25-04-22 Lecture 11

Last lectures

- Akerlof lemon's market
- Job market signaling

Today

Disclosure / evidence games

Sender-Receiver Games

Disclosure Games

Disclosure games

–Grossman (1981): The Informational Role of Warranties and Private Disclosure about Product Quality. *Journal of Law and Economics.*
–Milgrom (1981): Rational Expectations, Information Acquisition, and Competitive Bidding. *Econometrica.*

One Seller has a car of privately known quality type $\theta \in \{L, H\}$

Many Buyers with valuation v_θ, with $0 < v_L < v_H$

Game:

evidence

say nothing

1. Seller sends one message from **type-dependent** message set: $m \in M(\theta) = \{\theta, \emptyset\}$

reveal true type

2. Buyers observe message m and form belief $\mu(m)$ over $\{L, H\}$
3. Seller gets market price $p = \mathbb{E}_{\theta \sim \mu(m)}[v_\theta]$ (=expected buyer value)

Disclosure games

- If seller sends message $m=\theta$, buyers must know that θ is true type

$$\Rightarrow \quad p(m=\theta) = \mathbb{E}_{\tilde{\theta}}[V_{\tilde{\theta}} \mid m=\theta] = V_\theta$$

- For type H it is dominant strategy to reveal the truth.
- If H always sends $m=H$, then in equilibrium $\mu(m)$ for any $m \neq H$ must place prob. 1 on $\theta = L$.

$\Rightarrow$ only equilibrium is full separation : - H gets V_H
- L gets V_L

Other form of "evidence" in practice : warranty.

When θ becomes observable to buyer after purchase, H-seller can offer sale at $p = V_H$ with buy-back option.
(L-type sellers will not be willing to do that).

Disclosure games – Unravelling in the Milgrom-Grossman model

More than two types: Suppose $\Theta = \{\theta_1 < \cdots < \theta_n\}$ or $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$ (with increasing $v(\theta)$)

- For highest type, revealing own type is dominant strategy
- Given that highest type separates, 2nd highest type also prefers to separate

⋮

"unravelling" until full separation

Argument works with **other evidence structures:** e.g.,

$M(\theta) = \{$any subset $m \subset \Theta$ with $\theta \in m\}$ – any true statement can be sent

$M(\theta) = \{\theta' \in \Theta$ with $\theta' \leq \theta\}$ – only understatements are feasible

$\Rightarrow$ unravelling until full separation still occurs

Disclosure games – Dye-evidence

–Dye (1984): Disclosure of nonproprietary Information. *Journal of Accounting Research*

Suppose seller types are $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$ with increasing v_θ and evidence

$$M(\theta) = \begin{cases} \{\theta, \emptyset\} & \text{with prob. } \gamma \\ \{\emptyset\} & \text{with prob. } 1-\gamma \end{cases}$$

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Suppose seller types are $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$ with increasing v_θ and evidence

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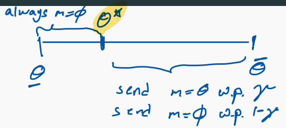
w. prob. γ

In equilibrium:

- **seller**-types in set $T \subset \Theta$ send message $m = \theta$ **if they can**
  other types, in $T^C = \Theta \setminus T$ send $m = \emptyset$ **always**
- **buyer**/market pays the seller the expected value
  - with message θ: $\mathbb{E}\left[v_{\tilde{\theta}} \mid m = \theta\right] = \mathbb{E}\left[v_{\tilde{\theta}} \mid \tilde{\theta} = \theta\right] = v_\theta$
  - with message $\emptyset$: $\mathbb{E}\left[v_{\tilde{\theta}} \mid m = \emptyset\right] = \mathbb{E}\left[v_{\tilde{\theta}} \mid \tilde{\theta} \text{ has no evidence } \bigcup \ \tilde{\theta} \in T^C\right]$

or

Disclosure games – Dye-evidence



Equilibrium

- Higher types reveal θ (if they can)
- Lower types pool with "no-evidence" types (some of which are high)

