Sender-Receiver Games

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Different classes of games with similar overall structure:

- Two players: sender (S) and receiver (R)
- S is privately informed about type $\theta \in \Theta$
- Actions:
  1. S sends message $m \in M$
  2. R sees message m and takes action $x \in X$
- Payoffs: $u_S(\theta, m, x)$ and $u_R(\theta, x)$

We distinguish the following classes

1. Signaling games: $u_S(\theta, m, x) \neq u_S(\theta', m, x)$ and
2. Disclosure games: message set M depends on type
3. Cheap talk: $u_S(\theta, m, x) = u_S(\theta, x)\ \forall m$
4. Bayesian Persuasion: cheap talk but sender commits to message for each type

Motivating Example – Akerlof (1978): A market for "lemons"

'If this is correct, economics would be different.' –editor of JPE (who rejected the paper)

Consider a market for cars with multiple sellers and buyers:

- quality $\theta \in \{L, H\}$ with $\lambda = \mathbb{P}[\theta = L]$
- seller's reserve price is r_θ with $0 < r_L < r_H$
- buyer's valuation is v_θ with $v_\theta > r_\theta$

  so sale is always efficient!

Think of a competitive market with excess demand so that buyers pay expected value

↳ If μ is the belief that $\theta = L$, then

$$p = \mathbb{E}[v_\theta \mid \mu] = \mu v_L + (1-\mu) v_H$$

Benchmarks:

- Complete information
- Incomplete symmetric information

Akerlof: A market for "lemons" – Symmetric info benchmarks

Complete information: Sellers and buyers know quality of cars

↳ All cars are sold at v_θ for $\theta \in \{L, H\}$

⇒ Efficient market outcome

Incomplete symmetric information: Nobody knows quality

- Expected value for buyers: $\lambda v_L + (1-\lambda) v_H \equiv v(\lambda)$
- Expected reserve price for sellers: $\lambda r_L + (1-\lambda) r_H$

In market with $p = v(\lambda)$ all cars are sold because $\lambda r_L + (1-\lambda) r_H < v(\lambda)$

Asymmetric information: Only sellers know quality of cars

- Supposing that all cars are sold in market, p must be $v(\lambda) = \mathbb{E}[v_\theta \mid \mu = \lambda]$
- But sellers know θ so H-type sellers will be willing to sell at p only if $p \geq r_H$

=> if $v(\lambda) \geq r_H$ -> all cars can be sold in market.
  ↳ "if only few bad cars"

=> However, if $v(\lambda) < r_H$, only L-type sellers are willing to sell at $p = v(\lambda)$.
  So buyers can infer that all offered cars must be Lemons
 -> price must be v_L
 -> in equilibrium, only L-types are sold. => This is inefficient because gains from trade of H-cars are not realised!

Sender-Receiver Games

Signaling

(Job Market) Signaling

Spence (1973): Job Market Signaling. *The Quarterly Journal of Economics.*

sellers have no valuation

Continuing with the same model, suppose $r_L = r_H = 0$ and seller can send message $m \in \mathbb{R}_+$ where cost of message depends on type:

$$c(m, \theta) = \frac{m}{\theta}$$

with $H > L$ → messages are cheaper to send for high types.

Buyers observe message m and form a belief $\mu(m) \in [0, 1]$ that $\theta = L$

Model buyers/market in reduced form and directly assume the seller is paid $p = \mathbb{E}_{\theta \sim \mu(m)}[v_\theta]$

remark: in Spence's paper our sellers are (potential) workers, our buyers are firms, and the costly message is education

Perfect Bayesian Equilibria

Eq. conditions: (i) the message sent by type θ, $m(\theta)$ must be

$$m(\theta) \in \arg\max_{m \in \mathbb{R}_+} \left\{ p(m) - \frac{m}{\theta} \right\}$$

(ii) $p(m) = \mathbb{E}_{\theta \sim \mu(m)} \left[V_\theta \right]$

(iii) $\mu(m)$ is consistent with Bayes rule whenever m is chosen with positive probability in the equilibrium.

① construct pooling equilibria (i.e. $m(L) = m(H)$)

note that we can freely choose beliefs for messages which are not sent in eq.

↳ we assign 'worst' belief for sender to off-path messages. → $\mu(m) = 1$ for $m \neq m(H) = m(L)$

⇒ (ii) prices: $p(m) = V(\lambda) = \lambda V_L + (1-\lambda) V_H$ for equilibrium message m

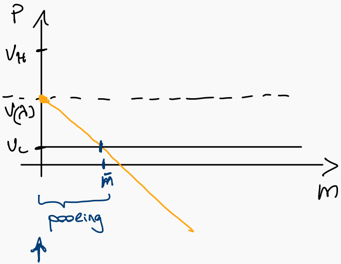
$p(m') = V(1) = V_L$ for off eq. messages.

