ELEC-C8201: Control Theory and Automation

Exercise 7

The problems marked with an asterisk (⋆) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Consider the feedback control system shown in Figure 1.

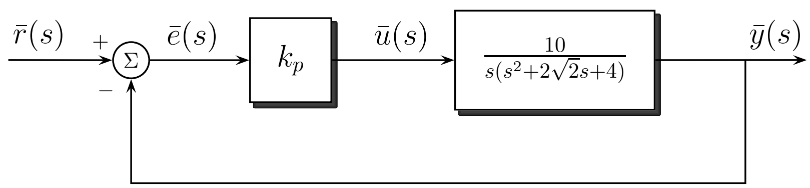

Figure 1: Feedback control system.

a) A part of the Nyquist diagram for the system is shown in Figure 2 (in the next page) for $k_p = 1$. Determine the gain and phase margins for the system. How would you expect the system to respond to changes in the desired signal $\bar{r}(s)$?

b) When $k_p = 0.5$, determine the range of frequencies for which

$$\left| \frac{1}{1 + k_p G(j\omega)} \right| \geq 1 ,$$

where $G(s) = \frac{10}{s(s^2+2\sqrt{2}s+4)}$. Explain the implications of this for system behavior.

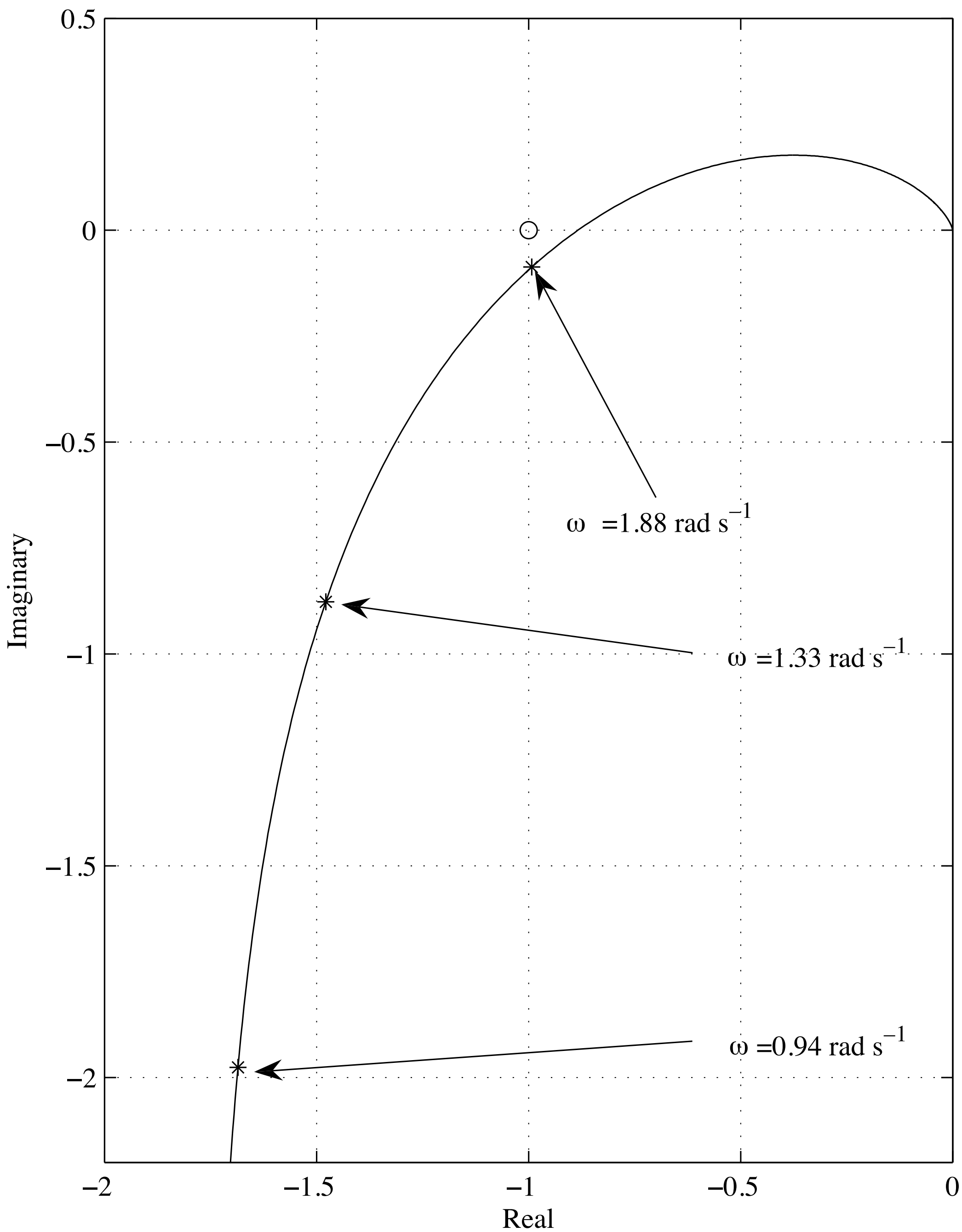

Figure 2: Incomplete Nyquist diagram.

Solution.

a) Gain margin is measured as $GM \approx 1/0.88 = 1.14$, see annotated figure. Phase margin is $\theta \approx 5^o$, again see annotated figure. These margins are very small, so we expect an oscillatory response to the changes in desired signal $\bar{r}(s)$.

b) When $k_p = 0.5$ the Nyquist diagram has the same phase but half the magnitude. In order to find the frequency where

$$\left| \frac{1}{1 + k_p G(j\omega)} \right| = 1$$

we note that $1 + k_p G(j\omega)$ is the complex vector joining $k_p G(j\omega)$ to the point $(-1, 0)$. On our graph, which corresponded to $k_p = 1$, this will therefore correspond to the vector joining $G(j\omega)$ and $(-2, 0)$. We look for the frequency where this vector has magnitude equal to 2, i.e., the frequency $0.94 rad/s$; see annotated figure.

