Microeconomics 4

Jan Knoepfle

Spring 2022

Organisation

Lectures

Instructor: Jan Knoepfle, jan.knoepfle@aalto.fi

Lectures:

- Mondays and Tuesdays 10-12h, Economicum seminar room 3-4
- streamed on Zoom for non-Helsinki students

Office hours:

- By appointment (email) either in person or over Zoom
- Feel free to reach out actively whenever we can help!

MyCourses Forum: To help your peers, post questions directly on forum whenever possible

Course material

Slides uploaded in advance (incomplete) and after the lectures (completed)

Textbooks:

- **Mailath: Modeling Strategic Behavior.**
- Borgers: An Introduction to the Theory of Mechanism Design.
- Mas-Colell, Whinston, Green: Microeconomic Theory.
- Krishna: Auction Theory.
- Salanié: The Economics of Contracts.

Problem Sets

TA: Eero Mäenpää, eero.maenpaa@aalto.fi

Exercises:

- 4 problem sets, posted on MyCourses one week before due date
- Exercise sessions with Eero Mondays 14-16h in Economicum seminar room 3-4

  Dates: 21.03, 28.03, ~~1~~4.04 [04.04.], 02.05
- Hand in your solutions to problem set on MyCourses before exercise session
- Model solutions uploaded after exercise sessions

Evaluation

Requirements:

- At least 50% of solutions to problem sets
- Pass final exam

Grades based on exam only

Information Economics

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Introduction

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- Micro 4: we want to design the optimal 'game' to achieve the 'best' outcome
- Asymmetric Information poses main problem
- Examples for such 'designed games'
  - Sales procedures
  - Voting mechanisms
  - Employment contracts

Information Economics

Two main classes of design problems:

1) Adverse Selection (hidden information)

    Uninformed party cannot see characteristic of informed party. Uninformed moves first.

    Concepts: Screening and Mechanism Design

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Will also discuss a class of 'fixed games' where asymmetric information is crucial:

(3) Sender-Receiver Games

   Uninformed party does not see characteristic of informed party. Informed moves first.

   Signaling games, (evidence) disclosure games, ...

Adverse Selection

Adverse Selection

Screening

Screening

Screening:

- Principal and one agent
- Agent has private information about his preferences
- Principal design (commits to) a mechanism

Simple example:

- One Seller, one Buyer
- Seller owns a phone, her own valuation 0, Buyer has valuation $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$
- Utilities from sale at price p: $u_S(p) = p$ and $u_B(\theta, p) = \theta - p$
- Buyer knows θ (we call θ his private type)
- Seller only knows that θ is drawn from distribution F. Assume that $F'(\theta) = f(\theta) > 0$

What is optimal selling procedure (mechanism) for the seller?

Screening – an example

Let's first consider a special and simple class of mechanisms: posted-price mechanisms

- Seller posts a price p and buyer decides whether to buy at this price or not
  - iff $p \le \theta$
- Seller's expected profit from price p: $\Pi(p) = \underbrace{(1 - F(p))}_{\mathbb{P}[\text{sale}]} \underbrace{p}_{\text{price}}$
- Seller-optimal posted-price must maximise $\Pi(p)$

what is marginal effect on $\Pi(p)$ if increasing p?

$$\frac{d\Pi(p)}{dp} = \underbrace{(1 - F(p))}_{\text{gain from all types who buy}} - \underbrace{f(p) \cdot p}_{\text{loss from marginal type } \theta = p}$$

$$\frac{d\Pi(p)}{dp} = 0 \iff p^* = \frac{1 - F(p^*)}{f(p^*)}$$

Screening – an example

Can the seller do better?

- Seller could bargain multiple rounds, offer lotteries at different prices, . . .

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What are the **fixed components** of our problem?

- Space of outcomes: allocation prob. $q \in [0, 1]$ and transfer $t \in \mathbb{R}$ from seller to buyer
- Preferences: seller: $-t$, buyer: $\theta q + t$ with $\theta \sim F$

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What are all **possible mechanisms**?

1. Seller commits to game:
   - space of strategies S
   - outcome functions $q \colon S \to [0, 1]$ and $t \colon S \to \mathbb{R}$
2. Buyer (knows θ and) chooses strategy $s(\theta) \in S$

Screening – Revelation Principal

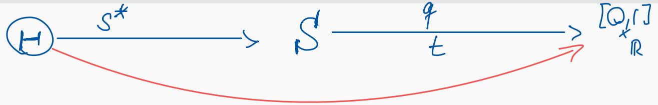
The set of all possible mechanisms of the form $\Gamma = (S, (q, t))$ is quite large!

Screening – Revelation Principal

Theorem (Revelation Principle)

Take any mechanism $\Gamma = \left(S, (q(s), t(s))_{s \in S}\right)$

and optimal agent strategy $s^*_\Gamma : \Theta \to S.$



Screening – Revelation Principal

Theorem (Revelation Principle)

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and optimal agent strategy $s^*_\Gamma \colon \Theta \to S.$

There is a ***direct*** *mechanism* $\hat{\Gamma} = \Big(\Theta, \big(\hat{q}(\theta), \hat{t}(\theta)\big)_{\theta \in \Theta}\Big)$

such that ***truthtelling*** $s^*_{\hat{\Gamma}} \colon \Theta \to \Theta$ *with* $s^*_{\hat{\Gamma}}(\theta) = \theta$

is an optimal strategy for the agent and the outcome is the same as in mechanism Γ.

