MEC-E1005

MODELLING IN APPLIED MECHANICS 2022

WEEK 16: BICYCLE WHEEL RIM RIGIDITY

Tue 10:15-12:00 Introduction and assignments (JF)

INTRODUCTION

LEARNING OUTCOMES

MEC-E1005 Modelling in Applied Mechanics is a project course, which combines modelling, numerical methods, experiments, and scientific writing in applied mechanics. Most projects are related with displacement, vibration, and stability analyses on toy structures but projects on rigid body dynamics and Arctic technology may be also offered. After the course a student

(1) understands the interplay of modelling and experiments in engineering work

(2) is able to use a numerical software in structural analyses

(3) knows how to write a technical report

Assessment: Participation in industry presentations (6p) and 3 reports (6p each)

EXPERIMENT VS. MODELLING

In design of a simple pendulum of a tall-case clock, the required information is the dependency of period T on mass m, initial angle ϕ_0 from the stable equilibrium position, acceleration by gravity g, and length L. The main options are

Straightforward experiment: Measurement of T on various physical structures (characterized by m and L), with various initial angles ϕ_0, and on various places on earth (characterized by g).

Dimension analysis: Application of generic principles of physics to get $T = \sqrt{L/g}\, f(\phi_0)$ and measurement of $T\sqrt{g/L}$ as the function of ϕ_0.

Mathematical modelling: Application of simplifying assumptions, the basic laws of mechanics, and rules of mathematics to get $T = 2\pi\sqrt{L/g}$.

MODELLING IN MECHANICS

- **Crop**: Decide the boundary of structure. Interaction with surroundings need to be described in terms of known forces, moments, displacements, and rotations.

- **Idealize**: Simplify the geometry. Ignoring the details, not likely to affect the outcome, may simplify analysis a lot.

- **Parameterize**: Assign symbols to geometric and material parameter of the idealized structure. Measure or find the values needed in calculations.

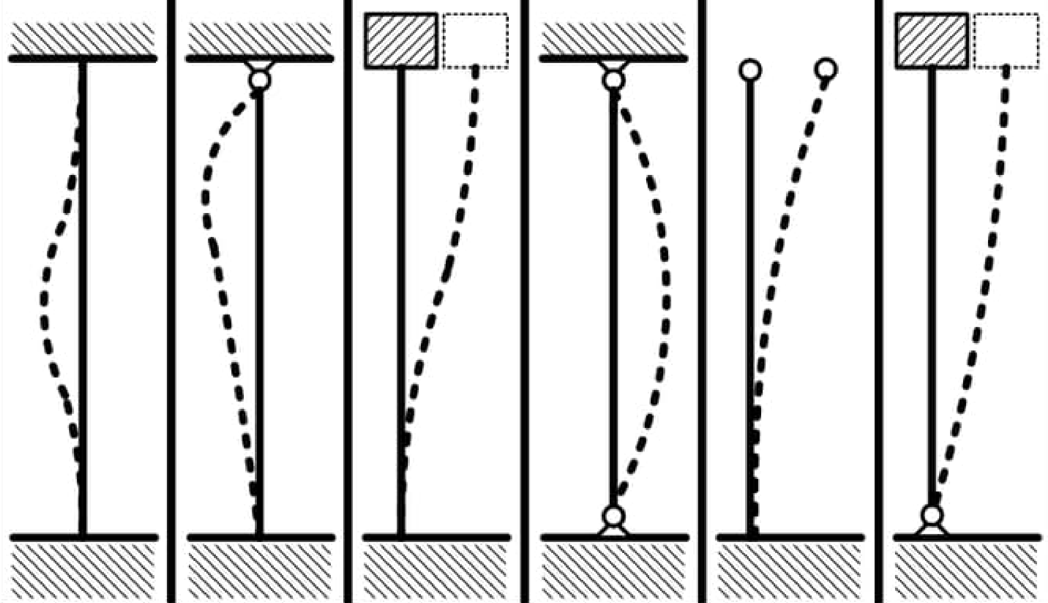
- **Model**: Write the mathematical description consisting of equilibrium equations, constitutive equations, and boundary conditions.

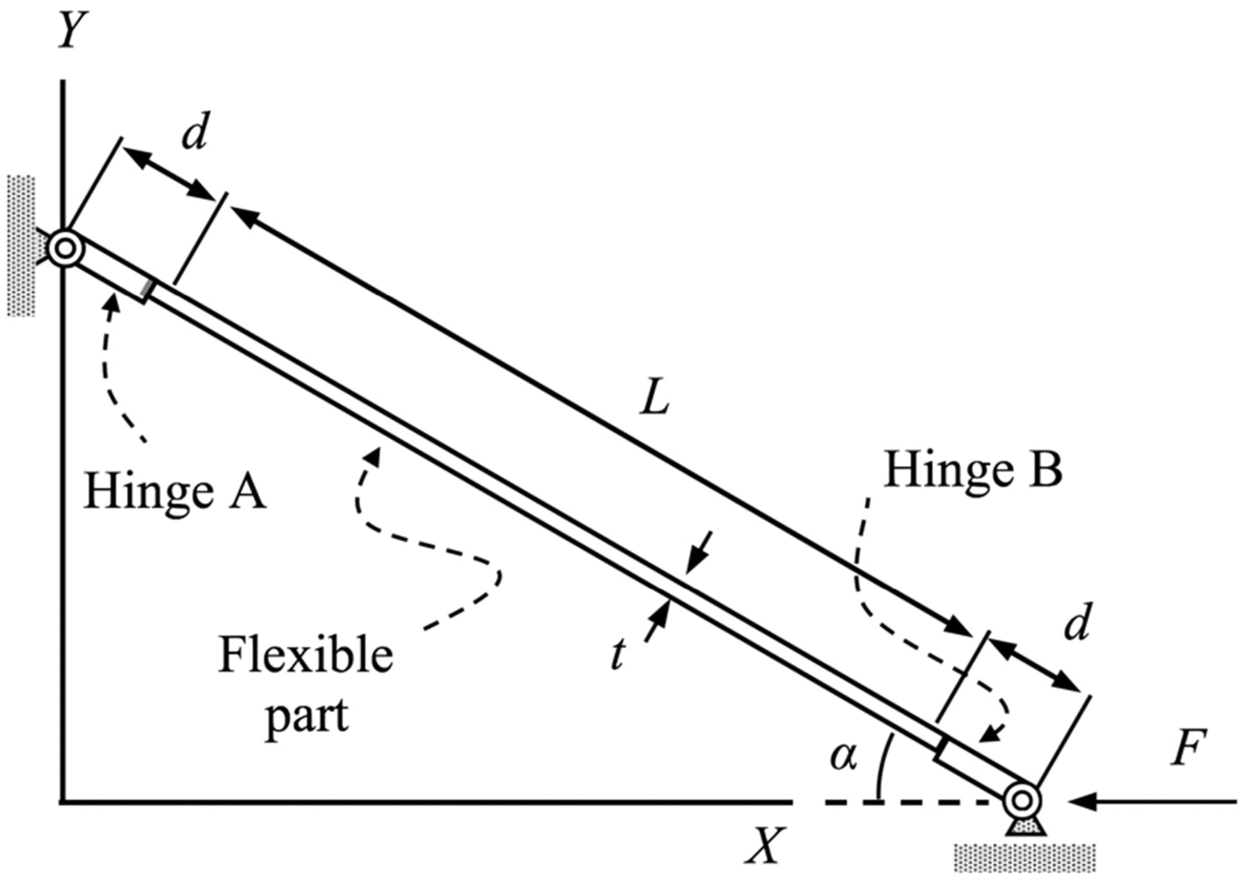
- **Solve**: Use an analytical or numerical method and hand calculations or software to find the solution.

