4-4-22 lecture 07

Last week

Salanié p. 119 ->
has a good intro

Intro to moral hazard

Mailath 346 ->

MWG 480 ->

- If effort is observable principal can implement any effort choice with "forcing contract"

MWG 482 ->

- If agent is risk neutral and wages unrestricted, then first best can be implemented (even with unobservable effort) by "selling the firm to agent".

Today

Salanié p. 138 ->

- Risk neutral agent and limited liability
- Risk averse agent

Mailath p. 348 ->

MWG p.483 ->

Moral Hazard – Limited Liability (and risk-neutral agent)

$c(e)$ is convex increasing

Suppose

- continuum of effort choices $e \in E = [0,1]$
- two possible outputs $\pi \in \{0, \bar{\pi}\}$ with $\mathbb{P}[\pi = \bar{\pi}|e] = e$ and $\mathbb{P}[\pi = 0|e] = 1 - e$
- risk-neutral agent: $v(w) = w$
- **limited liability:** $w(\pi) \geq 0$

The principal solves

$$\max_{e, w(0), w(\bar{\pi})} \{e\,(\bar{\pi} - w(\bar{\pi})) + (1-e)\,(0 - w(0))\} \quad \text{such that}$$

$$e \in \operatorname*{argmax}_{e'} \{e' w(\bar{\pi}) + (1-e')w(0) - c(e')\} \tag{IC}$$

$$e\, w(\bar{\pi}) + (1-e)w(0) - c(e) \geq 0, \tag{IR}$$

$$w(\bar{\pi}) \geq 0, \quad w(0) \geq 0 \tag{LL}$$

limited liability

Moral Hazard – Limited Liability (and risk-neutral agent)

2-step-procedure:

1. Take any effort level e, what is the optimal wage pair $(w_e(0), w_e(\pi))$ to implement e?

2. What is the optimal effort level e?

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1. Agent is willing to choose e' if it maximises

IC: $e' \cdot w(\pi) + (1-e') w(0) - c(e')$

→ (i) choose $w(0)$ as low as possible $w_e(0) = 0$

$\geq$ → by constraint (LL)

(ii) $w_e(\pi)$ from agent's FOC (of IC)

$w(\pi) - c'(e) = 0 \Rightarrow \boxed{w_e(\pi) = c'(e)}$

② optimal effort level

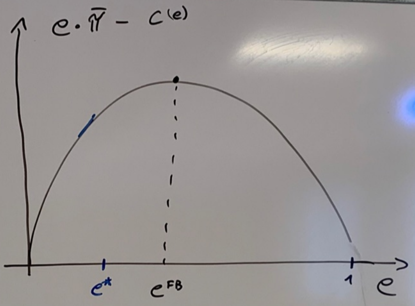
$$\max_e \left\{ e\cdot(\bar{\pi} - w_e(\bar{\pi})) + (1-e)(0 - w_e(0)) \right\} \overset{\text{from } ①}{=} \max_e \left\{ e(\bar{\pi} - c'(e)) + 0 \right\}$$

FOC: $\bar{\pi} - c'(e) - e c''(e) \overset{!}{=} 0 \Rightarrow$ solution $e^* < e^{FB}$

↳ because e^{FB} solves $\bar{\pi} - c'(e) = 0$

$\Rightarrow$ with limited liability, effort is distorted downward even if the agent is risk neutral.

