A practical method for torsional strength assessment of container ship structures


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Abstract

Container ship structures are characterized by large hatch openings. Due to this structural property, they are subject to large diagonal deformations of hatch openings and warping stresses under complex torsional moments in waves. This necessitates torsional strength assessment of hull girder of container ships in their structural design stage. In this paper, a practical method for torsional strength assessment of container ship structures with transparent and consistent background is discussed based on the results from up-to-date analyses. In order to estimate the torsional response characteristics as accurately as possible, three-dimensional Rankine source method, after being validated by tank tests, is employed for estimation of wave loads on a container ship, and FE analyses are conducted on the entire-ship model under the estimated loads. Then, a dominant regular wave condition under which the torsional response of the container ship becomes maximum is specified. Design loads for torsional strength assessment that give torsional response equivalent to the long-term predicted values of torsional response are investigated based on the torsional moments on several container ships under the specified dominant wave condition. An appropriate combination of stress components to estimate the total hull girder stress is also discussed.

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Keywords: Container ship; Torsional strength; Rankine source method; Tank test; Design load; Combined stress

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1. Introduction

According to economy of scale, the size of container ships is growing. The development of container ships seems to have been accelerated in recent years. The 5500 TEU class container ships that used be of the largest carrying capacity 10 years ago, are more or less standard at present. Nowadays, the largest post-Panamax container ships have a carrying capacity of over 8000 TEU and even a basic study on the design of a 12500 TEU container ship has been carried out [1].

Payer [2] discussed the development and transition of container ships both in technological and economical aspects. To the top of the technological problems associated with container ships comes the torsional responses of hull girder. That is, container ships, that are characterized by large hatch openings, are subject to considerable torsional deformations and warping stresses in waves. In this context, Sun and Soares [3] conducted a pioneering work in the ultimate strength of a hull girder with large openings under torsional moments. The warping stress components need to be taken into consideration for assessing the hull girder strength combined with other stress components like vertical bending stress and horizontal bending stress. The torsional deformations may cause concentrated stresses around hatch corners where structural design must be carried out in view of fatigue strength.

Many studies on container ship structures have dealt with torsional responses both from the structural and hydrodynamic viewpoints, e.g. [4–10]. Shimizu [5,8] modeled container ship structures as a beam with variable cross sections and conducted hydrodynamic and structural analyses. Nakata [6] and Umezaki [7] separately developed a total system in which ship motion analysis and structural analysis are conducted in combination with statistical analysis.

The torsional responses are influenced by the distribution of the torsional moments as well as the distribution of stiffness parameters of the ship’s structure whereas vertical bending stress or horizontal bending stress is determined only by the amplitude of the bending moments at a concerned position in relation to the section modulus at the section. Then, the entire-ship model is called for when analyzing the torsional response. Furthermore, torsional moments must be analyzed with high accuracy with respect not only to the magnitude but also to their distribution. As the phase differences among the vertical bending stress, the horizontal bending stress and the warping stress also have a significant influence on the estimation of the total hull girder stress, complex wave load analysis is also called for. Therefore, numerical wave load analysis and FE analysis have been used for the development of new and innovative design of container ships. In this sense, the development of container ship structures is largely dependent upon these numerical analyses.

On the other hand, it cannot be denied that design loads introduced by classification societies for structural strength assessment of hull girder have been utilized in the design of container ships as standard loads for the sake of convenience. This means that the accuracy of the estimation of the torsional response, and consequently the final dimensioning of the structure and the safety of the ship, depend largely on the accuracy of the design loads. Therefore, it is desired to develop the design loads for torsional strength assessment that reflect the accurate
results obtained from the up-to-date numerical analyses. Using such design loads will lead to more appropriate assessment of torsional strength of container ships without conducting complex wave load analyses and to the advancement of the structural safety of ships.

In order to make the design loads reliable and convincing for the ship designers, the process of the development is to be transparent and rational. A co-author [11] has succeeded in developing the practical estimation methods of design loads for primary structural members of tankers and bulk-carriers that have transparent and consistent backgrounds. Reference [11] discussed the relations among design sea states, design wave and design load and finally showed that the design loads can be set by following their proposed method so that responses under the design loads may be equivalent to the long-term predicted values of the responses.

Our present goal is to propose a practical method for torsional strength assessment with transparent and consistent background based on the results obtained through as accurate analyses as possible. Design loads and an appropriate combination of stress components for assessing the hull girder torsional strength of container ships are mainly discussed. The research is carried out in the following steps:

(1) Establishment of wave load estimation method: Though there are a few examples of experimental study on the torsional moments of container ships, there seems to be none on post-Panamax size container ships. A numerical method which have been developed by a co-author is first validated by tank tests.

(2) Accurate structural analysis: Torsional moments on container ships in waves are first calculated, and then, the loads are directly applied to an entire-ship FE model. Response functions of warping stress and relative deformations are determined. Short- and long-term predictions (exceedance probability $Q = 10^{-8}$) are also made.

(3) Proposal and verification of design loads: A dominant regular wave condition under which the torsional responses of the container ship become maximum is specified by referring to the response functions. Torsional moments under the dominant wave condition are calculated for 10 container ships of different sizes. Design loads for torsional strength assessment are proposed based on the results. The responses under the proposed design loads are compared with the long-term predicted values obtained in Step (2) for the container ships.