BUCKLING OF A BEAM

Does $p_{\mathrm{cr}} = \pi^2 \dfrac{EI}{(KL)^2}$ **predict the buckling force within engineering accuracy?**

BUCKLING LOAD OF BEAM $p_{cr} = \pi^2 \frac{EI}{(KL)^2}$

Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0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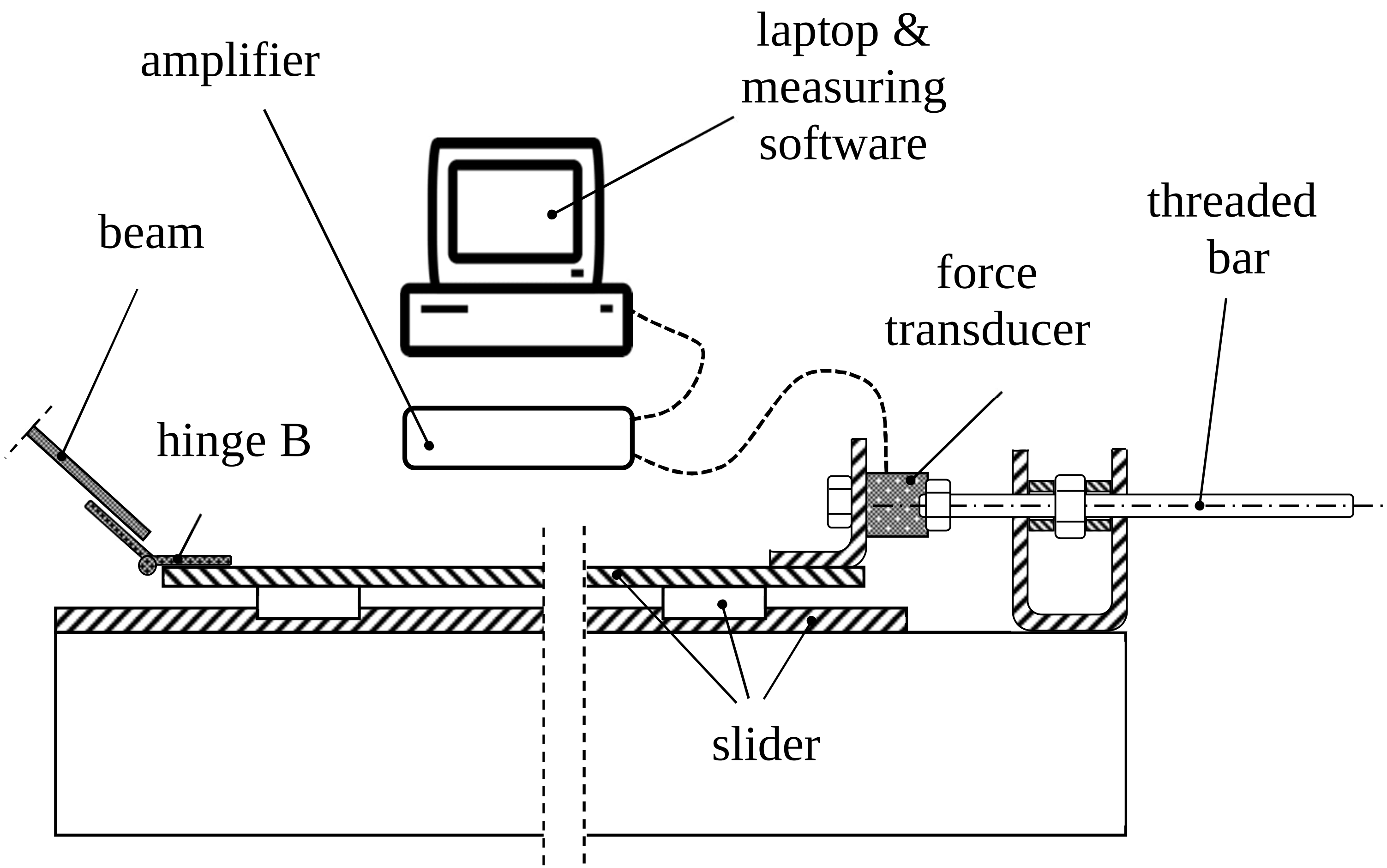
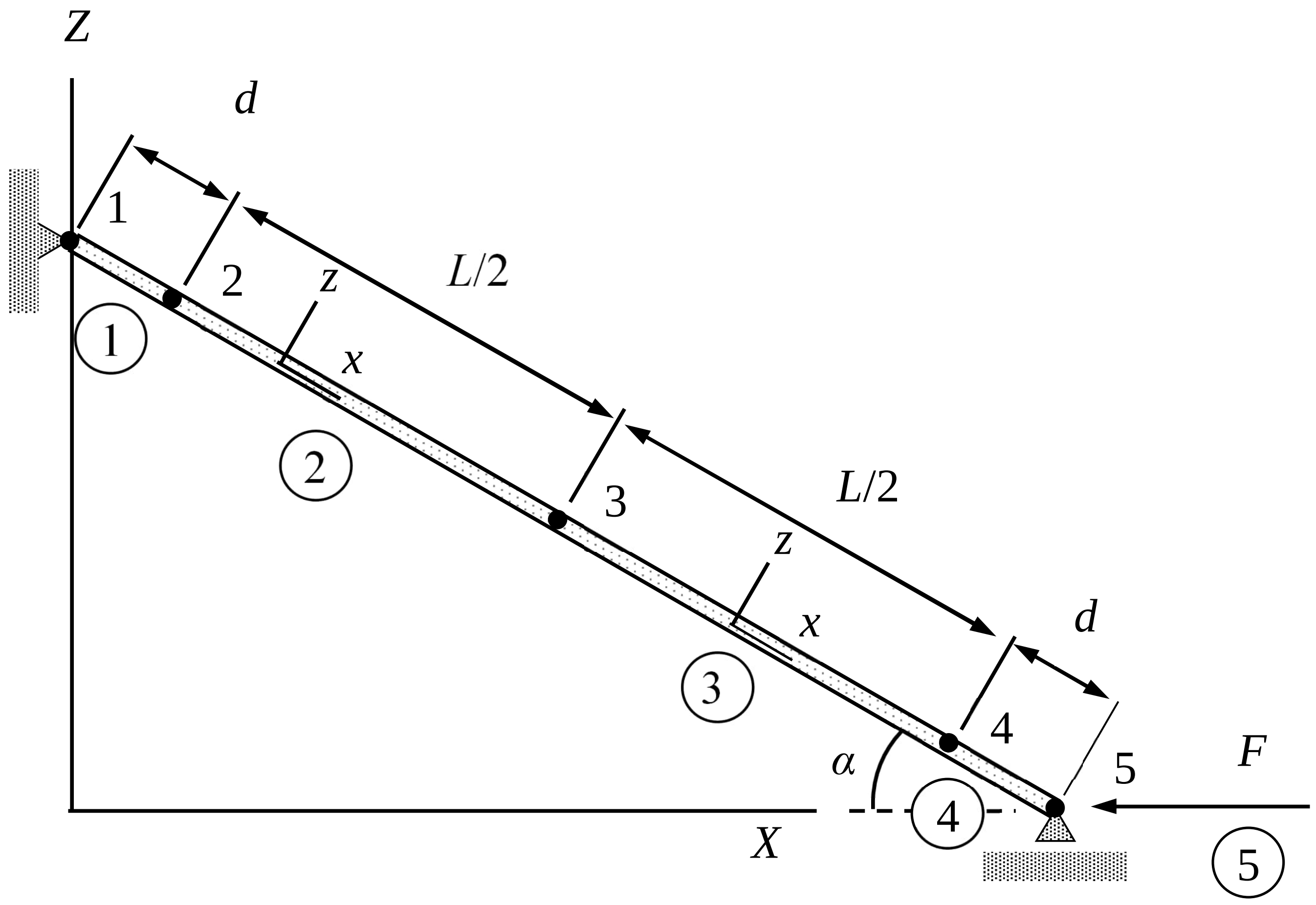
EXERIMENTAL METHOD