here : $e\bar{\pi} = \mathbb{E}[\pi | e]$



Moral Hazard – Risk sharing

Suppose

FOSD

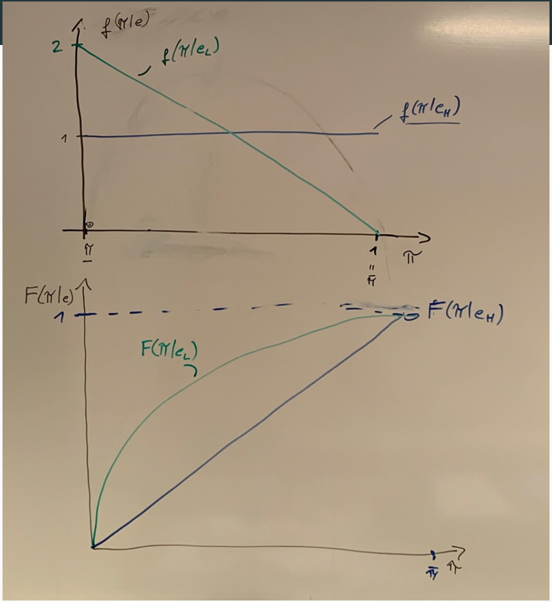
- binary effort choice $e \in E = \{e_L, e_H\}$ with $e_L < e_H$
- output $\pi \in [\underline{\pi}, \bar{\pi}]$ with distribution $F(\cdot|e)$ satisfying $F(\pi|e_H) \leq F(\pi|e_L)$ for all π
- risk-averse agent: $v(w)$ increasing and strictly concave
- effort cost $c(e_L) = 0$ and $c(e_H) = c_H > 0$

Principal solves

$$\max_{e \in \{e_L, e_H\}, w(\cdot)} \int_{\underline{\pi}}^{\bar{\pi}} (\pi - w(\pi))\, f(\pi|e)\, \mathrm{d}\pi \quad \text{such that}$$

$$e = \underset{e' \in \{e_L, e_H\}}{\operatorname{argmax}} \int_{\underline{\pi}}^{\bar{\pi}} v\left(w(\pi)\right) f(\pi|e')\, \mathrm{d}\pi - c(e'), \tag{IC}$$

$$\int_{\underline{\pi}}^{\bar{\pi}} v\left(w(\pi)\right) f(\pi|e)\, \mathrm{d}\pi - c(e) \geq 0 \tag{IR}$$



f(π|e)
2
f(π|e_L)
f(π|e_H)
1
0
π
1
π
F(π|e)
1
F(π|e_H)
F(π|e_L)
π
π

Principal solves

$$\max_{e\in\{e_L,e_H\},w(\cdot)} \int_{\underline{\pi}}^{\bar{\pi}} (\pi - w(\pi))\, f(\pi|e)\, d\pi \quad \text{such that}$$

IC $\qquad e = \operatorname*{argmax}_{e'\in\{e_L,e_H\}} \int_{\underline{\pi}}^{\bar{\pi}} v\,(w(\pi))\, f(\pi|e')\, d\pi - c(e'),$

IR $\qquad \int_{\underline{\pi}}^{\bar{\pi}} v\,(w(\pi))\, f(\pi|e)\, d\pi - c(e) \geq 0$

① optimal wage function for each effort

② optimal effort level

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①(i) Suppose principal wants to implement $e = e_L$

$IC_{e_L \succ e_H}$: $\boxed{\mathbb{E}_\pi[v(w(\pi)) \mid e = e_L]} - 0 \geq \boxed{\mathbb{E}_\pi[v(w(\pi)) \mid e = e_H]} - c_H$

IR_{e_L} : $\mathbb{E}_\pi[v(w(\pi)) \mid e = e_L] - 0 \geq 0$

Note: with a constant wage function $w(\pi) = \bar{w}$ for all π, then

$IC_{e_L \succ e_H}$ holds always because $\boxed{} = \boxed{}$

→ to implement lowest effort e_L, choose lowest wage to satisfy IR_{e_L} constraint

$w_{e_L}(\pi) = \bar{w}_{e_L}$: $v(\bar{w}_{e_L}) = 0$

① (ii) Suppose the principal wants to implement $e = e_H$

$$\max_{w(\cdot)} \left\{ \int_{\underline{\pi}}^{\overline{\pi}} \underbrace{-w(\pi)}_{-v^{-1}(u(\pi))} f(\pi | e_H) \right\}$$ such that