(4) Proposal and verification of a combination of stress components: Response functions of vertical bending stress, horizontal bending stress and the total hull girder stress in which phase differences among stress components are rigorously considered are calculated following Step (2). An appropriate combination of stress components is proposed by referring to the relations among the response functions. The hull girder stresses estimated by the proposed combination of stress components are compared with the long-term predicted values of total hull girder stresses in which the phase differences among stress components are rigorously considered.

(5) Proposal of a practical method for torsional strength assessment is based on the above.
2. Wave load estimation

2.1. Advantages and disadvantages of various numerical analysis methods

Various strip methods [12,13] have been developed and used to estimate wave-induced ship motions and wave loads including vertical bending moment and horizontal bending moment with sufficient accuracy for practical purposes. Strip methods are extensively used as a standard tool to predict the nonlinear loads and motions [14], sometimes considering the elasticity of ship structures today [15–17]. However, as they do not precisely consider hydrodynamic interference effects of reflected waves among strips lengthwise or three-dimensional effects, there are some concerns on the accuracy of the strip methods for estimation in short waves.

In order to enhance the accuracy of estimation especially in short waves, many numerical methods considering the three-dimensional effects have been proposed. Among them are three-dimensional Green function method [18] and Rankine source method [19–21] based on three-dimensional potential theory. These methods have advantages in that they take the three-dimensional effects into account, have good stability of computations and their computing time is moderate. Therefore, they are being anticipated as convenient design tools replacing the strip methods.

Although most of these three-dimensional methods were initially developed for frequency domain simulation, they were soon extended to analysis methods for time domain simulation [22,23] with the advancement of computing power. This facilitated the nonlinear effects, that appear with finiteness of amplitudes of waves and ship motions which may play an important role in the estimation of wave loads, to be considered in the time domain simulation. One of the disadvantages of these methods is, however, still the large time requirement for computations.

It is shown by a co-author [24] that Computational Fluid Dynamics (CFD) can be applied to the estimation of wave loads considering the finiteness of wave and motion amplitudes more precisely. Even highly nonlinear phenomena such as slamming and shipping of green water might be considered since the Navier–Stokes equation is directly and numerically solved in this method. Although the maturity level of the method is comparatively low as it has just started being developed, it is anticipated as one of the ultimate methods for the estimation of wave loads.

Advantages and disadvantages of the above methods are summarized in Table 1. Considering the accuracy, the stability and the time for computations and the compatibility with a long-term prediction method, an estimation method based on frequency domain and three-dimensional potential theory is adopted in this paper. The nonlinearity with the finiteness of wave and motion amplitudes are considered by utilizing the results of tank tests conducted under large wave conditions.

2.2. Rankine source method

Rankine source method was initially used to predict wave resistance in steady-state free surface flow on an advancing ship by Gadd [25] and Dawson [26]. It has been expanded to solve unsteady free surface flow problems. It is now being used to
directly estimate ship motions and loads on a ship advancing in waves in the linear frequency domain. This method formulates boundary value problems by integral equations taking a simple source as core function in frequency domain.

A co-author has also developed a numerical analysis code for the estimation of wave loads based on the three-dimensional Rankine source method [27,28]. In the numerical analysis code, the upstream finite difference operator is applied for radiation condition of Kelvin waves [26], while Rayleigh viscosity is employed for the radiation condition of ring waves [19,21].

2.3. **Tank test**

Tank tests were conducted at the Nagasaki Experimental Tank (length: 160 m, width: 30 m, depth: 3.5 m) of Mitsubishi Heavy Industries. A scale model was towed with a six free motion guide fixed on an XY carriage. The model used in the tank tests is a typical post-Panamax container ship which was designed by National Maritime Research Institute of Japan. **Table 2** shows the principal particulars of the model. The ship model is separated into four segments being divided at the cross-sections located respectively at the square stations (SS) 2.5, 5.0 and 7.5. Adjacent segments are connected via a force transducer so that the sectional forces and moments can be measured at these sections, as shown in Fig. 1.

Six degrees of motions, two shear forces (vertical and horizontal shear forces) and three moments (vertical and horizontal bending moments and torsional moments) at
the three cross-sections and hydrodynamic pressures were measured. Considering
that the weight distribution along the ship length is important for evaluating
torsional moments, a typical weight distribution of the full loading condition from
the actual loading manual was reproduced in the tank tests.

The wave condition of the tank tests is shown in Table 3. The tank tests were
carried out in regular waves for three different incident wave heights, 10 different
wave lengths, seven different incident wave angles from 180° (head sea) to 0°
(following sea) at 30° intervals and two different ship speeds. The tank tests in waves
15 m high could not be carried out in short wave range since such incident waves
break at this height.

2.4. Comparisons with numerical results

Figs. 2(a)–(c) show the comparisons of the amplitudes of the response functions,
or the so-called response amplitude operator (RAOs) of torsional moments, at the
three sections (SS 2.5, 5.0 and 7.5) in bow sea (120°). The abscissa indicates the
wavelength \( \lambda \) normalized by ship’s length \( L(\lambda/L) \), while the ordinate shows the
amplitude of torsional moments per unit wave amplitude. The tank test results were
analyzed by Fourier analysis, and thus, the tank test values show the amplitudes of
components having the same period as the encountering wave period. In the figure,
“Exp.(3.5 m)”, “Exp.(9.0 m)”, “Exp.(15.0 m)”, “STRIP” and “Rankine” represent
the tank test results in waves with three different incident wave heights, the
numerical results by the strip method and the Rankine source method, respectively.
The torsional moments were calculated about the vertical position of the force
transducer, or slightly below the still-water level.