Screening – direct IC mechanisms

Thanks to revelation principle, without loss to focus on direct truthful mechanisms

Seller's optimisation problem:

$$\max_{\substack{q:\Theta\to[0,1]\\ t:\Theta\to\mathbb{R}}} \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta)\,\mathrm{d}\theta \qquad \text{such that}$$

① buyer is willing to participate

② buyer is willing to reveal true type

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Seller's optimisation problem:

$$\max_{\substack{q:\Theta\to[0,1]\\ t:\Theta\to\mathbb{R}}} \int_{\underline{\theta}}^{\bar{\theta}} -t(\theta) f(\theta)\,\mathrm{d}\theta \qquad \text{such that}$$

$$\text{for all } \theta \in \Theta: \quad \theta q(\theta) + t(\theta) \geq 0 \qquad (IR_\theta)$$

$$\text{for all } \theta, \hat{\theta} \in \Theta: \quad \theta q(\theta) + t(\theta) \geq \theta q(\hat{\theta}) + t(\hat{\theta}) \qquad (IC_{\theta,\hat{\theta}})$$

individual rationality

incentive compatibility

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...still a lot of constraints

Screening – incentive compatible allocations and transfers

Type θ's utility from report $\hat{\theta}$ is

$$V(\theta, \hat{\theta}) = \theta q(\hat{\theta}) + t(\hat{\theta}), \qquad \text{with } V(\theta) \equiv V(\theta, \hat{\theta})\Big|_{\hat{\theta}=\theta}$$

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The IC constraints imply two important conditions:

1. $q(\cdot)$ must be weakly increasing in θ

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The IC constraints imply two important conditions:

1. $q(\cdot)$ must be weakly increasing in θ
2. $V'(\theta) = q(\theta)$ and we can integrate so that $V(\theta) - V(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s$

rough idea:

- Assume(!) that functions $q(\cdot)$ and $t(\cdot)$ are differentiable
- $IC_{\theta,\hat{\theta}}$ says that $V(\theta,\hat{\theta})$ must be max at $\hat{\theta}=\theta$

"FOC": $\frac{\partial}{\partial \hat{\theta}} V(\theta,\hat{\theta})\Big|_{\hat{\theta}=\theta} = 0 \iff \theta q'(\hat{\theta}) + t'(\hat{\theta})\Big|_{\hat{\theta}=\theta} = 0$

$\frac{d\, V(\theta,\theta)}{d\theta} = q(\theta) + \underbrace{\theta q'(\theta) + t'(\theta)}_{=0}$

Screening – change order of integration

Inserting $-t(\theta) = \theta q(\theta) - V(\theta) = \theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s - V(\underline{\theta})$ into max. problem gives

$$\max_{q:\Theta\to[0,1],V(\underline{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(s)\,\mathrm{d}s - V(\underline{\theta})\right] f(\theta)\,\mathrm{d}\theta \quad \text{s.t.} \quad V(\underline{\theta}) \geq 0, \text{ and } q(\cdot) \text{ increasing}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} q(s)\,ds\, f(\theta)\,d\theta$$

change order of integration

$\underline{\theta} \le s \le \theta \le \bar{\theta}$

$$\int_{\underline{\theta}}^{\bar{\theta}} \underbrace{\int_{s}^{\bar{\theta}} f(\theta)\,d\theta}_{1-F(s)} \cdot q(s)\,ds = \int_{\underline{\theta}}^{\bar{\theta}} (1-F(s))\, q(s)\,ds$$

$s \to \theta$

$$= \int_{\underline{\theta}}^{\bar{\theta}} \frac{1-F(\theta)}{f(\theta)}\, q(\theta) f(\theta)\,d\theta$$

Screening – virtual value

Changing the order of integration gives

$$\max_{q:\Theta\to[0,1],V(\underline{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] q(\theta)f(\theta)\,\mathrm{d}\theta - V(\underline{\theta}) \quad s.t.\ V(\underline{\theta}) \geq 0,\ q(\cdot)\text{ increasing}$$

- We can choose optimal $q(\cdot)$ pointwise (if result satisfies monotonicity constraint)
- $J(\theta) \equiv \theta - \frac{1-F(\theta)}{f(\theta)}$ is called the virtual valuation of type θ
- We say that distribution F is regular if $J(\theta)$ is increasing

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- We can choose optimal $q(\cdot)$ pointwise (if result satisfies monotonicity constraint)
- $J(\theta) \equiv \theta - \frac{1-F(\theta)}{f(\theta)}$ is called the virtual valuation of type θ
- We say that distribution F is regular if $J(\theta)$ is increasing
- If F regular, optimal allocation is $q^*(\theta) = \begin{cases} 0 & \text{if } \theta < \theta^* \\ 1 & \text{if } \theta \geq \theta^*, \end{cases}$ with $\theta^* = \frac{1-F(\theta^*)}{f(\theta^*)}$
- What if J is not monotone? → execise

Screening example – recap

We have used many tools today that we will develop in more detail further on:

- Revelation Principle
- Characterising IC in terms of allocation rule only
- Virtual valuations

These have allowed us to derive several results:

- We can solve for optimal mechanisms
- Utilities and transfers are pinned down almost entirely by IC allocation rule
- Posted price mechanisms are optimal!