

Density-Functional Theory for Practitioners - Lecture 1

Orlando Silveira Júnior, Adolfo Otero Fumega
and Ondřej Krejčí,
(Developed by Patrick Rinke and Milica Todorović)

Aalto University
School of Science
Department of Applied Physics

Getting to know each other

Go to the white board and write:

- your name
- your country of origin
- a hobby or past-time you enjoy

The course - Basic information

Dates: 5.9 - 1.12.2019

Credits: 5 ECR



MyCourses page:

<https://mycourses.aalto.fi/course/view.php?id=36086>

Time line

Period 1 (5.9. - 14.10.)

Theory
lectures

Mondays

Practical
hands-on
classes

Thursdays

Hands-on classes & check points:
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Period 2 (24.10. - 1.12.)

Project work

Check point

27.10.

Check point

10.11.

Check point

1.12.

Content

Must know	Should know	Nice to know
Basic principles of computational materials modelling	Relation between certain materials properties and different modelling approaches	Pros and cons of different modelling approaches
Basic principles of density-functional theory (DFT)	The exchange-correlation functional	Strengths and limitations of DFT

Content

Must know	Should know	Nice to know
How to use DFT in computational materials modelling	<ul style="list-style-type: none">• Equilibrium structures of materials (e.g., molecules, solids, surfaces)• Elastic properties of materials• Thermodynamics (phase diagrams)	<ul style="list-style-type: none">• Vibrations, phonons and vibrational spectroscopy• Band structures and photo-electron spectroscopy• Dielectric function and optical spectra
Basic familiarity with the DFT software package FHI-aims	FHI-aims and its features	Familiarity with Linux

Learning outcomes

After completion of the course you

- Have developed an understanding of DFT-based materials modelling.
- Are familiar with the FHI-aims DFT software package.
- Can solve simple physics, chemistry and material science problems with DFT.
- Can follow a presentation (e.g. conference or seminar) on DFT results.
- Can plan, execute, document and present a small research project.
- Can give peer feedback.

Time line



Virtual Winter School on Computational Chemistry



NEXT edition February 2023

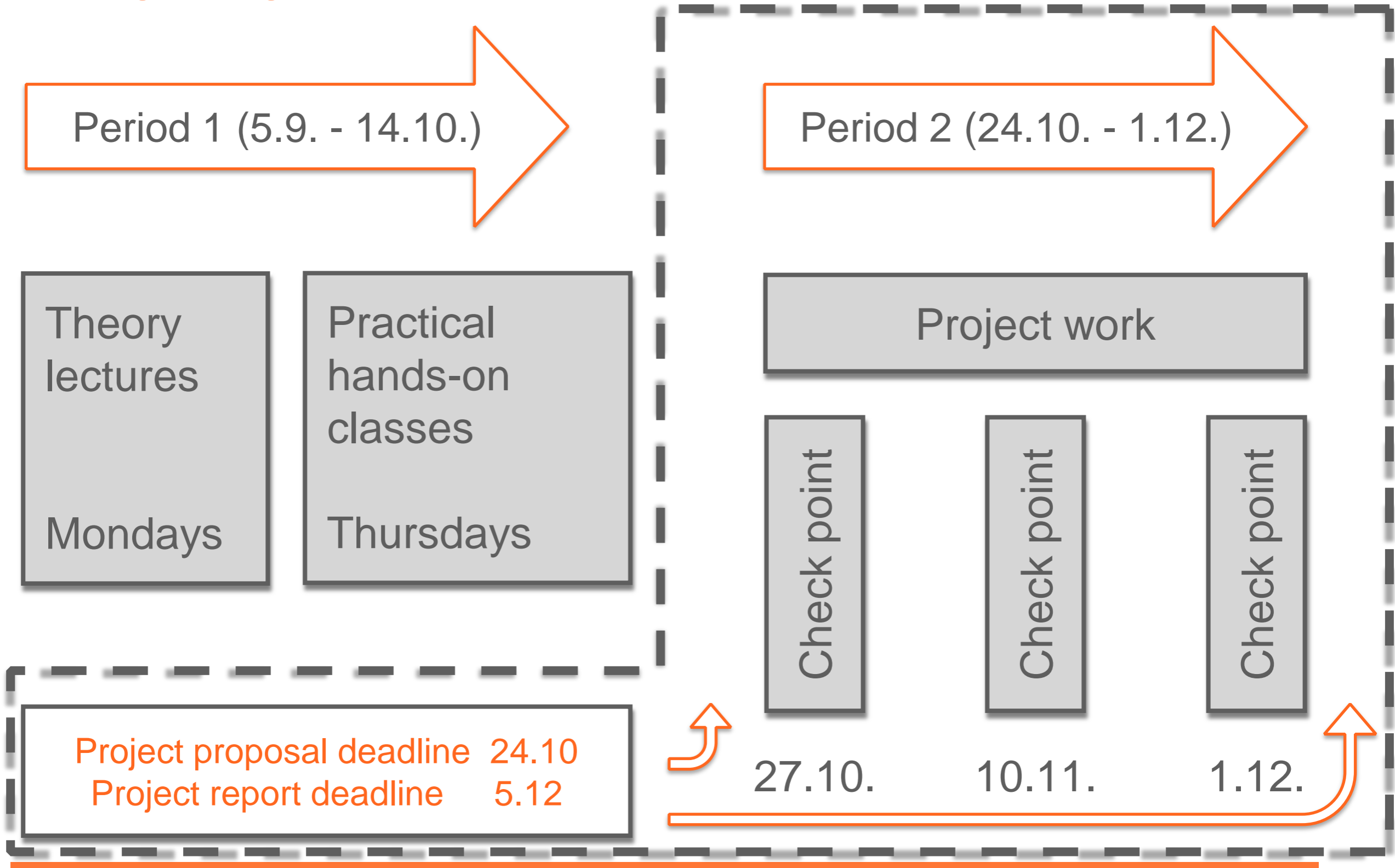
More info at:

<https://winterschool.cc/home/news/winterschool>



Aalto University
School of Science

Time line



Project work

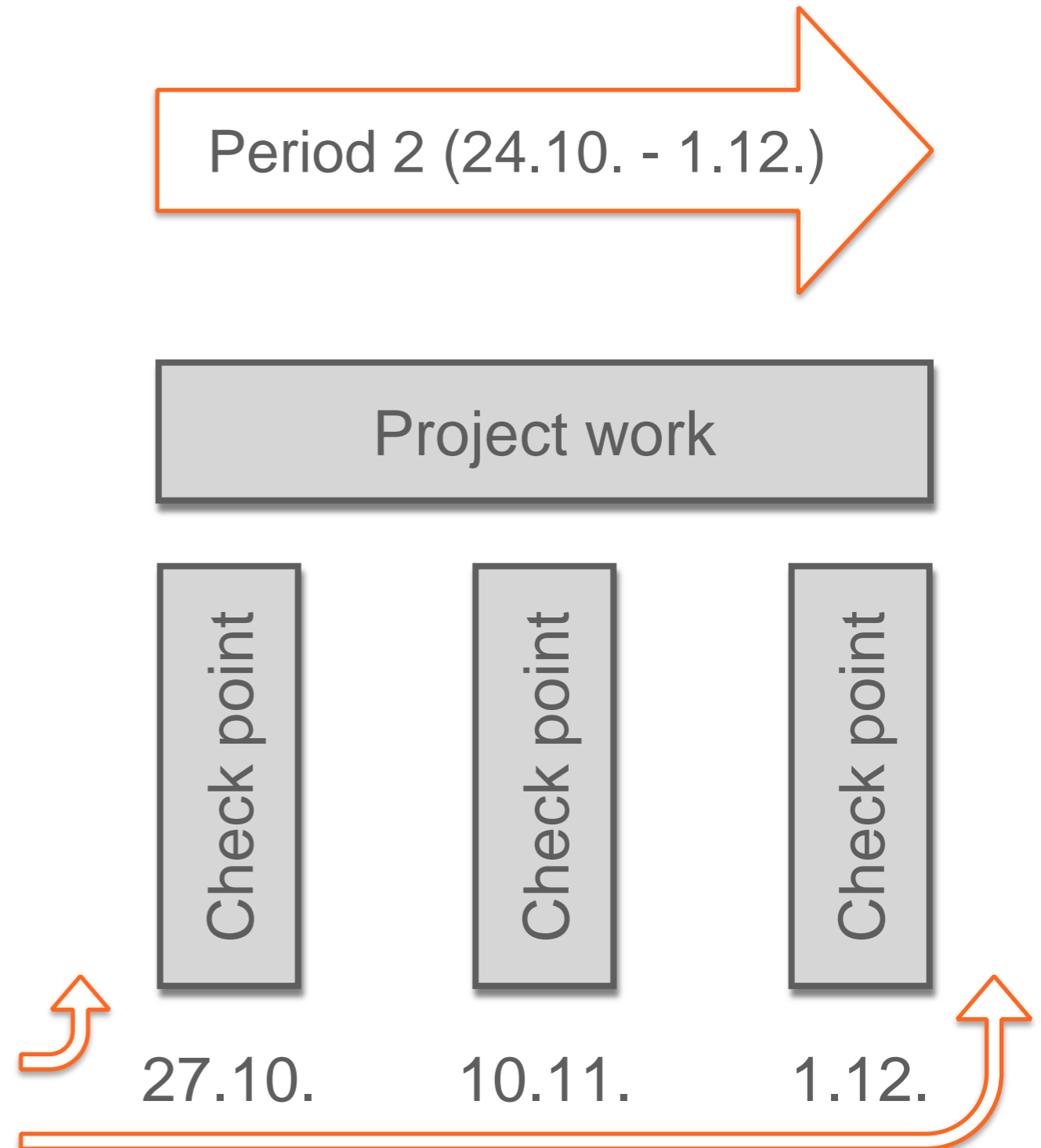
Project:

Propose and solve an original problem with DFT.

The problem can be:

- from your own research.
- something you are interested in.
- something you discover in this course and would like to pursue further.

Must be doable in 6 weeks!



Project timeline

24.10. (end of week 7):

Submit project plan

27.10. contact session:

Project presentations

10.11. contact session:

Progress report presentations

01.12. contact session:

Final results presentations

05.12. dead line

Submit final report

Assessment

assessed by teacher

feedback from teachers

peer feedback

peer/teachers feedback

assessed by teacher

Submit project plan

Project presentations

Progress report presentations

Final results presentations

Submit final report



Course requirements

The 5 ECR credits are awarded if you:

attend at least 5 out of the 6 practical sessions*

submit the project proposal

attend at least 2 out of the 3 project check points*

submit 3 project presentations

complete the project

submit the final project report

*if not fulfilled,
additional
assessment
possible

Work load

Activity	Time per week	Frequency	Total
Period 1			
Contact session	2h	6	12h
Reading for contact session	1h	6	6h
Hands-on session in computer lab	4h	6	24h
Time to think and reflect	3h	6	18h
Planning project work	3h	1	3h
Writing project plan	4h	1	4h
			67h
Period 2 – Project Work			
Contact Session	4h	3	12h
Preparing project plan and progress presentation	3h	3	9h
Project work	7h per week	6	42h
Writing final report	4h	1	4h
			67h
			134h

Questions ?

Period 1 (5.9. - 14.10.)

Theory lectures

Mondays

Practical hands-on classes

Thursdays

Project proposal deadline 24.10
Project report deadline 5.12

Period 2 (24.10. - 1.12.)

Project work

Check point

27.10.