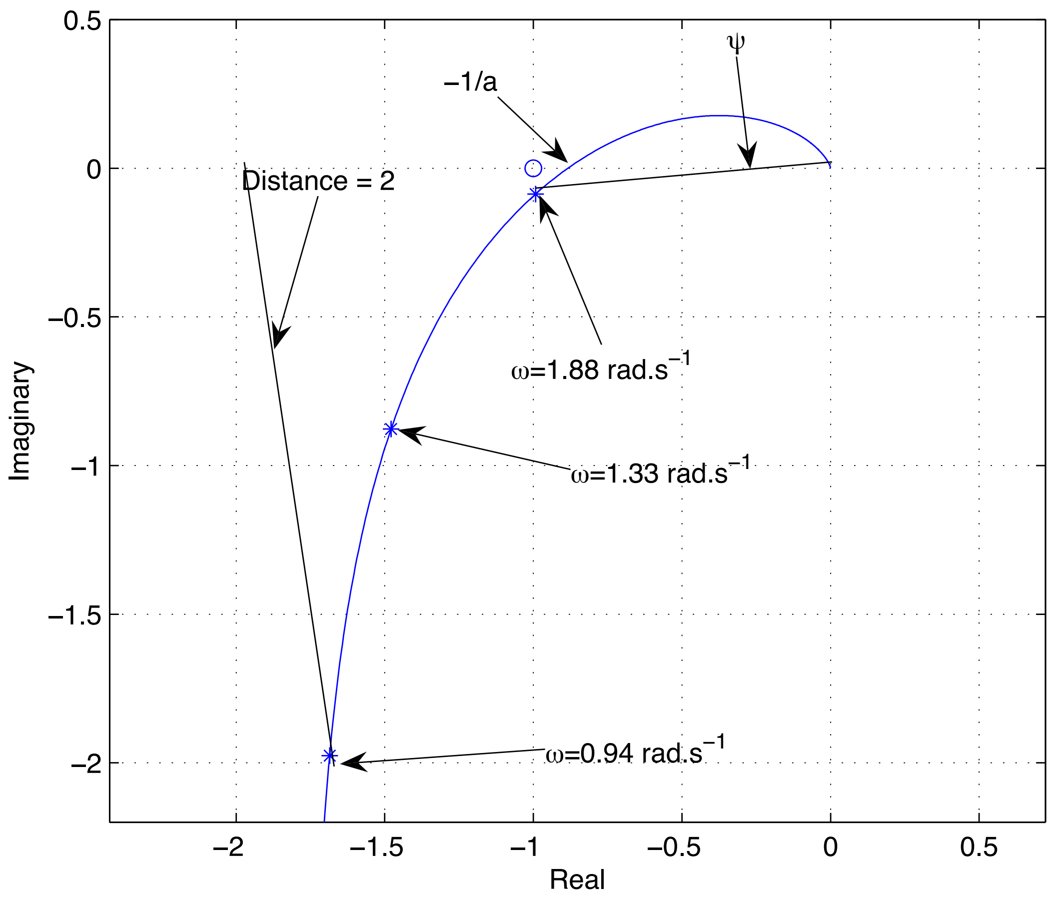

Figure 3: Annotated Nyquist diagram.

2. Consider the feedback control system shown in Figure 4.

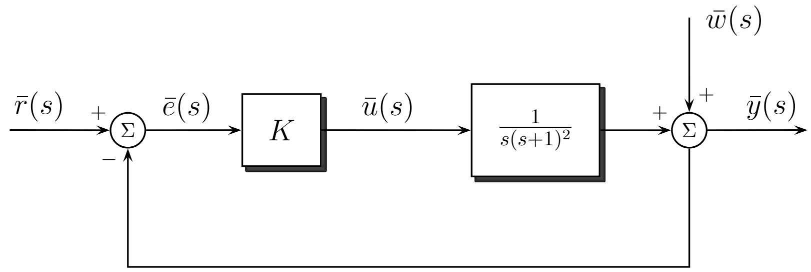

Figure 4: Feedback control system.

a) How would you determine experimentally the data necessary to plot its Nyquist diagram?

b) Determine the behavior of its Nyquist diagram as $\omega \to 0$ and $\omega \to \infty$.

c) Find the frequency where the imaginary part of $G(j\omega)$ becomes zero and hence complete the Nyquist diagram of $KG(s)$ on Figure 5 (in the next page) for $K = 1$.

d) Calculate the gain margin and estimate the phase margin of the feedback system. For what range of K is the feedback system stable?

e) For what range of frequencies (if any) will the system attenuate the effect of any disturbances $\bar{w}(s)$, if $K = 1$?

