Microeconomics 4

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Organisation

Instructor: Jan Knoepfle, jan.knoepfle@aalto.fi

Lectures:

- Mondays and Tuesdays 10-12h, Economicum seminar room 3-4
- streamed on Zoom for non-Helsinki students

Office hours:

- By appointment (email) either in person or over Zoom
- Feel free to reach out actively whenever we can help!

MyCourses Forum: To help your peers, post questions directly on forum whenever possible

Course material

Slides uploaded in advance (incomplete) and after the lectures (completed)

Textbooks:

- **Mailath: Modeling Strategic Behavior.**
- Borgers: An Introduction to the Theory of Mechanism Design.
- Mas-Colell, Whinston, Green: Microeconomic Theory.
- Krishna: Auction Theory.
- Salanié: The Economics of Contracts.

Problem Sets

TA: Eero Mäenpää, eero.maenpaa@aalto.fi

Exercises:

- 4 problem sets, posted on MyCourses one week before due date
- Exercise sessions with Eero Mondays 14-16h in Economicum seminar room 3-4

  Dates: 21.03, 28.03, 14.04, 02.05
- Hand in your solutions to problem set on MyCourses before exercise session
- Model solutions uploaded after exercise sessions

Evaluation

Requirements:

- At least 50% of solutions to problem sets
- Pass final exam

Grades based on exam only

Information Economics

Information Economics

- Micro 3: framework to analyse interaction in given game and predict outcome
- Micro 4: we want to design the optimal 'game' to achieve the 'best' outcome
- Asymmetric Information poses main problem
- Examples for such 'designed games'
    - Sales procedures
    - Voting mechanisms
    - Employment contracts

Information Economics

Two main classes of design problems:

1) Adverse Selection (hidden information)

   Uninformed party cannot see characteristic of informed party. Uninformed moves first.

   Concepts: Screening and Mechanism Design

2) Moral Hazard (hidden action)

   Uninformed party does not see action of informed party. Uninformed moves first.

   Concepts: Contract Theory

Will also discuss a class of 'fixed games' where asymmetric information is crucial:

(3) Sender-Receiver Games

   Uninformed party does not see characteristic of informed party. Informed moves first.

   Signaling games, (evidence) disclosure games, ...

Adverse Selection

Adverse Selection

Screening

Screening

Screening:

- Principal and one agent
- Agent has private information about his preferences
- Principal design (commits to) a mechanism

Simple example:

- One Seller, one Buyer
- Seller owns a phone, her own valuation 0, Buyer has valuation $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$
- Utilities from sale at price p: $u_S(p) = p$ and $u_B(\theta, p) = \theta - p$
- Buyer knows θ (we call θ his private type)
- Seller only knows that θ is drawn from distribution F. Assume that $F'(\theta) = f(\theta) > 0$

What is optimal selling procedure (mechanism) for the seller?

Screening – an example

Let's first consider a special and simple class of mechanisms: posted-price mechanisms

- Seller posts a price p and buyer decides whether to buy at this price or not
- Seller's expected profit from price p: $\Pi(p) = \underbrace{(1 - F(p))}_{\mathbb{P}[\text{sale}]} \underbrace{p}_{\text{price}}$
- Seller-optimal posted-price must maximise $\Pi(p)$

Screening – an example

Can the seller do better?

- Seller could bargain multiple rounds, offer lotteries at different prices, ...
- Problem: space of possible selling procedures is very large

What are the **fixed components** of our problem?

- Space of outcomes: allocation prob. $q \in [0, 1]$ and transfer $t \in \mathbb{R}$ from seller to buyer
- Preferences: seller: $-t$, buyer: $\theta q + t$ with $\theta \sim F$

What are all **possible mechanisms**?

1. Seller commits to game:
   - space of strategies S
   - outcome functions $q \colon S \to [0, 1]$ and $t \colon S \to \mathbb{R}$
2. Buyer (knows θ and) chooses strategy $s(\theta) \in S$

Screening – Revelation Principal

The set of all possible mechanisms of the form $\Gamma = (S, (q, t))$ is quite large!

Screening – Revelation Principal

Theorem (Revelation Principle)

Take any mechanism $\Gamma = \left(S, (q(s), t(s))_{s \in S}\right)$

and optimal agent strategy $s^*_\Gamma \colon \Theta \to S.$

There is a ***direct*** *mechanism* $\hat{\Gamma} = \left(\Theta, \left(\hat{q}(\theta), \hat{t}(\theta)\right)_{\theta \in \Theta}\right)$

such that ***truthtelling*** $s^*_{\hat{\Gamma}} \colon \Theta \to \Theta$ *with* $s^*_{\hat{\Gamma}}(\theta) = \theta$

is an optimal strategy for the agent and the outcome is the same as in mechanism Γ*.*

Screening – direct IC mechanisms

Thanks to revelation principle, without loss to focus on direct truthful mechanisms

Seller's optimisation problem:

$$\max_{\substack{q:\Theta\to[0,1]\\ t:\Theta\to\mathbb{R}}} \int_{\underline{\theta}}^{\bar{\theta}} -t(\theta) f(\theta)\,\mathrm{d}\theta \qquad \text{such that}$$

$$\text{for all } \theta \in \Theta: \quad \theta q(\theta) + t(\theta) \geq 0 \tag{IR_θ}$$

$$\text{for all } \theta, \hat{\theta} \in \Theta: \; \theta q(\theta) + t(\theta) \geq \theta q(\hat{\theta}) + t(\hat{\theta}) \tag{$IC_{\theta,\hat{\theta}}$}$$

...still a lot of constraints

Screening – incentive compatible allocations and transfers

Type θ's utility from report $\hat{\theta}$ is

$$V(\theta, \hat{\theta}) = \theta q(\hat{\theta}) + t(\hat{\theta}), \qquad \text{with } V(\theta) \equiv V(\theta, \hat{\theta})\Big|_{\hat{\theta}=\theta}$$