- After sending $m=\theta^*$, get V_{θ^*}
- After sending $m=\emptyset$, gets $\mathbb{E}_{\tilde{\theta}}\left[V_{\tilde{\theta}} \mid \{\text{no evidence}\} \cup \{\tilde{\theta} \le \theta^*\}\right]$

$$= \mathbb{E}_{\tilde{\theta}}\left[V_{\tilde{\theta}} \mid \{\text{no evidence}\} \cup \left(\{\tilde{\theta} \le \theta^*\} \cap \{\text{evidence}\}\right)\right]$$

these are disjoint events

θ^* determined by indifference:

$$V_{\theta^*} = \mathbb{E}_{\tilde{\theta}}\left[V_{\tilde{\theta}} \mid \{\text{no evidence}\} \cup \left(\{\tilde{\theta} \le \theta^*\} \cap \{\text{evidence}\}\right)\right] \Rightarrow$$

gives unique cutoff $\theta^* \in (\underline{\theta}, \bar{\theta})$ for any $\gamma \in (0,1)$

$\gamma \nearrow 1 \Rightarrow \theta^* \searrow \underline{\theta}$ (convergence to previous unravelling model)

$\gamma \to 0, \Rightarrow V_{\theta^*} = \mathbb{E}_{\tilde{\theta}}[V_{\tilde{\theta}} \mid m=\emptyset] = \mathbb{E}[V_{\tilde{\theta}}]$

Disclosure games – Dye-evidence – Application to stock market

- Firm value $\theta \in \{5, 10\}$ with $\lambda = \mathbb{P}[\theta = 5]$ based on Shin (2003) Econometrica: "Disclosure and Asset Returns"
- **Two periods:**

  t=1: With prob γ, manager learns θ and chooses to disclose $m = \theta$ or $m = \emptyset$

  With prob $1 - \gamma$, manager learns nothing and discloses $m = \emptyset$

  t=2: Firm value θ becomes public
- Share price p_t = expected value conditional on all public information at (end of) period t.
- Manager wants to maximise share price

Exercise:

a) What is the optimal choice for the manager in $t = 1$ conditional on θ?

b) What is the share price in $t = 1$ conditional on m?

c) Consider the change from p_1 to p_2. Are bad or good news followed by higher volatility?

d) Suppose the manager's info is always public. How do answers to (b) and (c) change?

a) optimal messages

- If manager learns $\theta = 10$, it is optimal to reveal it as it gives highest possible price
- If he learns $\theta = 5$, optimal not to reveal it and be pooled with uninformed (some of which are $\theta = 10$)

$m^*(10) = 10$, $m^*(5) = \phi$

b) $p_1(m)$

- $p_1(10) = 10$
- $p_1(5) = 5$ → won't occur in equilibrium
- $p_1(\phi) = \mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta} \mid \{\text{no info}\} \cup \{\tilde{\theta} = 5\}\right] = \mathbb{E}\left[\tilde{\theta} \mid \{\text{no info}\} \cup \{\text{info} \cap \tilde{\theta} = 5\}\right]$

disjoint events

$$= \frac{\Pr[\text{no info}] \cdot \mathbb{E}[\tilde{\theta}] + \Pr[\text{info} \cap \tilde{\theta} = 5]\, \mathbb{E}[\tilde{\theta} \mid \tilde{\theta} = 5]}{\Pr[\text{no info}] + \Pr[\text{info} \cap \tilde{\theta} = 5]}$$

$$= \frac{(1-\gamma)[\lambda 5 + (1-\lambda)10] + \gamma \cdot \lambda \cdot 5}{1-\gamma + \gamma\lambda} \in (5, 10)$$

c) <u>change from p_1 to p_2:</u>

Based on Shin (2003) Econometrica "Disclosure and Asset Returns".

In eq. only $m=10$ and $m=\phi$ will be sent (clearly $m=10$ is good news, $m=\phi$ is bad news)

if $m=10 \Rightarrow p_1=10$ $p_2=10$ w.p. 1 $\longrightarrow$ no volatility from $t=1$ to $t=2$

$m=\phi \Rightarrow p_1=p_1(\phi)$

$$p_2 \begin{cases} \nearrow 10 \\ \searrow 5 \end{cases}$$

more volatility after bad news

d) with public info

$m=5 \longrightarrow p_1=p_2=5$

$m=\phi \longrightarrow p_1=\lambda 5+(1-\lambda)10 \longrightarrow p_2 \begin{cases} \xrightarrow{\lambda} 5 \\ \xrightarrow{1-\lambda} 10 \end{cases}$

$m=10 \longrightarrow p_1=p_2=10$

no systematic difference in volatility after good or bad news.