Lecture 4: Quantum circuits 09.03.2022

Introduction to simulation and Multiphysics problem with COMSOL

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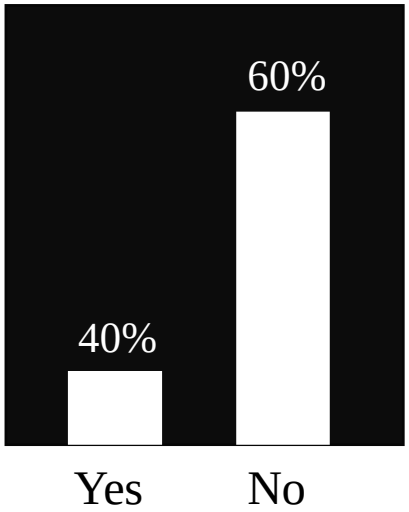
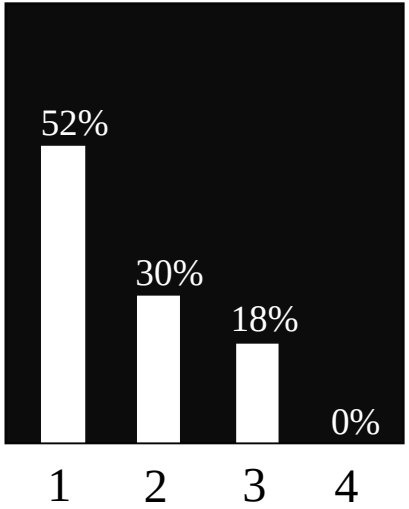
School of Science

Your response

Familiarity with simulation



Familiarity with COMSOL



1. Have never heard of COMSOL
2. Have heard of COMSOL
3. Have some experience with COMSOL
4. Have used COMSOL extensively

Need for simulation

- Deeper understanding of the physical system
- Engineering qubits
- Investigating EM fields in quantum circuits
- Limitation to theoretical models