The numerical results obtained by the Rankine source method show good
agreement with the tank test results in waves 3.5 m high, especially in the shorter
Table 3
Wave conditions of tank tests

<table>
<thead>
<tr>
<th>Incident wave height (m)</th>
<th>Wave length ( \lambda )/ship length ( L )</th>
<th>Encountering angle ( \chi ) (deg)</th>
<th>Ship speed (knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5, 9.0, 15.0</td>
<td>0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.6</td>
<td>0, 30, 60, 90, 120, 150, 180</td>
<td>18.4, 24.5</td>
</tr>
</tbody>
</table>

Fig. 2. Comparisons of response functions of torsional moments obtained by numerical methods with those obtained by tank tests: (a) at SS 2.5, (b) at SS 5.0 and (c) at SS 7.5.

wave range, as shown in Fig. 2 whereas the numerical results obtained by the strip method do not agree well with the tank test results.

It is also noticed from the figure that the results in the shorter wavelength range of bow seas \( (120^\circ) \) are the largest. Moreover, the values at stern section (SS 2.5) are larger than those at the other sections (SS 5.0 and 7.5) in the shorter wavelength range of bow sea \( (120^\circ) \).
2.5. Nonlinear characteristics of torsional moments in waves

The differences among the tank test results indicated as “Exp.(3.5m)”, “Exp.(9.0m)” and “Exp.(15.0m)” in Figs. 2(a)–(c) are caused by the nonlinear characteristics of torsional moments due to wave heights. The nonlinearity is clearly observed to appear with the torsional moments at bow section (SS 7.5) in such a way that the torsional moments per unit incident-wave amplitude becomes larger as the wave height increases whereas the three values in waves of different wave heights are almost identical with the torsional moments at stern section (SS 2.5) and midship (SS 5.0) which implies very weak nonlinearity.

Short- and long-term predictions of torsional moments about shear center with respect to the midship section were made by using the RAOs of torsional moments obtained in the experiment. When making the short- and long-term predictions, the ISSC-1964 wave spectra (directional distribution: \( \cos^2 \theta \)) and the IACS wave data (North Atlantic Ocean, whole year [29]) were used, respectively. Long-term predictions were made by using the method proposed by Fukuda [30].

Then, coefficients associated with nonlinearity of wave loads are determined quantitatively by comparing the long-term predicted values of torsional moments in large waves with those in linear small waves. The coefficient of nonlinearity \( C_{\text{nonlinear}} \) is defined as the ratio of the long-term predicted value of torsional moments in large waves to that in small waves that represent linear components,

\[
C_{\text{nonlinear}} = \frac{[X_{\max}]_{H_w = 9.0\,\text{m}}}{[X_{\max}]_{H_w = 3.5\,\text{m}}},
\]

where \( [X_{\max}]_{H_w = 9.0\,\text{m}} \) and \( [X_{\max}]_{H_w = 3.5\,\text{m}} \) are respectively the long-term predicted values of torsional moments obtained by using the RAOs in waves 9.0 m high at \( 10^{-8} \) probability level and those in waves 3.5 m high. Herein, it is assumed that the tank test results of the concerned responses in waves 3.5 m high are considered as the linear ones, and that the increase of torsional moments due to nonlinearity is nearly the maximum in waves 9 m high since the waves over 15 m high break in the short wave region where the torsional moments are significant.

Table 4 shows the coefficients of nonlinearity regarding the torsional moments at the three respective sections (SS 2.5, 5.0 and 7.5). From the table, it is found out that the nonlinear characteristics can hardly be significant at the midship and stern sections (SS 2.5 and 5.0) while the nonlinear coefficients of the torsional moments at bow section (SS 7.5) are sizeable.

3. Structural response estimation

3.1. General

As mechanics of materials show, torsional response and horizontal response are coupled in case of a beam with large open section. With a view to de-coupling the warping stress and the horizontal bending stress, the standard procedure is that
torsional moments are evaluated about the shear center with respect to the midship section and the warping stress is defined as a hull girder stress under the torsional moments. However, as the height of the shear center of a real container ship structure which can be expressed as an assembly of consecutive sections with different sectional properties varies along the ship length, the stress components cannot be separated perfectly.

On the other hand, the warping stress $\sigma_{WT}$ might also be defined as the hull girder stress under horizontal shear forces and torsional moments with horizontal bending stress deducted, which is conceptually expressed as $\sigma_{WT} = \sigma_{HS} + \sigma_{TM} - \sigma_{WH}$ where $\sigma_{HS}$, $\sigma_{TM}$ and $\sigma_{WH}$ are hull girder stress due to horizontal shear forces, torsional moments and horizontal bending moments, respectively. In this definition, an assumption of the height of shear center is not required.

Throughout this research, the response functions of the warping stress are obtained following the first definition as far as the results from the two definitions are almost identical. The second definition is adopted for evaluating the warping stress at around bow and stern parts.

3.2. Analysis procedure of torsional response

Torsional moments on a post-Panamax size container ship ($L \times B \times D \times d = 287 \times 40 \times 24 \times 13 \text{ m}^4$) in regular waves are analyzed by employing the three-dimensional Rankine source code explained in the previous section. The moments are estimated about shear center.

