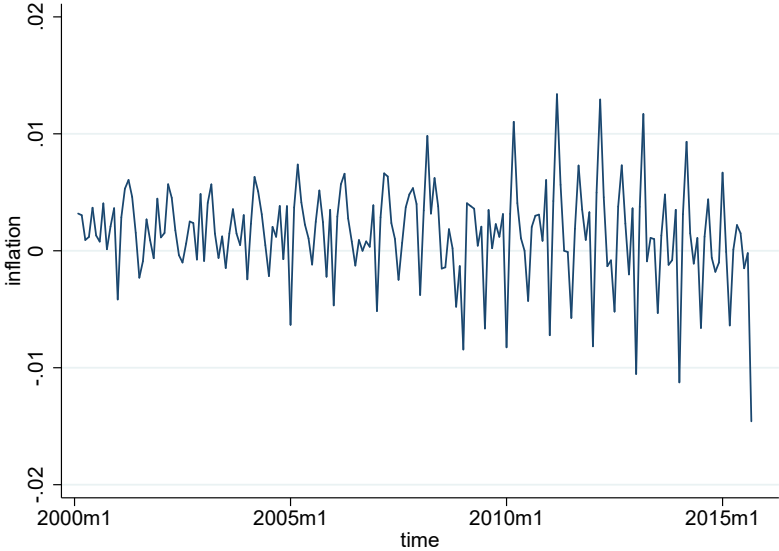
Inflation

Unemployment

Change in unemployment
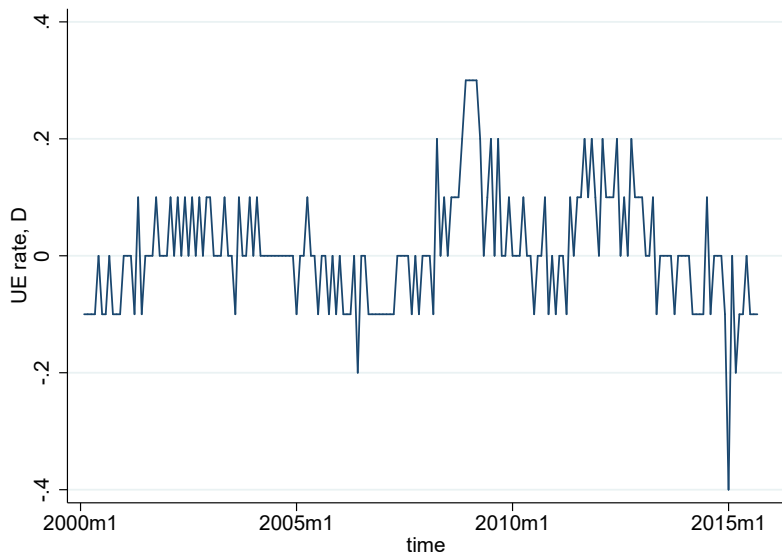
VAR on inflation and UE

Stata code

```
1  use "mytimeseries.dta", clear
2  forvalues t = 1/10{
3    dfuller ue_euro, lags('t')
4    }
5  forvalues t = 1/10{
6    dfuller d.ue_euro, lags('t')
7    }
8  forvalues t = 1/10{
9    dfuller dlnpind_euro, lags('t')
10   }
```

DF tests

```
. forvalues t = 1/10{
  2.        dfuller ue_euro, lags(`t')
  3.        }

Augmented Dickey-Fuller test for unit root          Number of obs   =      187

                    ───────── Interpolated Dickey-Fuller ─────────
               Test          1% Critical       5% Critical      10% Critical
            Statistic           Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)        -0.724           -3.481            -2.884            -2.574
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.8405
```
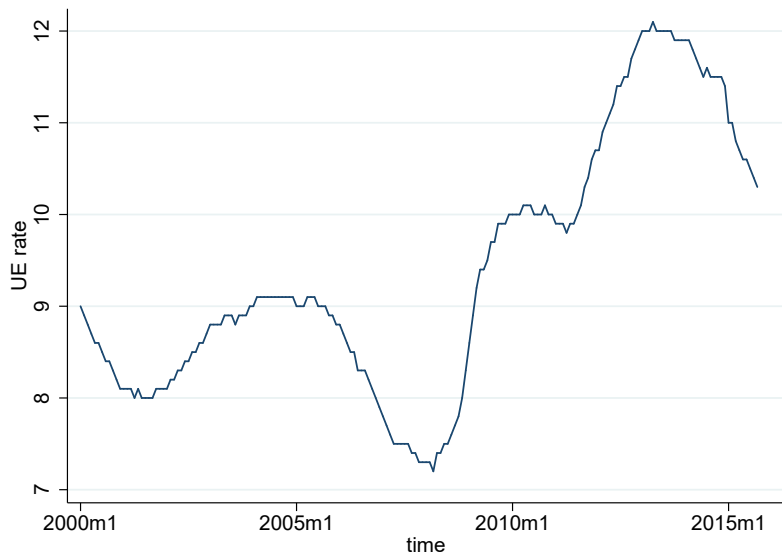
```
. forvalues t = 1/10{
  2.        dfuller d.ue_euro, lags(`t')
  3.        }

Augmented Dickey-Fuller test for unit root          Number of obs   =      186

                    ───────── Interpolated Dickey-Fuller ─────────
               Test          1% Critical       5% Critical      10% Critical
            Statistic           Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)        -4.188           -3.481            -2.884            -2.574
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0007
```