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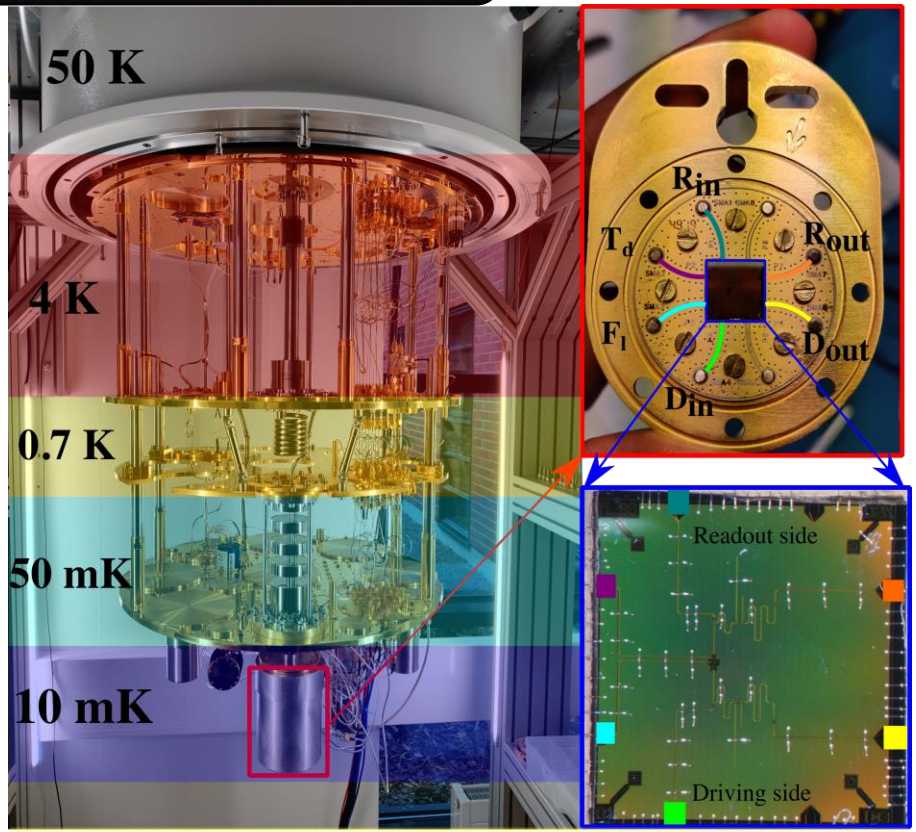
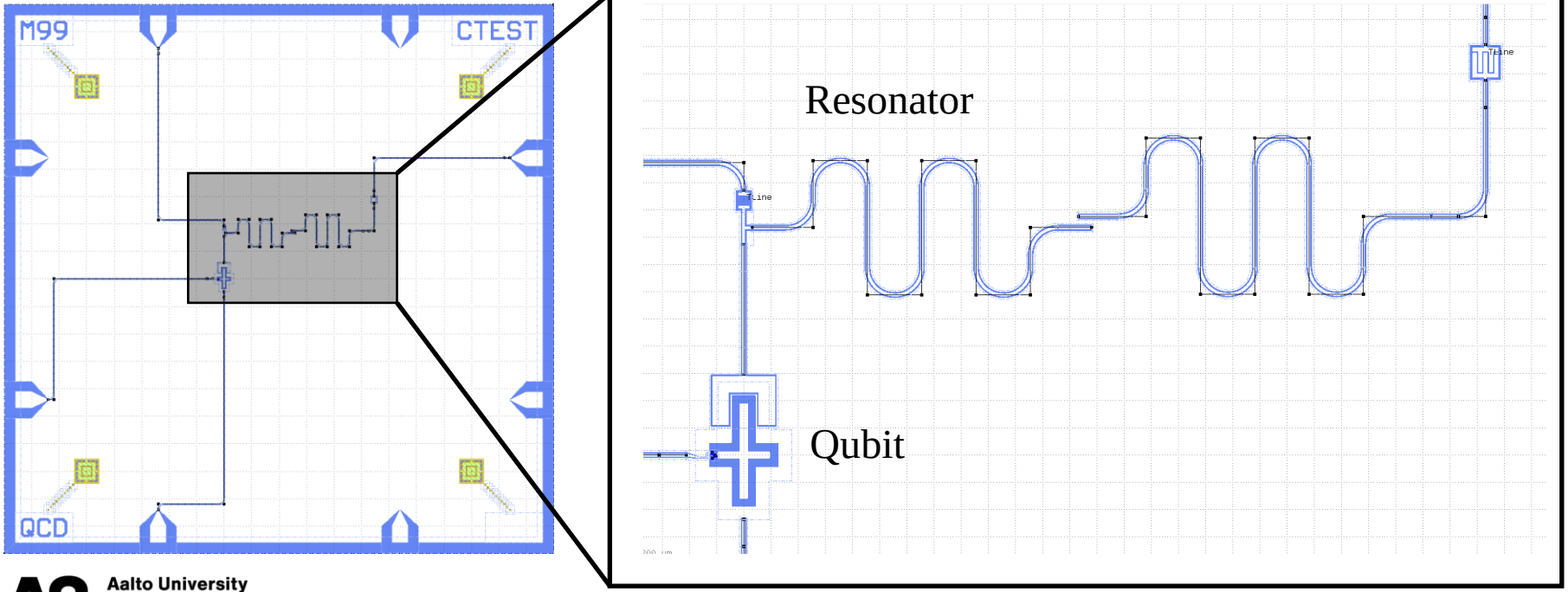
Need for numerical simulation



R_{in} - Readout input line
R_{out} - Readout output line
T_d - Classical transmon drive line
D_{in} - Drive input line
D_{out} - Drive output line
F_l - Flux line