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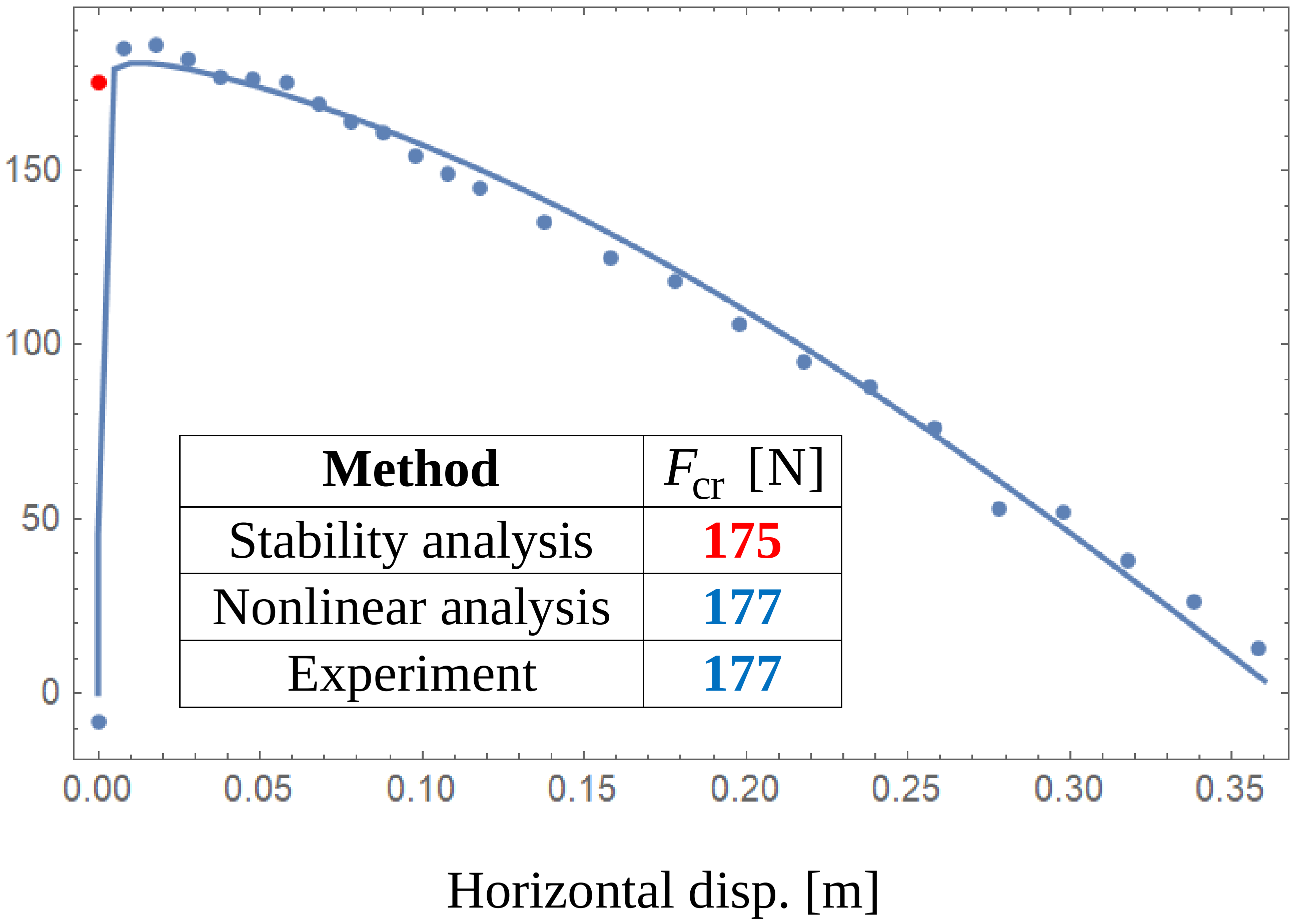
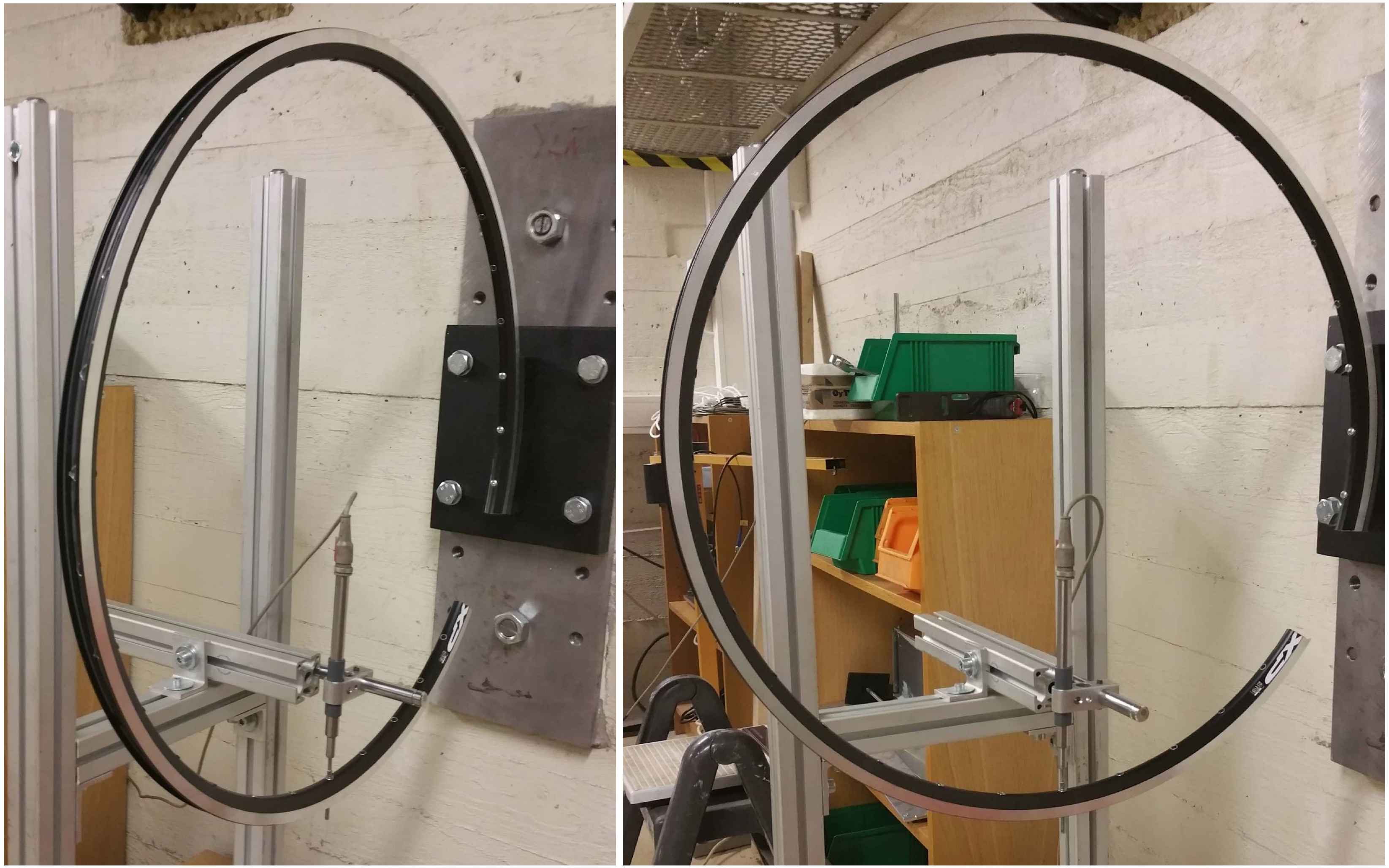
EXPERIMENT VS. MODEL