Structural response of the container ship under the torsional moments in waves is analyzed by using a global FE-structural model shown in Fig. 3. It consists of about 50,000 of relatively coarse elements, most of which are plate elements and rod elements. The girder or floor spacing is taken as the standard length of one side of the element. Effects of stiffeners are considered by lumping and modeling in the nearest mesh line so that all the global structural properties, such as vertical bending rigidities, horizontal bending rigidities, torsional rigidities and torsional-bending rigidities, may be reproduced in the FE-model.

An appropriate combination of constraints is made so that the translational and rotational displacements as a rigid body may be constrained. As the torsional moments analyzed by use of the Rankine source code are self-balanced, reaction forces at constrained points are sufficiently small.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>$\theta = 120^\circ$</th>
<th>All heading</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS 2.5</td>
<td>1.04</td>
<td>1.02</td>
</tr>
<tr>
<td>SS 5.0</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>SS 7.5</td>
<td>1.18</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Now a certain regular wave is selected, and the torsional moments calculated under the regular wave condition are directly applied to the FE-model taking the distribution shape of the torsional moments into account. An application method of the torsional moments is illustrated in Fig. 3 as an example. The difference in the torsional moments between consecutive cross sections at bulkheads is taken, then a set of equivalent vertical shear loads is applied at each bulkhead. It is necessary to make FE-analysis for the given regular wave only twice, that is, for one instant and the next instant shifted by 90° in phase.

Response functions of the warping stress are obtained by repeating the analysis described above under regular waves with different wavelengths and wave heading angles. In the same manner, response functions of the relative deformations of hatch openings are calculated. The relative deformation of hatch opening is defined as the difference in longitudinal displacements between hatch sides (Fig. 4).

3.3. Results

Figs. 5(a)–(c) show the RAOs of torsional moments at SS 2.5, 5.0 and 7.5, respectively. The abscissa shows the wave length \( \lambda \) normalized by the ships length \( L \) and the ordinate shows the non-dimensional torsional moments about shear center per unit wave amplitude. The curves of the RAOs of torsional moments are characterized by one mild peak that appears in the short wave region. The peak is the highest when the wave encountering angle is 120°.
Fig. 4. Definition of relative deformation of hatch openings.

Fig. 5. Response amplitude operators of torsional moments about shear center: (a) at SS 2.5, (b) at SS 5.0 and (c) at SS 7.5.
The RAOs of the warping stress in front of the engine room and those of the relative deformation of a hatch opening are shown in Figs. 6 and 7, respectively. The warping response and the relative deformation are also characterized by one mild peak curve whose peak appears in the comparatively short wave region when the wave encountering angle is $120^\circ$.

4. Design Load for torsional strength assessment

4.1. General

Design loads may generally be defined as loads that give the response values equivalent to the long-term predicted values (e.g. exceedance probability $Q = 10^{-8}$.
corresponding to ship’s design life) of response. In this context, design loads for torsional strength assessment are discussed considering that they give the values of warping stress and relative deformation equivalent to the respective long-term predicted values.

If the values of long-term predictions are mostly represented by response values in a certain regular wave (hereafter termed as dominant wave), the torsional moments as design loads can be discussed based on the torsional moments under the dominant wave condition.

The largest response which a ship may encounter once or a few times during her lifetime will be attained in extreme waves. In such extreme waves, nonlinear effects appear in various hull responses such as ship motions, hull girder moments and pressures. Thus, the effects of nonlinearity are to be considered when setting the final design loads.

4.2. Procedure for determination of design loads

The design loads for torsional strength assessment are investigated in the following steps:

(1) Specify a dominant regular wave condition under which the amplitudes of torsional response become maximum by referring to the response functions.
(2) Determine the distribution shape of the design torsional moments based on 10 container ships of different sizes under the specified wave condition.
(3) Determine the magnitude of the design torsional moments based on the long-term prediction of the torsional moments calculated for the container ships.
(4) Verify the proposed design loads by comparing the response under the proposed design loads and the long-term predicted values of the response.
(5) Consider the effects of nonlinearity.

4.3. Specification of dominant wave condition

The RAOs of warping stress in front of the engine room become maximum when the incident wave length is relatively short and the wave encountering angle is 120° (see Fig. 6). The set of wave length and wave encountering angle can be a candidate of a dominant wave condition. Such sets were examined with respect to the warping stress at 26 points on the upper deck level along the ship length. The number of the sets which coincide with the respective wave condition (wave length and wave encountering angle) was counted.

The results are summarized in Table 5. It is confirmed that most of the RAOs become maximum under the same wave condition: the wave length $\lambda$ is about 0.35$L$ and the wave encountering angle $\chi$ is 120°.

Dominant wave conditions were also examined with respect to the relative deformations of hatch openings and the torsional moments along the ship length. It was confirmed that the RAOs of relative deformation of hatch opening become
maximum under the same wave condition, and the RAOs of torsional moments at
most square-stations about shear center also become maximum under the same wave
condition. Thus, a regular wave with wave length $\lambda = 0.35L$ and wave encountering
angle $\chi = 120^\circ$ was specified to be the dominant regular wave condition.