Need for numerical simulation

Single-qubit quantum processor



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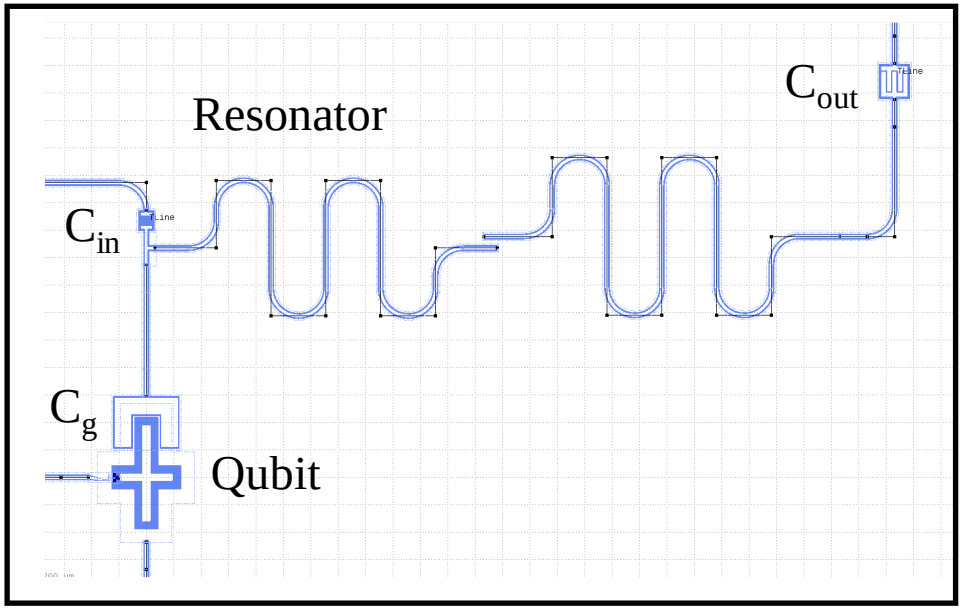
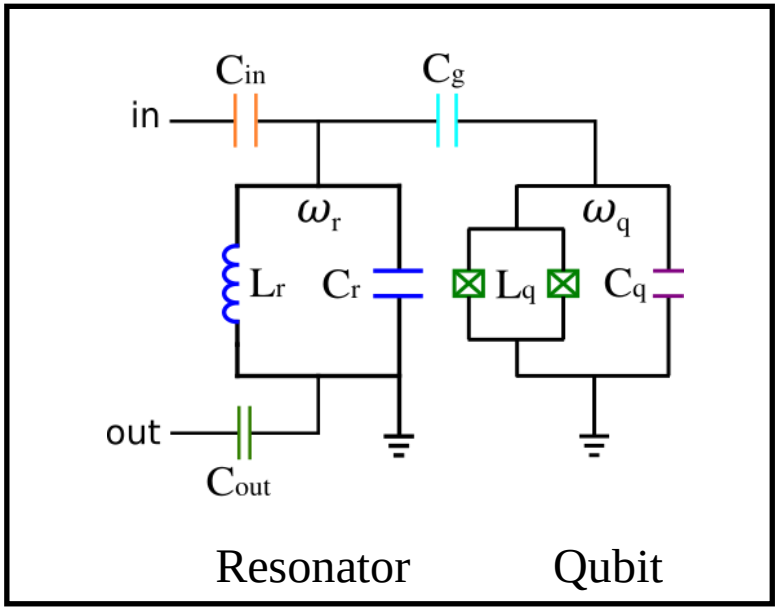
Need for numerical simulation

Single-qubit quantum processor



Lumped model



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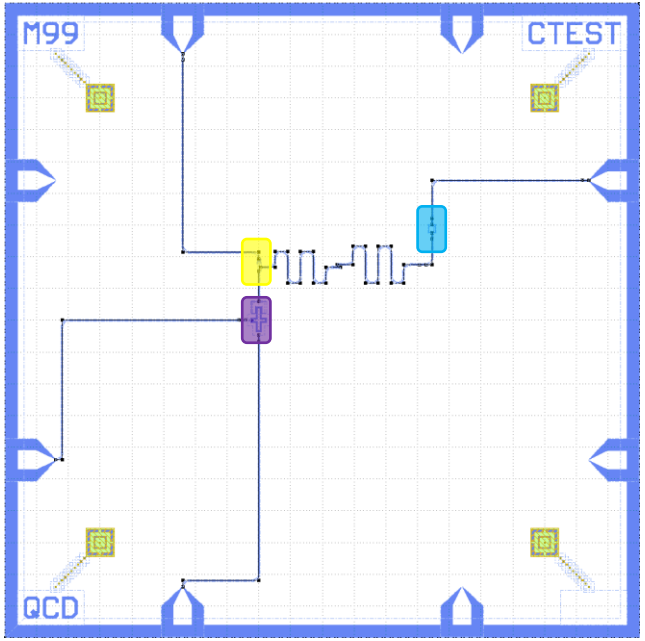
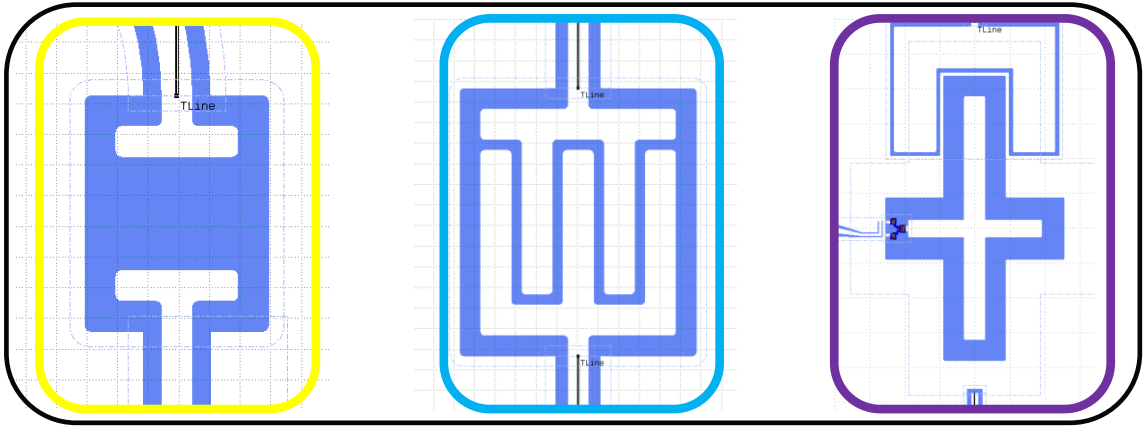
Bare minimum for a single-qubit quantum processor!