Method	F_{cr} [N]
Stability analysis	175
Nonlinear analysis	177
Experiment	177

BICYCLE WHEEL RIM DESIGN

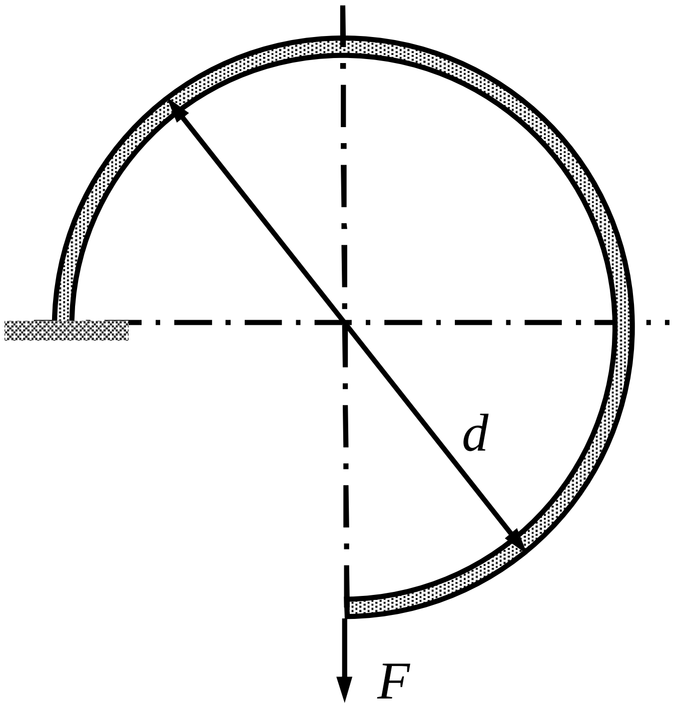


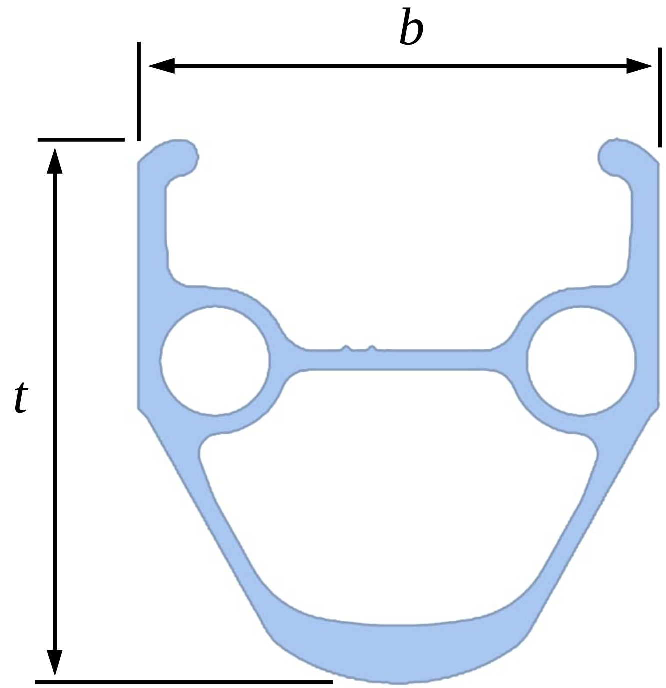
ASSIGNMENT 1

Rigidity of a wheel rim depends on the shape of cross-section, radius of the rim and material used. In design of the rim, a model is used to find a good combination of geometric and material parameters. In this modelling assignment, you will validate a bicycle wheel rim model by comparing rigidities given by a model and experiment on a bicycle wheel rim.

The outcome is a report that should explain the methods used, compare the results obtained from the three methods, and discuss any possible discrepancies between these methods. The report should be detailed enough to enable a reader having some background in experiments and engineering models to follow the derivations and to repeat the same steps.