4.4. Proposal of design loads for torsional strength assessment

Torsional moments about shear center under the dominant wave condition
specified in Section 4.3 were calculated by using the Rankine source code on 10 ships
of different sizes listed in Table 6. In Fig. 8, the longitudinal distributions of
torsional moments at a certain instant are shown. It seems that the distribution
shapes resemble each other and the distribution curves may be approximated by a
representative curve. In Fig. 9, the longitudinal distributions of torsional moments at
the consecutive instant shifted by $90^\circ$ in phase are also shown. The curves are also
approximated by another representative curve.

These representative curves are taken as the tentatively proposed design load
distributions, and are expressed by the following equations:

\[
C_{T1} = 1.1 \left[ \sin \left( 2\pi \frac{x}{L} \right) + 0.1 \sin^2 \left( \frac{\pi x}{L} \right) \right]
\times \exp \left( -0.7 \frac{x}{L} \right) \exp \left( -8 \left( \frac{x/L - 0.5}{0.5} \right)^{10} \right),
\]

\[
C_{T2} = 0.5 \left[ -\sin \left( 3\pi \frac{x}{L} \right) + 0.8 \sin^3 \left( \frac{\pi x}{L} \right) \right]
\times \exp \left( -0.6 \frac{x}{L} \right) \exp \left( -8 \left( \frac{x/L - 0.5}{0.5} \right)^{10} \right).
\]

As shown later in Fig. 14, there are two peaks respectively at around SS 2.5 and SS
7.5 in the longitudinal distribution of the amplitude of torsional moments along the
ship length. The peak at around SS 2.5 is about 1.5 times as high as that at SS 7.5,
which is considered to be caused by the unsymmetric hull shape and weight
distribution along the ship length.

The magnitude of the design loads is determined according to the long-term
predicted values of torsional moments at SS 2.5 obtained by using the Rankine
source code. It is shown that the long-term predicted values are well approximated

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**Table 5**
Dominant wave conditions along the ship length in terms of wave length $\lambda$ and the wave encountering angle $\chi$ under which the RAOs with respect to warping stress become maximum

<table>
<thead>
<tr>
<th>Position ($x/L$)</th>
<th>0.15</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda/L$</td>
<td>0.39</td>
<td>0.35</td>
<td>0.31</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.39</td>
<td>0.51</td>
</tr>
<tr>
<td>Angle $\chi$</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>
by Eq. (3),

\[ M_{WT} = 1.3 C_1 L d_f C_b (0.65d + e) + 0.2 C_1 L B^2 C_w (\text{kNm}), \]

where \( L, B, d, e, C_w \) and \( C_b \) are the ship’s length (m), breadth (m), draft (m), the distance to shear center from baseline (m), the water plane coefficient and the block coefficient, respectively. \( C_f \) is the wave height coefficient corresponding to the IACS vertical bending moment and is given as

\[
C_1 = \begin{cases} 
10.75 - \left( \frac{200-L}{100} \right) & \text{for } L \leq 300 \, \text{(m)}, \\
10.75 & \text{for } 300 < L \leq 350 \, \text{(m)}, \\
10.75 - \left( \frac{L-350}{150} \right)^{1.5} & \text{for } 350 < L \, \text{(m)}. 
\end{cases}
\]
There are two components in Eq. (3). One from torsional moment due to shear force multiplied by the distance to shear center from the working point. And the other mainly from moment due to the bottom pressure distribution. A comparison with the long-term predicted values of the first component is shown in Fig. 10(a). The abscissa shows the long-term predicted values of horizontal shear force while the ordinate shows the values calculated from Eq. (3). The comparison of the results calculated from the formula with the long-term predicted values is shown in Fig. 10(b). The former represents the larger part of the total magnitude of the design torsional moments, as shear center is far below the baseline of the ship structure (about 0.5D, D: moulded depth of ship).

4.5. Verification of the proposed design loads

Response functions of the warping stresses and the relative deformations of hatch openings were calculated for the container ships listed in Table 6 following the analysis procedure described in Section 3.1. The dominant wave condition discussed in Section 4.3 was again specified for the respective ships. It was confirmed that the dominant wave condition is common among the investigated ships.

Next, short- and long-term predictions were made, and the long-term predicted values were compared with the response predicted by using the proposed design loads in Eqs. (5) and (6) for verification:

\[ M_{WT1} = M_{WT} C_{T1}, \]  
\[ M_{WT2} = M_{WT} C_{T2}. \]

Here, no corrections due to nonlinearity regarding torsional moments discussed in the next section are made on the design torsional moments since the long-term
predicted values were obtained based on the linear results. That is, the distribution shape of the design torsional moments as in Eqs. (2a) and (2b) were employed for the comparative study.

Examples of the results are shown in Fig. 11(a) and (b). The curves denoted as ‘CT1’ and ‘CT2’ show the longitudinal distribution of the warping stress under the design torsional moments expressed as in Eq. (2a) and Eq. (2b), respectively. The curve denoted as 'proposal' shows the longitudinal distribution of the warping stress amplitude obtained by taking the root mean square of the two warping stress values, which is intended to be equivalent to the distribution of the long-term predicted values of warping stress denoted as ‘long-term prediction’ in the figure. The results of the long-term predicted values of the warping stress are reproduced by using the proposed design loads along the entire ship length.

In the same manner, comparisons were also carried out with respect to the relative deformations of hatch openings. The results are shown in Fig. 12(a) and (b). The estimations from the design loads are in good agreement with the long-term predicted values.