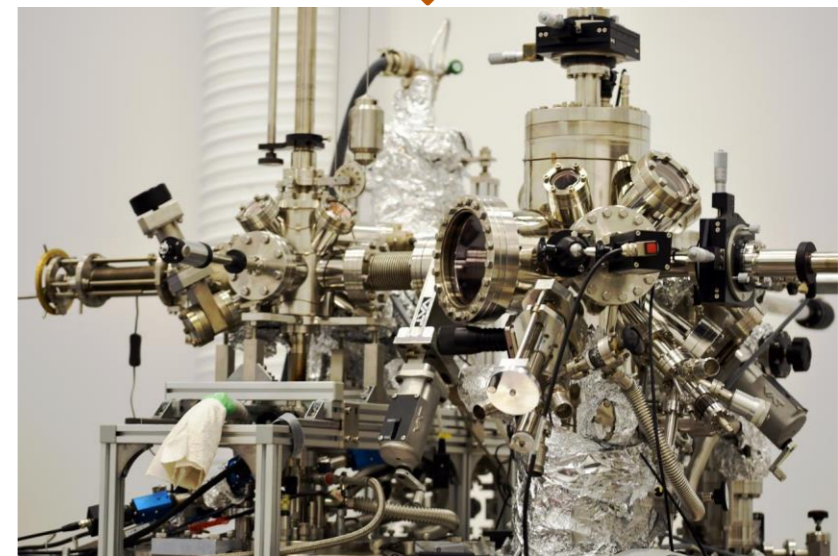
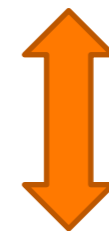
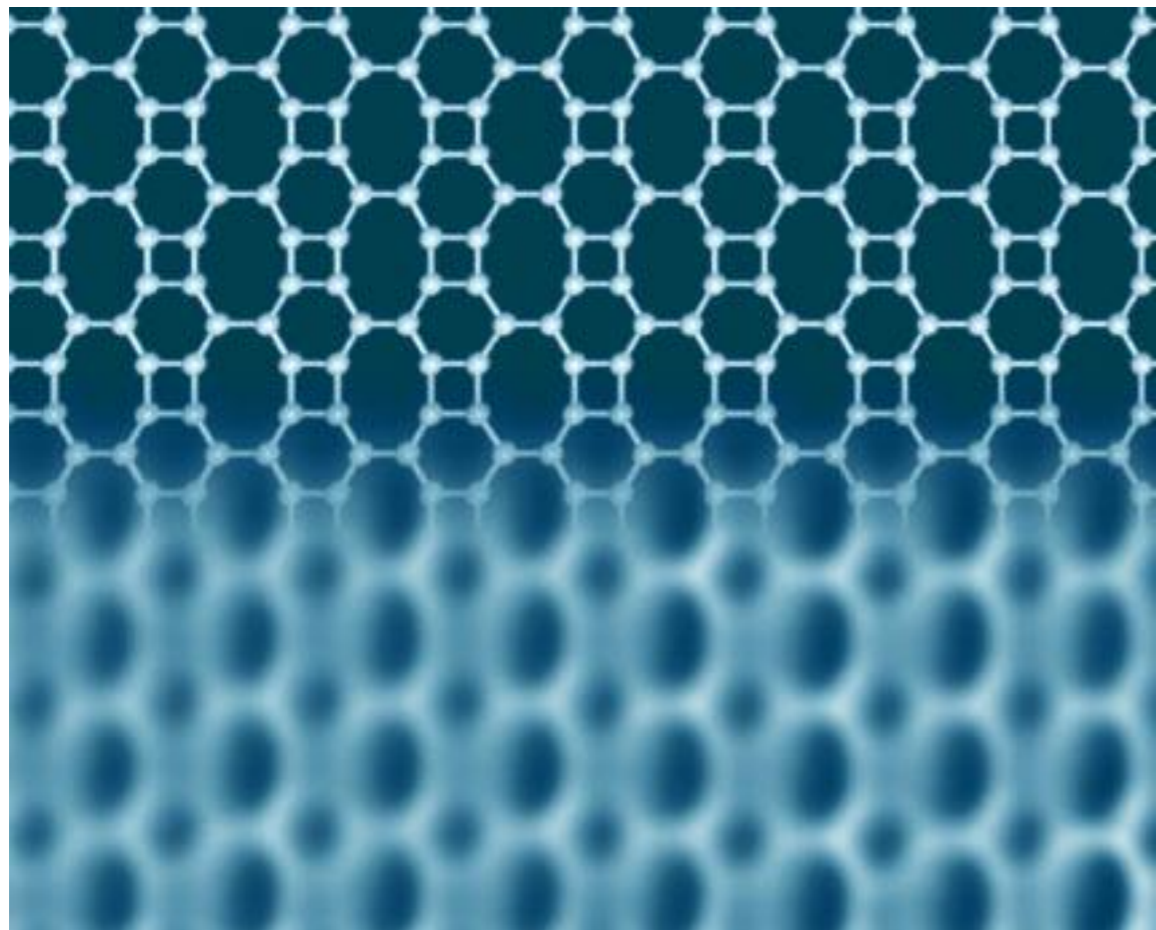
Check point

10.11.

Check point

1.12.