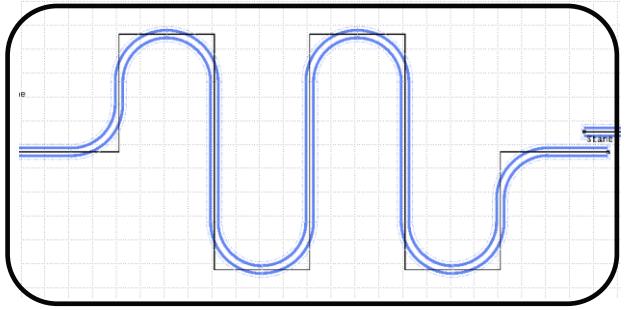
Need for numerical simulation

Single-qubit quantum processor



Capacitors





Coplanar waveguide (CPW) resonator

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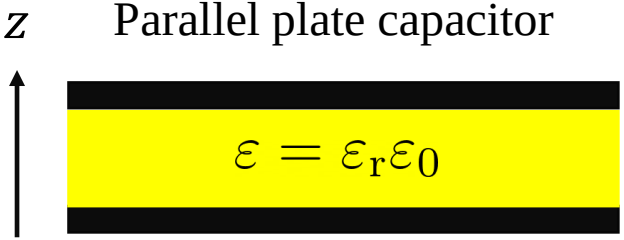
Capacitors

Partial differential equation (PDE)

- Laplace equation $\nabla^2 V = \frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0$

- Boundary value problem

$$\begin{cases} V(z=d) = V_0 \\ V(z=0) = 0 \end{cases}$$



$$\frac{d^2V}{dz^2} = 0 \Rightarrow V(z) = \frac{V_0}{d} z$$

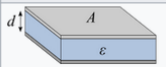
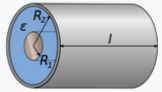
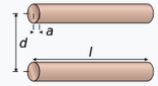
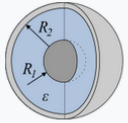
$$\vec{E} = -\nabla V = -\frac{V_0}{d}\hat{z}$$

$$\oiint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon} \iint \rho \, dA$$