4.6. Consideration of nonlinearity of torsional moments in large waves

So far, our discussions have been limited to the investigations of design loads based on linear wave loads which are obtained by use of the linear Rankine source theory. In linear theory, it is assumed that the wave amplitude and ship motion amplitude are small. However, the effect of nonlinearity in wave loads must be considered when proposing the design loads since the sea states in consideration is severe, and the wave height and ship motion amplitude are expected to be large in such sea states. Some corrections are made on the tentative design loads discussed above.

In Section 2.5, discussions were made on the nonlinearity of torsional moments in irregular waves. According to Table 4, nonlinearity has small effect on the
magnitude of torsional moments at SS 2.5, where the magnitude is referred to for determining the magnitude of the design loads for torsion. Therefore, no corrections are made on the magnitude of design loads expressed in Eq. (3).

However, the nonlinearity of wave loads has considerable effect on wave torsional moments at SS 7.5. This means that the distribution shape is to be modified. Interpolating linearly among the tank test results, the distribution of coefficient of nonlinearity regarding torsional moments is obtained, and is shown in Fig. 13.

Multiplying the tentatively proposed distributions given in Eqs. (2a) and (2b) by coefficients of nonlinearity in Fig. 11, then the final distributions of the design torsional moments is obtained. The final distributions of the design torsional moments are shown in Fig. 14. In this figure, the distributions given in Eqs. (5a) and

Fig. 11. (a,b) Comparison of longitudinal distribution of warping stresses under design torsional moments with that of the long-term predicted values.
and the envelope curves are shown for comparison:

\[
C_{T1} = 1.0 \left[ \sin \left( 2 \pi \frac{x}{L} \right) + 0.1 \sin^2 \left( \pi \frac{x}{L} \right) \right] \\
\times \exp \left( -0.35 \frac{x}{L} \right) \exp \left( -8 \left( \frac{x/L - 0.5}{0.5} \right)^{10} \right),
\]  

(7a)

\[
C_{T2} = 0.5 \left[ -\sin \left( 3 \pi \frac{x}{L} \right) + 0.65 \sin^3 \left( \pi \frac{x}{L} \right) \right] \\
\times \exp \left( -0.4 \frac{x}{L} \right) \exp \left( -8 \left( \frac{x/L - 0.5}{0.5} \right)^{10} \right).
\]  

(7b)
5. Estimation of total hull girder stress

5.1. General

When assessing the torsional strength of the hull girder of a container ship, vertical bending stress, horizontal bending stress and stress in still-water are combined with warping stress to estimate the total hull girder stress. The total hull girder stress is then checked to be below the allowable stress as acceptance criterion. An appropriate method to combine the stress components to estimate the total hull girder stress is discussed in this section.

Prior to the discussions, response functions of vertical bending stress and horizontal bending stress were calculated on the container ships listed in Table 6.
Response functions of the combined stress, or total hull girder stress were also calculated by summing up the three stress components of vertical bending stress, horizontal bending stress and warping stress, considering the phase difference among these stress components. Short- and long-term predictions of these stresses were also made.

There are two methods to combine the stress components. One method is to sum up the stress components with weighting factors determined based on the design wave [11]. For example, the combined value of two components $A$ and $B$ is in general expressed by using weighting factors $\alpha$ and $\beta$ (in most cases, either $\alpha$ or $\beta$ is 1.0), as $\alpha A + \beta B$. The other method is to combine the stress components in terms of a quadratic expression with correlation coefficients [31]. If the two components that have the correlation coefficients $\rho$ between them are added up, the combined value is expressed as $\sqrt{A^2 + 2\rho AB + B^2}$.

The latter method was adopted here. Because each of the stress components has significant contributions and the dominant wave conditions with respect to the total hull girder stresses could not be specified into one or a few conditions although the former method is effective when one of the stress components in consideration is dominant compared to the other stress components, and a dominant wave condition regarding the total hull girder stresses can be specified. Then, the correlation coefficients among vertical bending stress, horizontal bending stress and warping stress must be determined.

5.2. Correlation coefficients among stress components

Horizontal shear loads become maximum in oblique waves while vertical shear loads become maximum in head seas. Therefore, the maximum response of a ship to horizontal shear loads and the maximum response to torsion occur in oblique waves; so does the maximum response to vertical shear loads in head seas. The lengths of the waves under which the respective responses become maximum are also different. In this sense, the dominant wave conditions are completely different for the responses to the respective loads. In actual seas, which are supposed to be reproduced by a summation of elementary waves of various directions and wave lengths with random phases, it can be assumed that the maximum combined response occurs when the ship is encountered by waves represented by a summation of the two uncorrelated dominant waves under which the respective response become maximum. This assumption might yield rather rough, even less conservative estimation on the combined response in some cases since the waves cannot completely be represented only by the two dominant waves. However, it is validated by several examples conducted in the study as shown later.