(i) IC_{e_H, e_L} $$\int_{\underline{\pi}}^{\overline{\pi}} \underbrace{v(w(\pi))}_{u(\pi)} f(\pi | e_H)\, d\pi - c_H \geq \int_{\underline{\pi}}^{\overline{\pi}} \underbrace{v(w(\pi))}_{u(\pi)} f(\pi | e_L)\, d\pi - 0$$

(ii) IR_{e_H} $$\int_{\underline{\pi}}^{\overline{\pi}} \underbrace{v(w(\pi))}_{u(\pi)} f(\pi | e_H)\, d\pi - c_H \geq 0$$

Problem: In this formulation both sides of the IC constraint are concave in $w(\pi)$

$\Rightarrow$ Not clear whether FOC are sufficient for optimum.

Solution: Transformation of variable: $u(\pi) = v(w(\pi))$, then $w(\pi) = v^{-1}(u(\pi))$

$\Rightarrow$ now have $\max_{u(\pi)}$ concave function with linear constraints

Principal's FOC wrt $w(\pi)$

$$0 = -f(\pi|e_H) + \mu\, v'(w(\pi))\, f(\pi|e_H) - \mu\, v'(w(\pi))\, f(\pi|e_L) + \gamma\, v'(w(\pi))\, f(\pi|e_H)$$

$$\frac{1}{f(\pi|e_H)} \quad \frac{1}{v'(w(\pi))}$$

$$\Leftarrow \quad \boxed{\frac{1}{v'(w(\pi))} = \mu\left(1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right) + \gamma}$$

which constraints bind? IC constraint must bind ($\mu>0$) (if $\mu=0$ then wage would be constant)

IR constraint also bind ($\gamma>0$) (if $\gamma=0$, then would have $\frac{1}{v'(w(\pi))} = 0$ for some π

$$w_{e_H}(\pi) \begin{cases} < w^* & \text{if } \frac{f(\pi|e_H)}{f(\pi|e_L)} < 1 \\ = w^* & \text{if } \frac{f(\pi|e_H)}{f(\pi|e_L)} = 1 \\ > w^* & \text{if } \frac{f(\pi|e_H)}{f(\pi|e_L)} > 1 \end{cases}$$

let w^* : $v'(w^*) = \frac{1}{\gamma}$

$$\boxed{\frac{1}{u'(w(\pi))} = \mu\left(1 - \frac{f(\pi \mid e_L)}{f(\pi \mid e_H)}\right) + \gamma}$$

Is $w_{e_H}(\pi)$ increasing in π generally?

Not generally, only if $\underbrace{\frac{f(\pi \mid e_H)}{f(\pi \mid e_L)} \text{ is increasing in } \pi}$ $\hat{=}$ monotone likelihood ratio property (MLRP)

Note: MLRP is stronger than FOSD

i.e. MLRP $\Rightarrow$ FOSD but not vice versa

(exercise).

$c_L = 0$

Example:

- $[\underline{\pi}, \bar{\pi}] = [0, 1]$ with distribution $f(\pi|e_L) = 2 - 2\pi$ and $f(\pi|e_H) = 1$ for all π
- $v(w) = \log(w)$

What is the optimal wage rule $w(\pi)$ to implement e_L?

$w_{e_L}(\pi) = \bar{w}$: $v(\bar{w}) = \log(\bar{w}) = \underset{c_L}{0}$ $\quad \bar{w} = v^{-1}(\overset{c_L}{0}) = 1.$

What is the optimal wage rule $w(\pi)$ to implement e_H? $\rightarrow$ as a function of π, μ, γ

$$\frac{1}{v'(w(\pi))} = \mu\left(1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right) + \gamma$$

$$= w_{e_H}(\pi) = \mu\left(1 - \frac{2-2\pi}{1}\right) + \gamma = \gamma + \mu(2\pi - 1)$$

$\rightarrow$ wage is linearly increasing in output.