DF tests

```
. forvalues t = 1/10{
  2.        dfuller dlnpind_euro, lags(`t')
  3.        }

Augmented Dickey-Fuller test for unit root          Number of obs    =       186

                                  ————————— Interpolated Dickey-Fuller —————————
                    Test          1% Critical       5% Critical      10% Critical
                 Statistic           Value             Value             Value
————————————————————————————————————————————————————————————————————————————————
 Z(t)             -11.302           -3.481            -2.884            -2.574
————————————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.0000
```

VAR on inflation and UE

Stata code

```
1  var dlnpind_euro d.ue_euro
2  var dlnpind_euro d.ue_euro , lags(1 2 3)
3  var dlnpind_euro d.ue_euro
4  fcast compute f_ , step(12)
5  fcast graph f_dlnpind_euro , ///
6      title("Forecast for Euro area inflation") ///
7      graphregion(color(white)) bgcolor(white)
8    graph export "L_II_10_8_fcast_inflation.pdf", replace
9
10 fcast graph f_D_ue_euro , ///
11     title("Forecast for Euro area UE") ///
12     graphregion(color(white)) bgcolor(white)
13   graph export "L_II_10_9_fcast_ue.pdf", replace
```

VAR estimation results

- HQIC = Hannan-Quick information criterion

```
. var dlnpind_euro d.ue_euro

Vector autoregression

Sample: 2000m4 - 2015m9                    Number of obs    =        186
Log likelihood =   978.682                 AIC              =  -10.41594
FPE            =   1.03e-07                 HQIC             =  -10.34566
Det(Sigma_ml)  =   9.22e-08                 SBIC             =  -10.24251

Equation           Parms      RMSE     R-sq      chi2     P>chi2

dlnpind_euro          5      .004021   0.0567   11.18402   0.0246
D_ue_euro             5      .077844   0.3899   118.8916   0.0000
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]
dlnpind_euro					
dlnpind_euro					
L1.	.0607082	.0745786	0.81	0.416	-.0854631 .2068795
L2.	-.2318106	.0746343	-3.11	0.002	-.3780911 -.08553
ue_euro					
LD.	-.0019017	.0033073	-0.57	0.565	-.008384 .0045806
L2D.	.0024801	.0033123	0.75	0.454	-.0040119 .0089722
_cons	.0016858	.0003333	5.06	0.000	.0010326 .002339
D_ue_euro					
dlnpind_euro					
L1.	1.173706	1.443759	0.81	0.416	-1.65601 4.003421
L2.	-.1060152	1.444838	-0.07	0.942	-2.937846 2.725816
ue_euro					
LD.	.2300881	.0640266	3.59	0.000	.1045982 .355578
L2D.	.4883861	.064123	7.62	0.000	.3627074 .6140648
_cons	.0006486	.0064515	0.10	0.920	-.0119962 .0132933

VAR estimation results

```
. var dlnpind_euro d.ue_euro, lags(1 2 3)

Vector autoregression

Sample: 2000m5 - 2015m9                    Number of obs   =       185
Log likelihood =  975.7829                 AIC             = -10.39765
FPE            =  1.05e-07                  HQIC            = -10.29889
Det(Sigma_ml)  =  8.99e-08                  SBIC            = -10.15395

Equation           Parms      RMSE     R-sq      chi2     P>chi2

dlnpind_euro          7    .004039   0.0641   12.67015   0.0486
D_ue_euro             7    .077409   0.4029   124.8098   0.0000
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
dlnpind_euro						
dlnpind_euro						
L1.	.0393011	.0773214	0.51	0.611	-.112246	.1908483
L2.	-.2281727	.0747801	-3.05	0.002	-.3747389	-.0816064
L3.	-.0917675	.0771614	-1.19	0.234	-.2430011	.059466
ue_euro						
LD.	-.0015937	.0038003	-0.42	0.675	-.0090422	.0058548
L2D.	.0027843	.0034209	0.81	0.416	-.0039206	.0094892
L3D.	-.0011178	.0038034	-0.29	0.769	-.0085723	.0063366
_cons	.0018605	.0003637	5.12	0.000	.0011478	.0025733
D_ue_euro						
dlnpind_euro						
L1.	.6617554	1.481956	0.45	0.655	-2.242825	3.566336
L2.	.1285749	1.433248	0.09	0.929	-2.68054	2.93769
L3.	-.9287684	1.478889	-0.63	0.530	-3.827338	1.969801
ue_euro						
LD.	.1499964	.0728378	2.06	0.039	.0072368	.2927559
L2D.	.452871	.0655663	6.91	0.000	.3243634	.5813785
L3D.	.1552676	.0728961	2.13	0.033	.0123939	.2981414
_cons	.0023387	.0069703	0.34	0.737	-.0113229	.0160004