Therefore, it is estimated that the correlations between stress induced by horizontal shear loads and stress induced by vertical shear loads are zero. Or the correlation coefficients between warping stress and vertical bending stress and those between horizontal bending stress and vertical bending stress are zero. The combined stress is then estimated by taking the root mean square of the respective maximum values of the two stresses.
On the other hand, strong correlation is expected to exist between the warping stress and the horizontal bending stress, both of which are induced by horizontal shear loads. Fig. 15 shows the RAOs of horizontal bending stress at midship. It is observed that the maximum response occurs under the dominant wave condition (wave length $\lambda = 0.35L$, wave encountering angle $\chi = 120^\circ$) which was specified for warping stress in Section 4.3. Then, the correlation coefficients between the warping stresses and the horizontal bending stresses are assumed to be determined by those under the dominant wave condition of torsional response. The correlation coefficient $C_{\text{COR}}$ is represented by the cosine of the phase difference between the warping stress and the horizontal bending stress. The phases are obtained by referring to the response functions under the dominant wave condition:

$$C_{\text{COR}} = \cos(\theta_{WH} - \theta_{WT}),$$

where $\theta_{WT}$ is the phase of warping stress under the dominant wave condition and $\theta_{WH}$ is the phase of horizontal bending stress under the dominant wave condition.

The longitudinal distributions of the correlation coefficients $C_{\text{COR}}$ between the warping stress and the horizontal bending stress at upper deck under the dominant wave condition were calculated. The results are shown in Fig. 16. The characteristic of the longitudinal distribution is such that the coefficients are almost $-1.0$ at position from $0.15L$ to $0.75L$ while the coefficients come closer to $1.0$ at bow part and stern part, which forms a ‘bath-tub’ shape. There is a hump in this figure at position around $0.2L$–$0.3L$. This irregularity is due to the effects of constraint of the rigid engine room structure.

The ‘bath-tub’ characteristics of correlation coefficients mean that the warping stress and the horizontal bending stress counteract each other at upper deck side except at the bow part and stern part. On the other hand, the warping stress and the horizontal bending stress overlap each other at the bottom side since the phase difference between the warping stress at the upper deck side and that at the bottom side is $180^\circ$ while the phase difference between the horizontal bending stresses is $0^\circ$. 

![Fig. 15. Response amplitude operator of horizontal bending stress at mid-ship.](image-url)
Attention must be paid to the buckling of the bilge part structure due to the hull girder response including torsional response. The proposed values of correlation coefficients $\text{COR}$ are given in Table 7. The proposed distribution of the correlation coefficients is shown in Fig. 17.

Finally, the following combination rules of the stress components to estimate the total hull girder stress are proposed.

\[
\sigma_T = \sqrt{(\sigma_{\text{WV}})^2 + (\sigma_{\text{WH}}^2 + 2\text{COR} \sigma_{\text{WH}} \sigma_{\text{WT}} + \sigma_{\text{WT}}^2)} \quad \text{for upper deck side,} \quad (9a)
\]

\[
\sigma_T = \sqrt{(\sigma_{\text{WV}})^2 + (\sigma_{\text{WH}}^2 - 2\text{COR} \sigma_{\text{WH}} \sigma_{\text{WT}} + \sigma_{\text{WT}}^2)} \quad \text{for bottom side,} \quad (9b)
\]

where $\sigma_{\text{WV}}$, $\sigma_{\text{WH}}$ and $\sigma_{\text{WT}}$ are the long-term predicted values of the vertical bending stress, the horizontal bending stress and the warping stress in waves, respectively.

5.3. Verification of the proposed combination of stress components

Examples of the longitudinal distribution of the long-term predicted values of each stress component at upper deck are shown in Fig. 18(a). The longitudinal distribution of the long-term predicted values of the total hull girder stress is also plotted in this figure. The curves show such characteristics that the vertical bending stress is dominant among all the stress components at any position all over the ship length, especially around the midship where the total hull girder stress is almost equal to the vertical bending stress although each component has considerable contributions. The warping stress and the horizontal bending stress overlapped with the vertical bending stress boost up the stress level at the bow and stern parts, and the part in front of the engine room.

For validating the proposed combination rules of stress components, the combined stresses are obtained accordingly from Eq. (9a) or (9b) and the long-term predicted values of total hull girder stress are compared with the combined
stresses. In Fig. 18(a), the combined stress obtained from Eq. (9a) is shown as ‘proposal’. The results of long-term prediction of the total hull girder stress marked as ‘total’ and the estimated total hull girder stress marked as ‘proposal’ are in good agreement.

The comparisons at hatch coaming top level and bilge part are shown in Fig. 18(b) and (c), respectively. At the bilge part, the total hull girder stress around the midship which is calculated according to Eq. (9b) is significantly larger than the vertical bending stress as the stress is increased by the horizontal bending stress and the warping stress which overlap each other.

It is concluded that long-term predicted values of total hull girder stress can be reproduced by combining the long-term predicted values of respective stress components according to the combination rules proposed as in Eqs. (9a) and (9b) with good accuracy at both upper deck and bottom sides along the ship length. This validates the combination rules.

6. A practical method for torsional strength assessment

There are five components to be considered for evaluating total hull girder stress: vertical bending stress in waves, vertical bending stress in still water, horizontal bending stress, warping stress in waves and warping stress in still water.
Fig. 18. (a, b, c) Longitudinal distribution of long-term predicted value of each stress component: (a) upper deck, (b) hatch coaming top and (c) bilge part.
Assuming that the long-term predicted values of vertical bending moment are given by distributions of vertical bending moments expressed in a formula proposed by IACS, the resulting stresses under the distributions of vertical bending moment correspond to the long-term predicted values of vertical bending stress. As for the horizontal bending moment, design loads which are equivalent to the long-term predicted values of horizontal moments are already proposed [11]. Accordingly, the long-term predicted values of horizontal bending stress are readily calculated. The long-term predicted values of warping stress are calculated by using FEM under the design torsional moments in Section 4. The warping stress $\sigma_{WT}$ is calculated by the following equation:

$$\sigma_{WT} = \sqrt{\sigma_{WT1}^2 + \sigma_{WT2}^2},$$  \hspace{1cm} (10)

where $\sigma_{WT1}$ and $\sigma_{WT2}$ are warping stress obtained from global analysis under the torsional moments $M_{WT1}$ and $M_{WT2}$, respectively.