The IC constraints imply two important conditions:

1. $q(\cdot)$ must be weakly increasing in θ
2. $V'(\theta) = q(\theta)$ and we can integrate so that $V(\theta) - V(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s$

Screening – change order of integration

Inserting $-t(\theta) = \theta q(\theta) - V(\theta) = \theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s - V(\underline{\theta})$ into max. problem gives

$$\max_{q:\Theta\to[0,1],V(\underline{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s - V(\underline{\theta})\right] f(\theta)\,\mathrm{d}\theta \quad \text{s.t.} \quad V(\underline{\theta}) \geq 0, \text{ and } q(\cdot) \text{ increasing}$$

Screening – virtual value

Changing the order of integration gives

$$\max_{q:\Theta\to[0,1],V(\underline{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] q(\theta)f(\theta)\,\mathrm{d}\theta - V(\underline{\theta}) \quad s.t.\ V(\underline{\theta}) \geq 0,\ q(\cdot) \text{ increasing}$$

- We can choose optimal $q(\cdot)$ pointwise (if result satisfies monotonicity constraint)
- $J(\theta) \equiv \theta - \frac{1-F(\theta)}{f(\theta)}$ is called the virtual valuation of type θ
- We say that distribution F is regular if $J(\theta)$ is increasing
- If F regular, optimal allocation is $q^*(\theta) = \begin{cases} 0 & \text{if } \theta < \theta^* \\ 1 & \text{if } \theta \geq \theta^*, \end{cases}$ with $\theta^* = \frac{1-F(\theta^*)}{f(\theta^*)}$
- What if J is not monotone?

Screening example – recap

We have used many tools today that we will develop in more detail further on:

- Revelation Principle
- Characterising IC in terms of allocation rule only
- Virtual valuations

These have allowed us to derive several results:

- We can solve for optimal mechanisms
- Utilities and transfers are pinned down almost entirely by IC allocation rule
- Posted price mechanisms are optimal!

Screening – a few simple extensions

Divisible quantity instead of single indivisible good:

- Nothing changes
- Interpret $q \in [0, \bar{q}]$ as quantity instead of probability $q \in [0, 1]$

Production costs for the seller:

- Suppose seller incurs cost $c(q)$ when producing quantity q
- Seller's objective is now $-t(\theta) - c(q(\theta))$
- For the buyer nothing changes
- Optimality condition for pointwise maximisation (if c convex increasing):

$$\theta - \frac{1 - F(\theta)}{f(\theta)} - c'(q(\theta)) = 0.$$

Screening – incentive compatibility (formally)

We made our lives easy at several steps of the example:

- Did not proof that q must be increasing to fulfil IC
- Did not proof formally that $V'(\theta) = q(\theta)$ must hold to fulfil IC
- Buyer's linear valuation $\theta q + t$ seems like (very simple) special case

Let's see how to generalise result if buyer's utility is $u(\theta, q) + t$ and provide a complete proof.

Screening – incentive compatibility – three results

We consider three fundamental results:

1. Envelope Theorem
2. Revenue Equivalence Theorem
3. 'Incentive Compatibility Characterisation' Theorem

Screening – Envelope Theorem

Theorem (Envelope Theorem)

Assume that X is compact, and $\Theta = [\underline{\theta}, \overline{\theta}]$ and $g : \Theta \times X \to \mathbb{R}$ is differentiable in θ with uniformly bounded derivative. Suppose the selection $x^*(\theta)$ solves

$$V(\theta) = \max_{x \in X} g(\theta, x).$$

Then we have

$$V'(\theta) = \frac{\partial}{\partial \theta} g(\theta, x^*(\theta)) \ \ \textit{a.e.}$$

and

$$V(\theta) = V(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial \theta} g(s, x^*(s)) \, \mathrm{d}s.$$

Screening – Envelope Theorem – proof

Screening – Revenue Equivalence

Theorem (Revenue Equivalence)

Fix a function $q : \theta \to Q$*. Suppose that* Q *is compact and* $\Theta = [\underline{\theta}, \overline{\theta}]$*.*
Let the agent's utility be $u(\theta, q) + t$*, where* u *is differentiable in* θ *with uniformly bounded derivative. Any incentive compatible mechanism that implements* $q(\theta)$ *gives agent payoff*

$$V(\theta) = V(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial \theta} u(s, q(s)) \,\mathrm{d}s,$$

transfers must satisfy $-t(\theta) = u(\theta, q(\theta)) - V(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial \theta} u(s, q(s)) \,\mathrm{d}s.$

- By IC, the allocation rule almost completely pins down agent's and principal's payoff
- Only 'degree of freedom' is the constant $V(\underline{\theta})$

Screening – characterisation of IC

The Revenue Equivalence Theorem provides a necessary condition:

'If mechanism is incentive compatible, then (q, t) *satisfies...'*

Two issues remain:

- (When) are these conditions sufficient for incentive compatibility?
- In our example we said q had to be increasing, where did that come from?

Screening – characterisation of IC

Theorem

Suppose that Q is compact and $\Theta = [\underline{\theta}, \overline{\theta}]$. Let the agent's utility be $u(\theta, q) + t$, where u is differentiable in θ with uniformly bounded derivative.

If $\frac{\partial^2 u(\theta,q)}{\partial q \partial \theta} > 0$, then $(q(\theta), t(\theta))$ is IC if and only if

$$q(\theta) \text{ is non-decreasing}$$

and

$$-t(\theta) = u(\theta, q(\theta)) - V(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial \theta} u(s, q(s)) \, \mathrm{d}s.$$

Screening – characterisation of IC – proof