STRUCTURE IDEALIZATION





TRIANGLE REPRESENTATION

crd.xlsx, tri.xlsx

ANALYTICAL METHOD

Assuming a planar beam, clamping and the center of the rim on the same horizontal line, and $L = 3\pi R / 2$ (curvilinear $xy-$coordinate system, x along the rim at the area centroid, x directed to the center of circle):

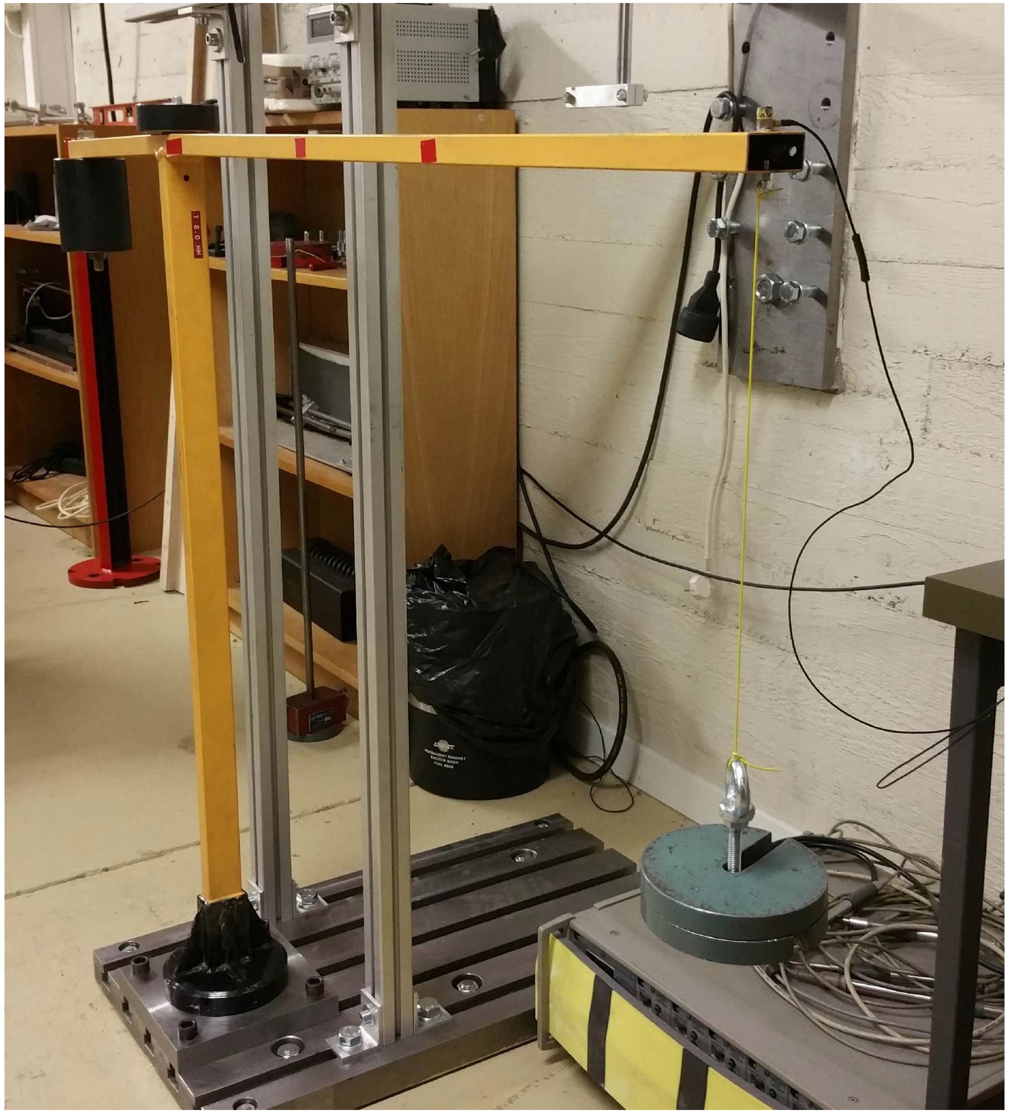
Equilibrium: $\frac{dN}{dx} - \frac{1}{R} Q = 0$, $\frac{dQ}{dx} + \frac{1}{R} N = 0$, $\frac{dM}{dx} + Q = 0$,

Constitutive: $N = EA(\frac{du}{dx} - \frac{1}{R} v)$, $Q = GA(\frac{dv}{dx} + \frac{1}{R} u - \psi)$, $M = EI \frac{d\psi}{dx}$

BC:s at the free end: $N(L) = 0$, $Q(L) + F = 0$, $M(L) = 0$

BC:s at the clamped end: $u(0) = 0$, $v(0) = 0$, $\psi(0) = 0$

CRANE CABLE SNAP ACCIDENT

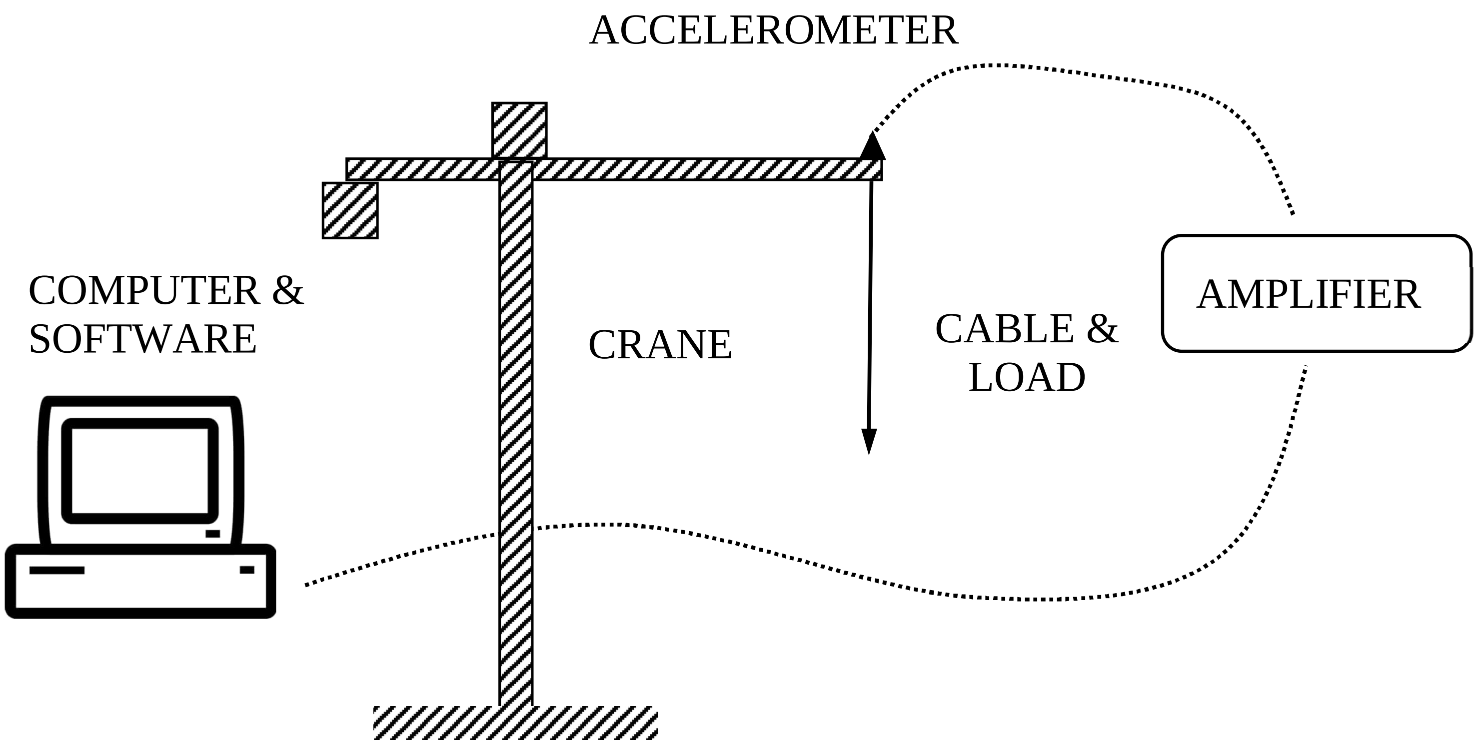


ASSIGNMENT 2

Breaking of a crane cable may result into structural damage. As a full-scale experiment is too expensive, a model is used to predict the structural response in design of the crane. In the modelling assignment, you will validate the crane model by comparing the three lowest eigenfrequencies of vibrations given by the model and measurements on a miniature crane. Also, accelerations due to the sudden snap of cable are compared.