By combining all the stress components according to Eq. (11a) or (11b), the longitudinal distribution of the long-term predicted values of total hull girder stress is calculated. In Eqs. (11a) and (11b), static components ($\sigma_{SV}$: vertical bending stress in still water, $\sigma_{ST}$: warping stress in still water) are superimposed to the dynamic components in Eqs. (9a) and (9b). The acceptance criteria is that values of total hull girder stress are lower than the allowable stresses in terms of yielding stress and buckling strength.

$$\sigma_T = \sqrt{(\sigma_{WY})^2 + (\sigma_{WH}^2 + 2C_{COR}\sigma_{WH}\sigma_{WT} + \sigma_{WT}^2) + \sigma_{SV} + \sigma_{ST}}$$

for upper deck side, \hspace{1cm} (11a)

$$\sigma_T = \sqrt{(\sigma_{WY})^2 + (\sigma_{WH}^2 - 2C_{COR}\sigma_{WH}\sigma_{WT} + \sigma_{WT}^2) + \sigma_{SV} + \sigma_{ST}}$$

for bottom side. \hspace{1cm} (11b)

A block diagram describing the procedure of the practical method for torsional strength assessment is shown in Fig. 19.

7. Conclusions

The following conclusions are drawn from the wave load analysis and tank tests on a post-Panamax size container ship:

(1) Torsional moments estimated by using the three-dimensional Rankine source method agree well with those obtained from the tank tests.

(2) According to the tank test results in waves of different wave heights, the nonlinearity of the torsional moments exists along the fore part of ships.

A practical method for torsional strength assessment of container ship structures was discussed based on the results of torsional response of the ship obtained from
The conclusions are summarized as follows:

(3) The dominant wave condition for torsional response of container ships was specified. The wave length is $0.35L$ and the wave encountering angle $\chi$ is $120^\circ$.

(4) Design loads for torsional moments were tentatively proposed based on the torsional moments obtained for container ships of different sizes under the specified dominant wave condition.

(5) The proposed design loads were applied to a range of container ships and the response values under the loads were compared with the long-term predicted values of torsional response. It was shown that the values of torsional response under the tentatively proposed design loads are equivalent to the long-term predicted values.

(6) The design loads were finally proposed by correcting the shape of distribution of the tentatively proposed design loads to consider the effect of nonlinearity in extreme waves based on the tank test results.

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**Design Loads:**

Refer to Eqs. (5) and (6) for wave-induced torsional moment

<table>
<thead>
<tr>
<th>Still water Vertical Bending Moment</th>
<th>Wave-induced Vertical Bending Moment</th>
<th>Wave-induced Horizontal Bending Moment</th>
<th>Still water Torsional Moment</th>
<th>Wave-induced Torsional Moment</th>
</tr>
</thead>
</table>

**Structural Analysis:**

Structural Analysis is conducted using an entire-ship model

Vertical and horizontal bending stresses may be calculated by I/Y calculation

<table>
<thead>
<tr>
<th>Still water Vertical Bending Stress</th>
<th>Wave-induced Vertical Bending Stress</th>
<th>Wave-induced Horizontal Bending Stress</th>
<th>Still Water Warping Stress</th>
<th>Wave-induced Warping Stress</th>
</tr>
</thead>
</table>

**Stress Combination:**

The total hull-girder stress is obtained by combining the stress components; Eq. (11)

**Strength Assessment:**

Hull-girder torsional strength assessment is conducted from the view points of yielding and buckling

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Fig. 19. Procedure of a practical method for torsional strength assessment.
(7) An appropriate combination of stress components for the estimation of the total hull girder stress was proposed in terms of a quadratic expression using coefficients to correlate the stress components.

(8) The combination of stress components were applied to evaluate the total hull girder stress of six container ships. It was shown that the long-term predicted values of hull girder stress are reproduced by summing up the long-term predicted values of vertical bending stress, the warping stress and the horizontal bending stress according to the proposed combination.

Although only the torsional strength assessment of hull girder of container ships is treated in this paper, good accuracy is also expected if the design loads and the combination rule are applied to the fatigue strength assessment of hatch corner parts subject to stress concentration due to torsional deformations. The research results summarized in this paper are reflected in the ClassNK guidelines for container ship structures which were published in November 2003 [32].

It must be also noted that the proposed practical method for torsional strength assessment has some limitations of applicability according to the nature of the development procedure. As it has been developed based on the results for ships ranging from 1500TEU to 8000TEU whose structural arrangements are conventional in such a point that the engine room is located at around SS 2.5, appropriate strength assessments are expected for ships within the above range. However, the applicability of the proposed practical method must be investigated in case of applying to a ship whose dimension or capacity is far larger or to a ship of new and innovative design. Even in that case, the direct load analysis and direct structural analysis that have been utilized in this paper to develop the practical method can be applied for the assessment of the torsional strength of the ship.

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