

ELEC-C8201: Control Theory and Automation

Exercise 8

The problems marked with an asterisk (\star) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Consider the *single-input, single-output* system described by

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t), \\ y(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}$$

Determine whether the system is controllable and observable when:

a) $A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [0 \ 2]$.

b) $A = \begin{bmatrix} -10 & 0 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = [1 \ 0]$.

c) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $C = [1 \ 0]$.

2. Consider the second-order system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} u(t), \\ y(t) &= [1 \ 0] \mathbf{x}(t)\end{aligned}$$

For what values of k_1 and k_2 is the system completely controllable?

3. Consider the third-order system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -5 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u(t), \\ y(t) &= [2 \ -4 \ 0] \mathbf{x}(t)\end{aligned}$$

- a) Verify that the system is observable.
- b) Determine the observer gain matrix required to place the observer poles at $s_{1,2} = -1 \pm j$ and $s_3 = -5$.

★ 4. Suppose that the vector differential equation describing the inverted pendulum is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 9.8 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} u(t)$$

Assume that all state variables are available for measurement and use state variable feedback. Place the system characteristic roots at $s = -2 \pm j$, -5 , and -5 .

5. A system is represented by the differential equation

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = \frac{du}{dt} + u$$

where y = output and u = input.

- a) Develop a state variable representation and show that it is a controllable system.
- b) Define the state variables as $x_1 = y$ and $x_2 = dy/dt - u$, and determine whether the system is controllable.

6. Consider the second-order system

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} 1 & 2 \\ -6 & 12 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -5 \\ 1 \end{bmatrix} u(t), \\ y(t) &= [4 \quad -3] \mathbf{x}(t) \end{aligned}$$

- a) Verify that the system is observable and controllable.
- b) Design a full-state feedback law and an observer by placing the closed-loop system poles at $s_{1,2} = -1 \pm j$ and the observer poles at $s_{1,2} = 12$.