The outcome is a report that should explain the methods used, compare the results obtained from the three methods, and discuss any possible discrepancies between these methods. The report should be detailed enough to enable a reader having some background in experiments and engineering models to follow the derivations and to repeat the same steps.

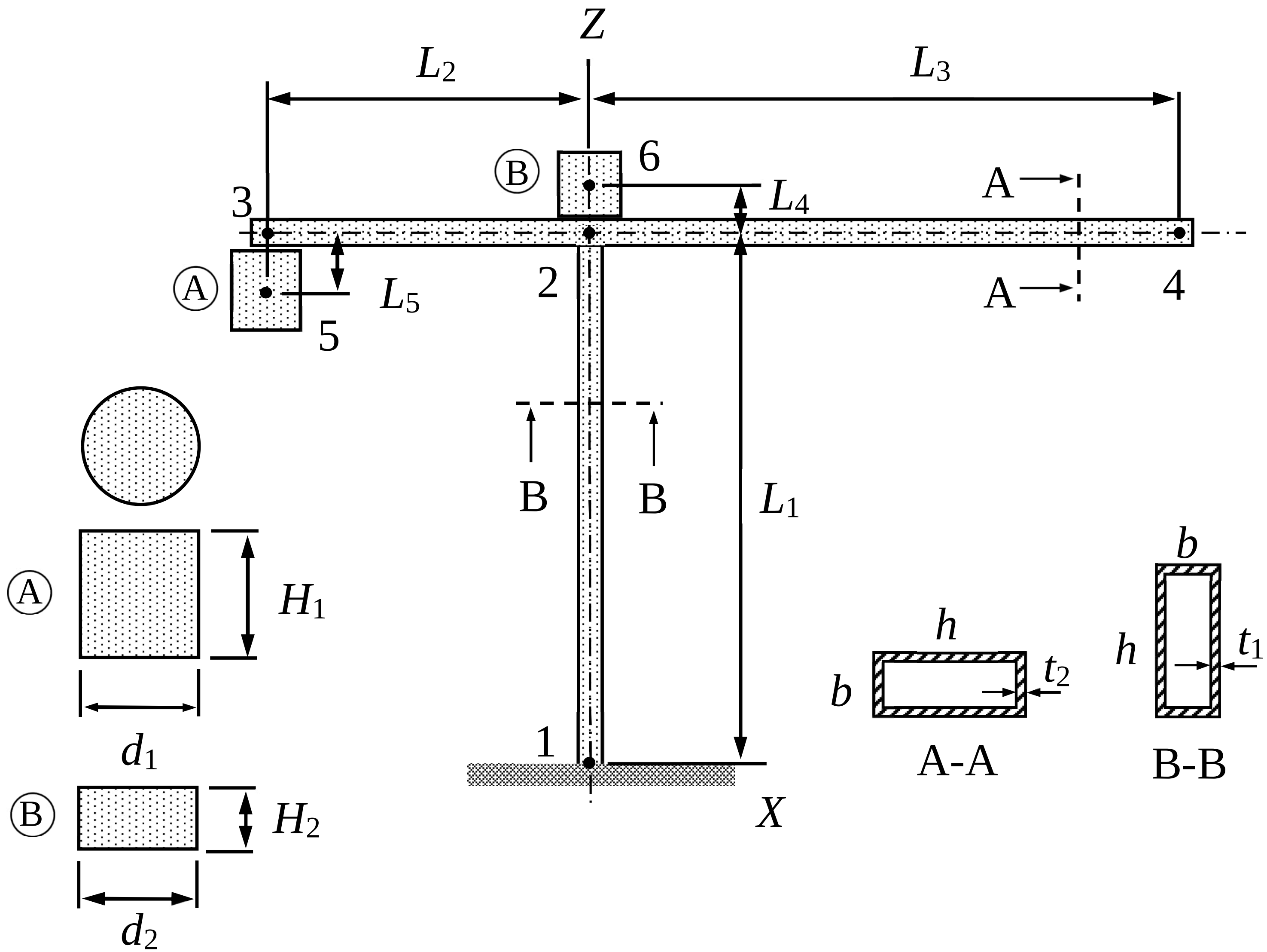
EXPERIMENTAL METHOD

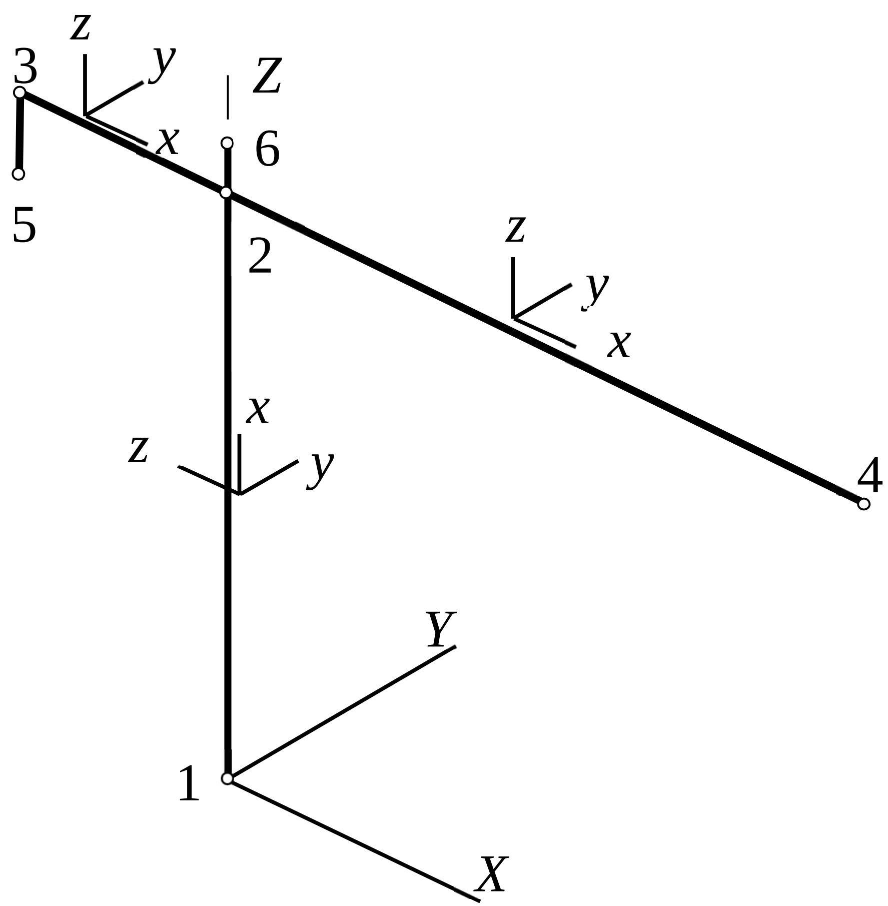


1. Hit the structure without the cable & load to start vibration

2. Simulate cable snap by burning

IDEALIZATION AND PARAMETRIZATION




z
y
3
Z
x
6
5
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y
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x
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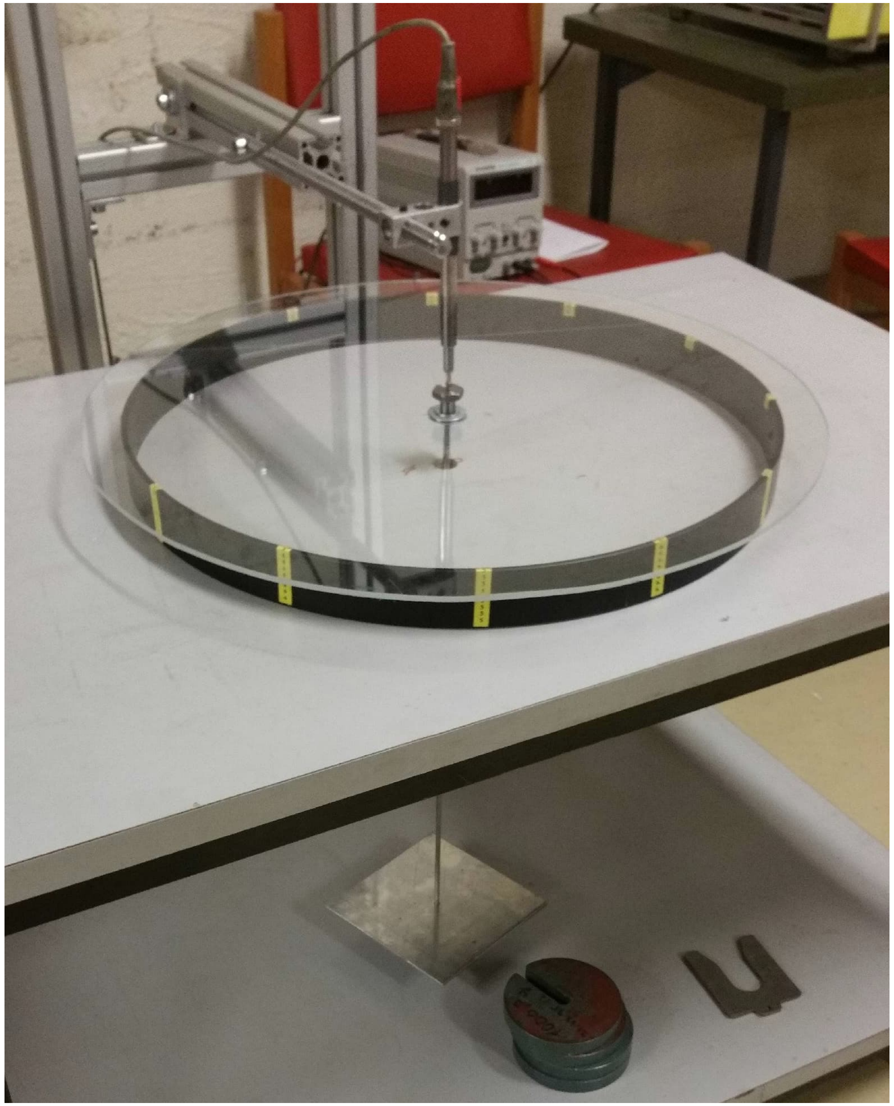
y
4
Y
1
X