$$\Rightarrow \quad C = \varepsilon \frac{A}{d}$$

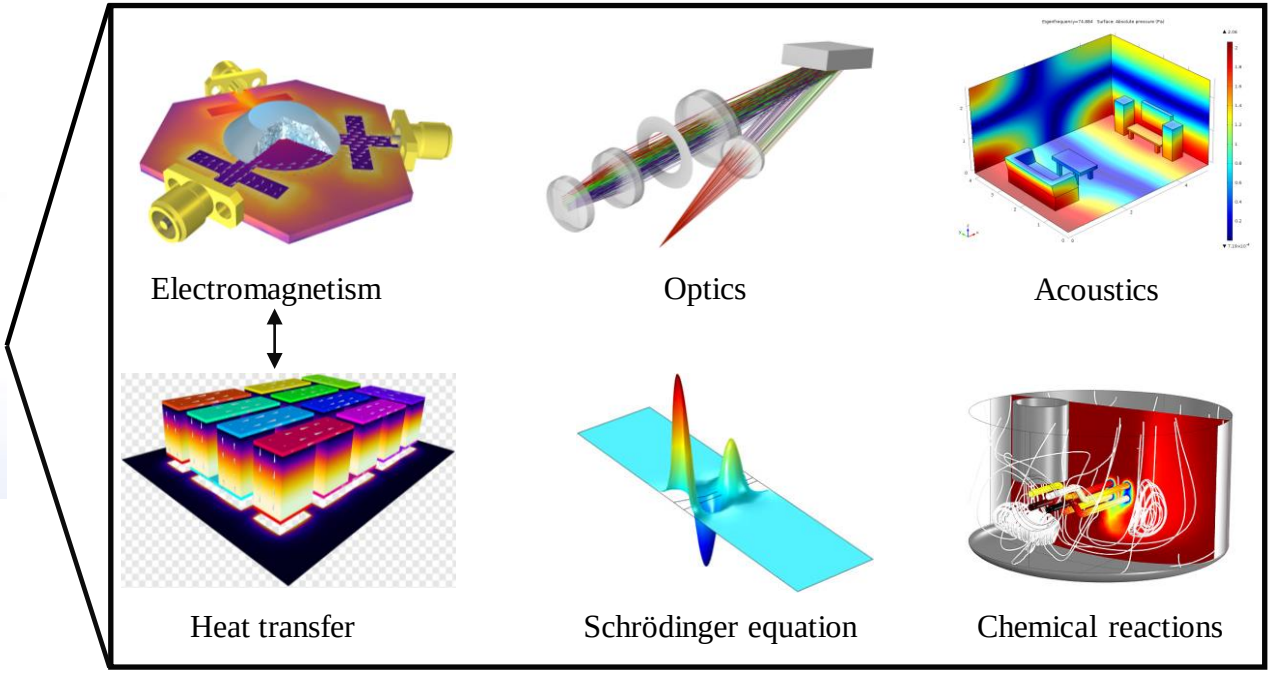
Nice closed form solution!

Capacitors

Type	Capacitance	Comment
Parallel-plate capacitor	$\varepsilon A/d$	 ε: Permittivity
Concentric cylinders	$\frac{2\pi\varepsilon\ell}{\ln(R_2/R_1)}$	 ε: Permittivity
Pair of parallel wires[13]	$\frac{\pi\varepsilon\ell}{\operatorname{arcosh}\left(\frac{d}{2a}\right)} = \frac{\pi\varepsilon\ell}{\ln\left(\frac{d}{2a}+\sqrt{\frac{d^2}{4a^2}-1}\right)}$	
Wire parallel to wall[13]	$\frac{2\pi\varepsilon\ell}{\operatorname{arcosh}\left(\frac{d}{a}\right)} = \frac{2\pi\varepsilon\ell}{\ln\left(\frac{d}{a}+\sqrt{\frac{d^2}{a^2}-1}\right)}$	a: Wire radius d: Distance, $d > a$ ℓ: Wire length
Two parallel coplanar strips[14]	$\varepsilon\ell\frac{K\left(\sqrt{1-k^2}\right)}{K\left(k\right)}$	d: Distance w_1, w_2: Strip width k_m: $d/(2w_m+d)$ k^2: k_1k_2 K: Complete elliptic integral of the first kind ℓ: Length
Concentric spheres	$\frac{4\pi\varepsilon}{\frac{1}{R_1}-\frac{1}{R_2}}$	 ε: Permittivity

@ wikipedia

COMSOL Multiphysics



@ Comsol

COMSOL Multiphysics

Physics modules

- Electromagnetics
- Fluid Dynamics
- Solid Mechanics

⋮

Mathematical models

- Partial Differential Equations (PDEs)
- Ordinary Differential Equations (ODEs)
- Differential-Algebraic Equations (DAEs)

⋮

Numerical models

- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Finite Difference Method (FDM)

⋮

COMSOL Workshop

You can download the software from https://download.aalto.fi

Check out the installation instruction from
mycourses webpage *Announcement* forum