VAR estimation results

Forecast for Euro area UE

VAR estimation results

Forecast for Euro area inflation
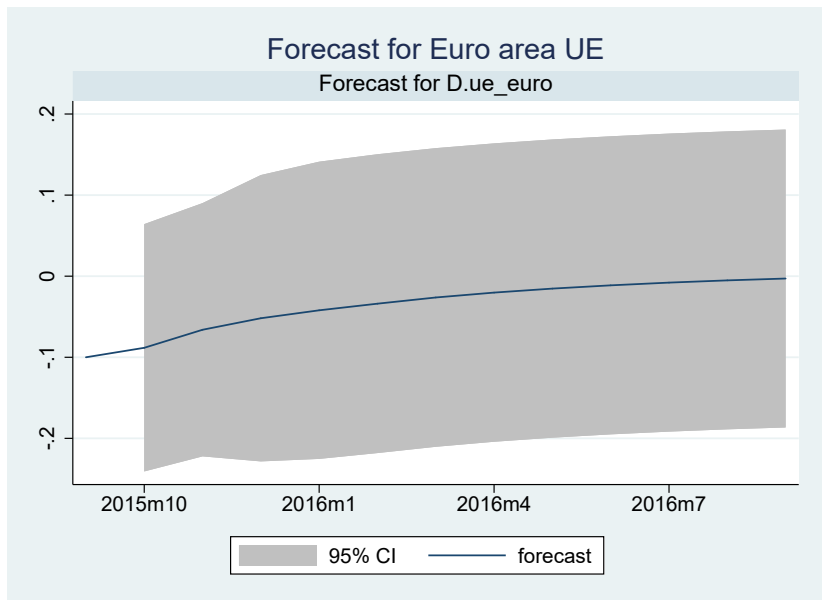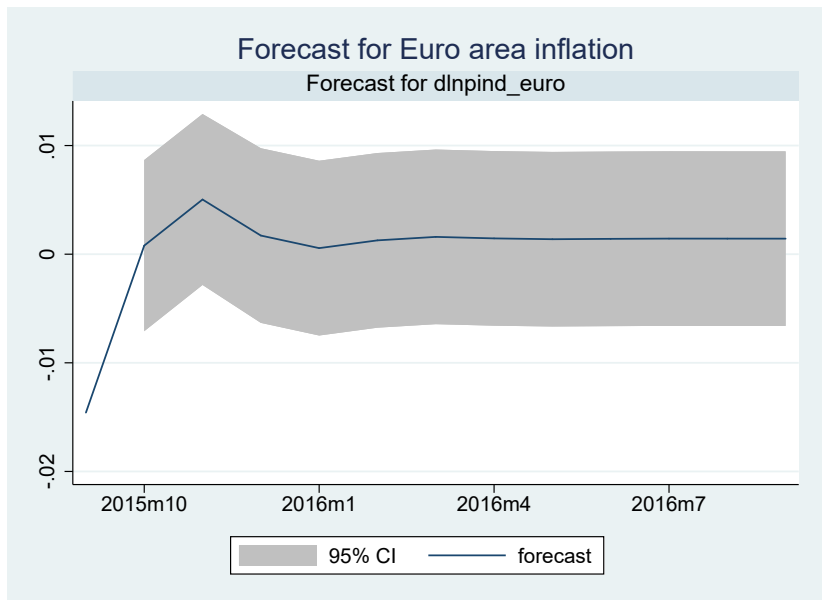
Granger causality

- Do lagged values of X predict Y_t, or the other way round?

- $H0$: the coefficients on all the lagged values of X in the Y regression are jointly insignificant.

- Granger causality $\neq$ causality.

- Does it make sense to assume the future does not affect the past?

VAR on inflation and UE

Stata code

```
1 var dlnpind_euro d.ue_euro
2 vargranger
```

Testing Granger causality

```
. vargranger

  Granger causality Wald tests
```

Equation	Excluded	chi2	df	Prob > chi2
dlnpind_euro	D.ue_euro	.63417	2	0.728
dlnpind_euro	ALL	.63417	2	0.728
D_ue_euro	dlnpind_euro	.66253	2	0.718
D_ue_euro	ALL	.66253	2	0.718