(i) when is equilibrium message optimal for sender?

$\theta = L: \quad \lambda V_L + (1-\lambda) V_H - \frac{m}{L} \geq V_L - 0$

$\theta = H: \quad \lambda V_L + (1-\lambda) V_H - \frac{m}{H} \geq V_L - 0$

$\Rightarrow$ pooling at m is an equilibrium if $m \leq (1-\lambda)(V_H - V_L) \cdot L$



for any $m > 0$, the pooling eq. "burns money"

② Separating equilibria: $m(H) = m_H \neq m(L) = m_L$

Again, assign worst belief after any message $m \notin \{m_L, m_H\}$

prices:

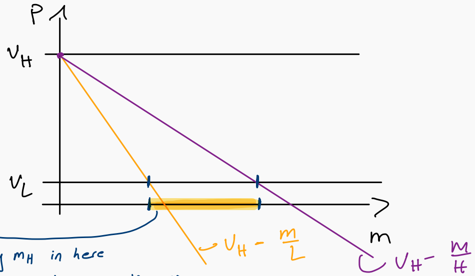
$$p(m) \begin{cases} v_H & \text{if } m = m_H \\ v_L & \text{if } m = m_L \\ v_L & \text{if } m \notin \{m_L, m_H\} \end{cases}$$

note that m_L must be 0 because $m=0$ induces belief $\mu = 1$

- $v_L - 0 \geq v_H - \frac{m_H}{L}$
- $v_H - \frac{m_H}{H} \geq v_L$

⇓

$$(v_H - v_L)L \leq m_H \leq (v_H - v_L)H$$



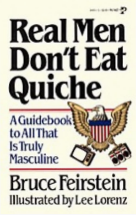
for any m_H in here and $m_L = 0$ is separating eq.

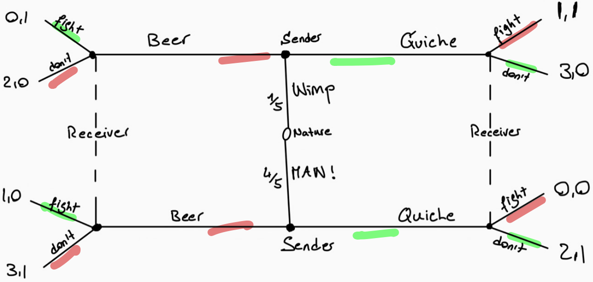
Who is better off in which type of eq.?

- L-type is better off in any pooling equilibrium. In pooling eq. the deviation to $m=0$ gives the same payoff as in any sep. eq.
- For H-type, not clear because cost of separating may exceed benefit.

Next: which equilibria are more sensible?

Excursion to equilibrium refinements





payoffs:

Sender: +1 for 'right' breakfast

+2 if no fight

Receiver +1 for fighting wimp

+1 for not fighting MAN

2 (pooling) equilibria:

1. (Beer, Beer), (Don't, Fight), $\mu(\text{wimp}|\text{Beer}) = 1/5$, $\mu(\text{wimp}|\text{Quiche}) \geq 1/2$ ⟹ Both survive test I

2. (Quiche, Qicle) (Fight, Don't), $\mu(\text{wimp}|\text{Beer}) \geq \frac{1}{2}$, $\mu(\text{wimp}/\text{Quiche}) = 1/5$ ⟶ Quiche does not survive test II

Eq. Refinement excursion

Definition I: Given a signaling game with sender payoff $u_S(m, \theta, x)$, a message ==m is dominated for type θ== if there is $m' \neq m$ s.t. $\min_x u_S(\theta, m', x) > \max_x u_S(\theta, m, x)$

m = message
θ = type
x = action of Receiver.

Definition II: Given a PBE in a signaling game, a message m is ==equilibrium dominated for type θ== if eq. payoff of θ (denoted $u_S^*(\theta)$) is s.t. $u_S^*(\theta) > \max_x u_S(\theta, m, x)$.

Test of dominated messages: Can equilibrium be supported if $\mu(m)$ puts prob. 0 on all types for which m is dominated?

Test of eq. dominated messages = ==intuitive criterion==: Can equilibrium be supported if $\mu(m)$ puts prob. 0 on all types for which m is equilibrium dominated?