LINEAR THEORY MODELLING ERROR

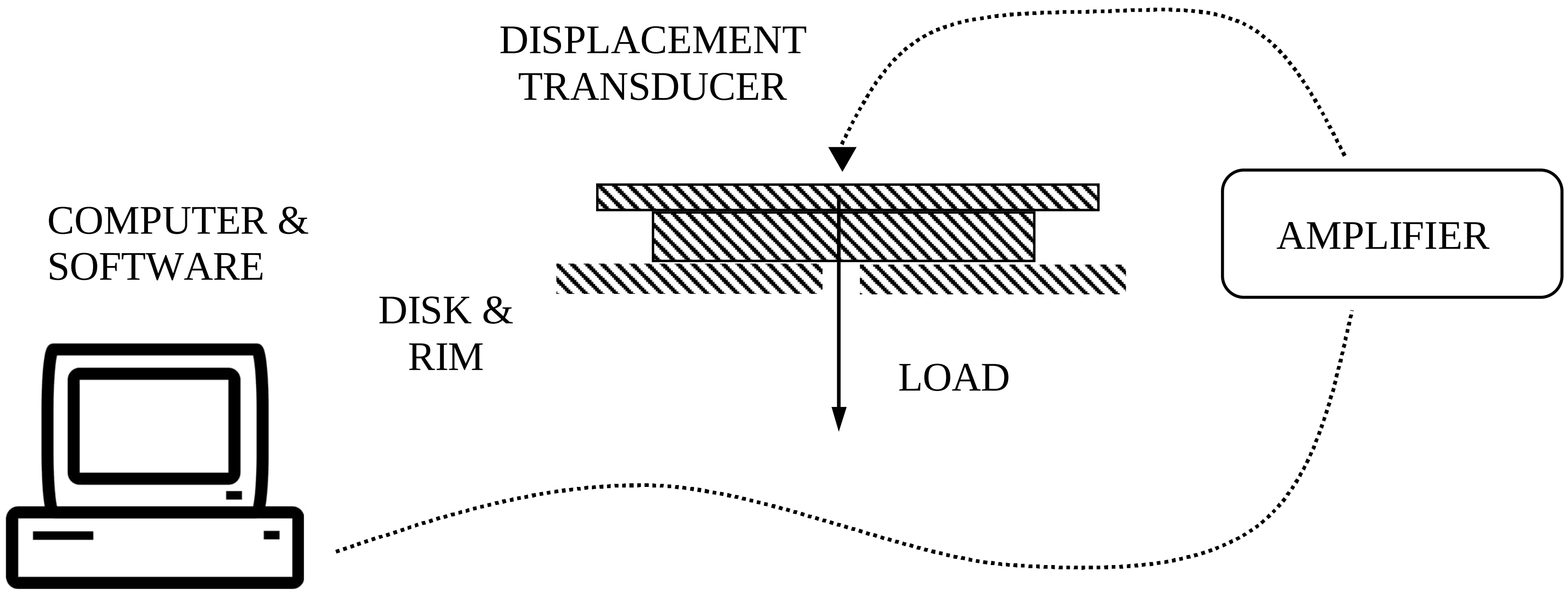


ASSIGNMENT 3

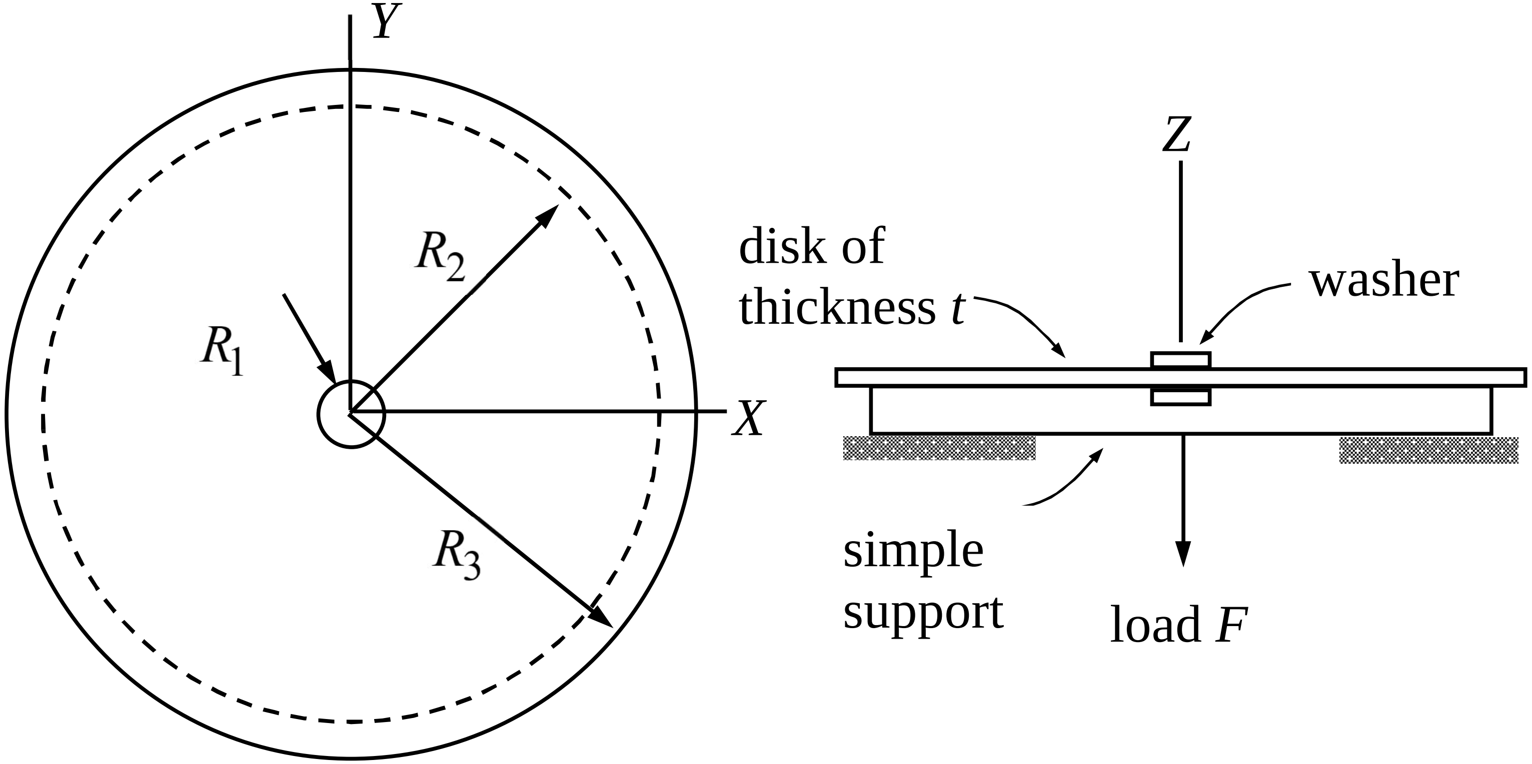
According to linear theory, rigidity of a circular plate is a constant whose value depends on the thickness, radius of the plate and material properties. Experiments indicate, however, that rigidity increases rapidly in the transverse displacement. In the modelling assignment, you will compare the rigidities given by a linear plate model and experiment to find the maximal displacement for an acceptable modelling error. Also, predictions by linear and non-linear models are compared for a picture about the source of the modelling error. A simply supported circular plexiglass plate is used in comparison.