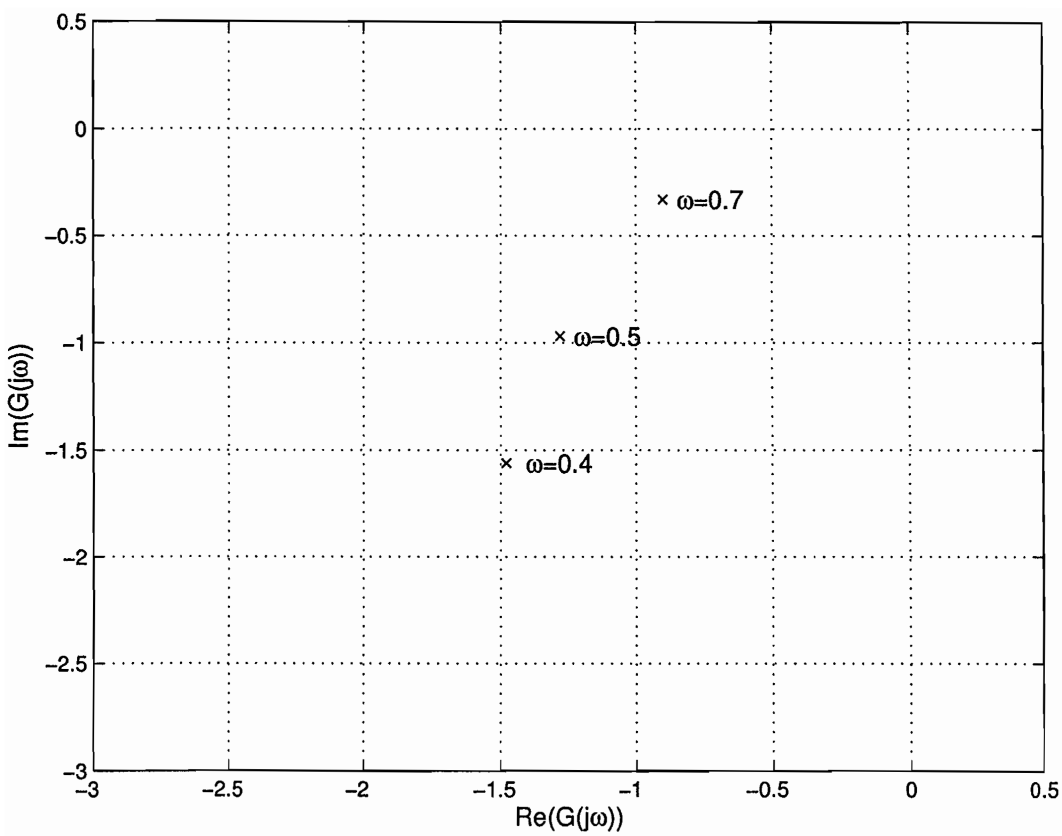

Figure 5: Incomplete Nyquist diagram.

Solution.

a) If we input harmonic functions of different frequencies into the open-loop system, then the response will oscillate on the same frequency. Once the transients are gone, the measured ratio of the output to input amplitudes will give us the magnitude of the transfer function, while the measured phase difference between input and output is the argument of the transfer function for the corresponding frequency

b) <u>When $\omega \to 0$:</u>

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2} \approx \frac{1}{j\omega}.$$

Hence, the magnitude approaches ∞ at an angle $-\pi/2$. If ω is small. we can use a Taylor series expansion for $G(j\omega)$. Towards that, we find the Taylor series expansion of $1/(j\omega+1)$ as follows

$$\frac{1}{1+j\omega} = \frac{1}{1-(-j\omega)} = 1 + (-j\omega) + (-j\omega)^2 + (-j\omega)^3 + \ldots \approx 1 - j\omega$$

Hence, $G(j\omega)$ becomes

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2} = \frac{1}{j\omega(j\omega+1)(j\omega+1)} = \frac{(1-j\omega)(1-j\omega)}{j\omega} \approx \frac{1}{j\omega} - 2$$

Therefore, the real part of the transfer function approaches -2.

<u>When $\omega \to \infty$:</u>

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2} \approx \frac{1}{j\omega(j\omega)^2}.$$

Hence, the magnitude approaches 0 at an angle of $\frac{\pi}{2}$.

c) For the imaginary part of $G(j\omega)$ to be 0, the imaginary part of the denominator has to be 0 (since the numerator has to be a real number):

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2} = \frac{1}{j\omega(2j\omega+1-\omega^2)}$$
$$= \frac{1}{-2\omega^2 + j\omega(1-\omega^2)}$$

This happens when $\omega = 1$.

d) When $\omega = 1$, $|G(j\omega)| = 1/2$ (simple substitution). As a result, the gain margin is 2. For the phase margin, a circle of radius 1 meets the diagram close to the point $\omega = 0.7$ and the phase margin is approximately 20^o. For the system to remain stable

$$0.5K < 1 \Rightarrow K < 2$$

e) The condition for effective feedback is

$$\left|\frac{1}{KG(j\omega)}\right| < 1.$$

A circle of radius 1 centered at $(-1,0)$ meets the Nyquist plot at $\omega = 0.5$. Therefore, it can be inferred that the feedback will reduce errors for frequencies lower than $0.5 rad/s$.

