

Exercise: Regulating a monopolist

A monopolist operates in an industry with inverse demand function $p(q) = 1 - 2q$.

The monopolist has cost $c(\theta, q) = \frac{2}{3}\theta q$, where her private type θ is uniformly distributed on $[0, 1]$. In addition, producing q units of the good on a technology of type θ causes environmental harm $\frac{1}{3}\theta q$.

A regulator is trying to maximise the expectation of consumer surplus plus taxes net of environmental harm:

$$q^2 + \tau - \frac{1}{3}\theta q,$$

by offering a menu $(q(\cdot), \tau(\cdot))$ of the quantity the monopolist must produce and taxes paid by the monopolist. The monopolist can shut down to get payoff of 0 after learning θ .

- State the regulator's problem that characterises the optimal direct mechanism.
- Show that if $(q(\cdot), \tau(\cdot))$ is incentive compatible, then $q(\cdot)$ must be decreasing and $V(\theta) = V(1) + \int_{\theta}^1 \frac{2}{3}q(s) ds$.
- Solve for the optimal mechanism.