The outcome is a report that should explain the methods used, compare the results obtained using the three methods, and discuss any possible discrepancies between the methods. The report should be detailed enough to enable a reader with some background in experiments and engineering models to follow the derivations and repeat the steps.

EXPERIMENTAL METHOD



IDEALIZATION AND PARAMETERIZATION

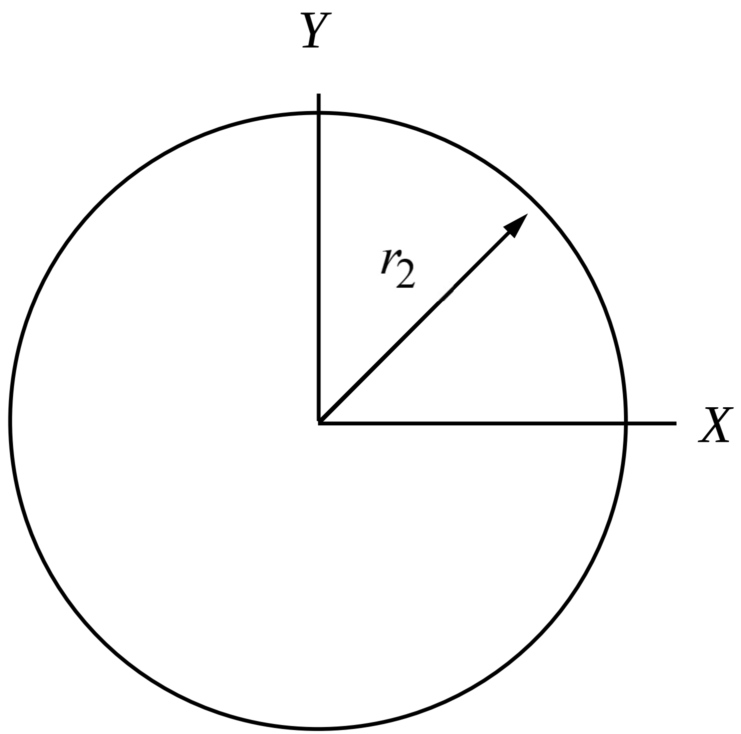


ANALYTICAL METHOD

$$w(r) = a + br^2 + cr^2(1 - \log r) + d \log r$$

$$w - \underline{w} = 0 \quad \text{or} \quad nQ - \underline{Q} = 0$$

$$\theta - \underline{\theta} = 0 \quad \text{or} \quad nM - \underline{M} = 0$$



Rotation: $\theta = -\dfrac{dw}{dr}$

Bending moment: $M = -D\left(\dfrac{d^2w}{dr^2} + \nu \dfrac{1}{r}\dfrac{dw}{dr}\right)$

Shear force: $Q = -D\left(\dfrac{d^3w}{dr^3} + \dfrac{1}{r}\dfrac{d^2w}{dr^2} - \dfrac{1}{r^2}\dfrac{dw}{dr}\right)$

NUMERICAL METHOD