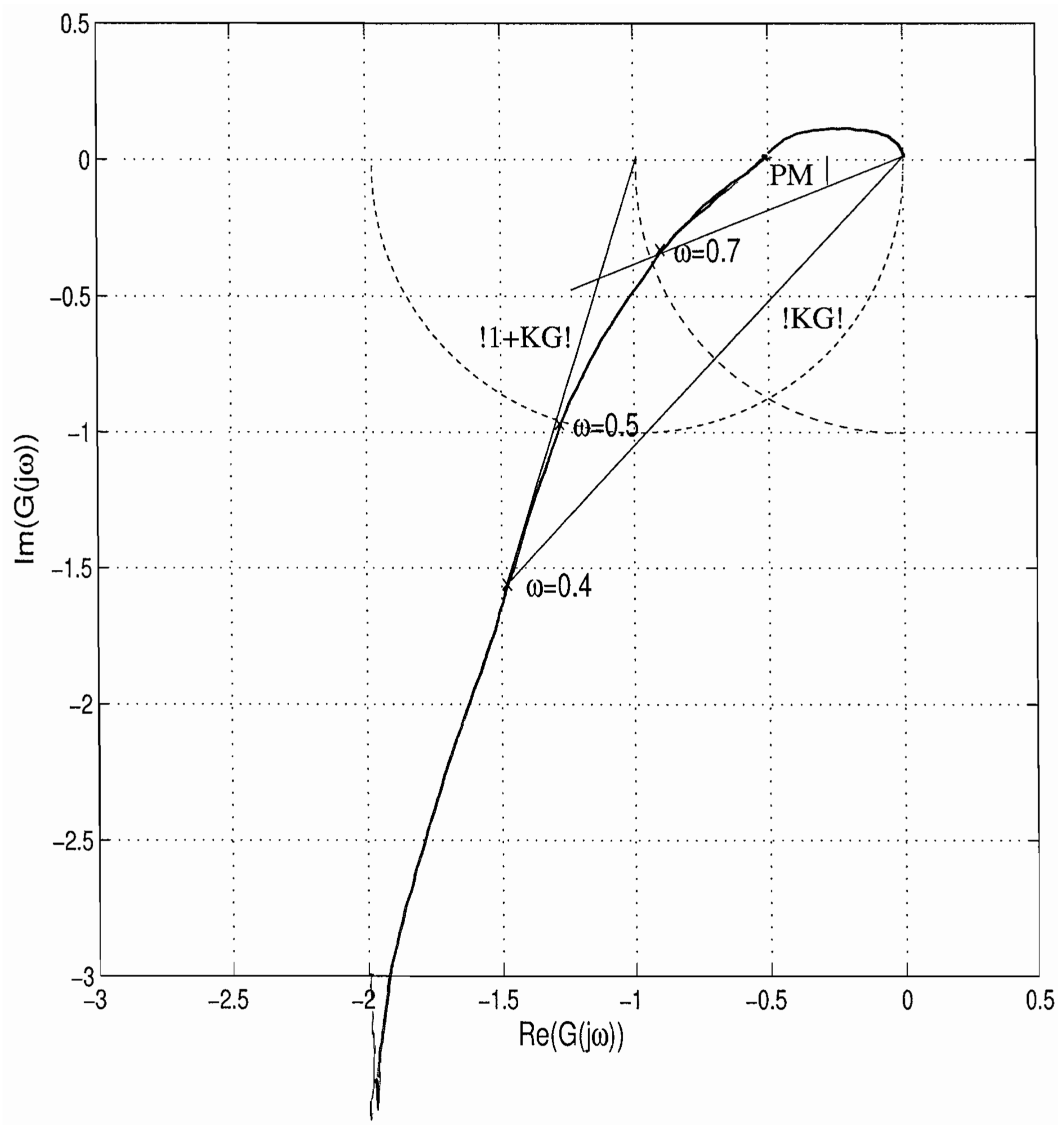

Figure 6: Annotated Nyquist diagram.