Materials Modelling



This class - content

Must know	Should know	Nice to know
Basic principles of computational materials modelling	Relation between certain materials properties and different modelling approaches	Pros and cons of different modelling approaches
The three “principles” of density-functional theory (DFT)	Variational principle	Quantities that can be calculated with DFT

This class - learning outcomes

After the class today you

- have a first impression of materials modelling as a context for DFT.
- know the three “principles of DFT”.

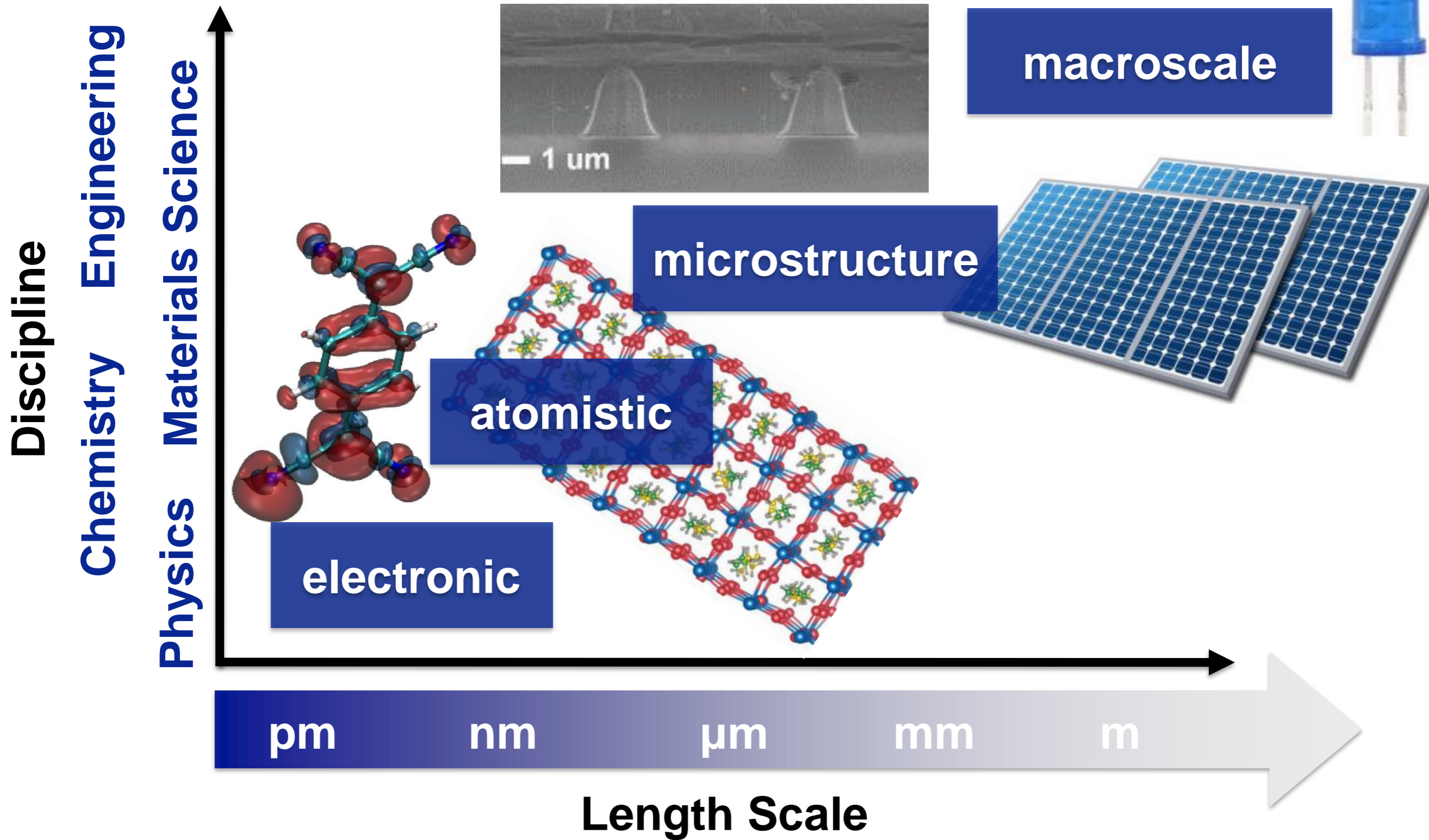
Materials modeling - a discussion

Discuss in your group:

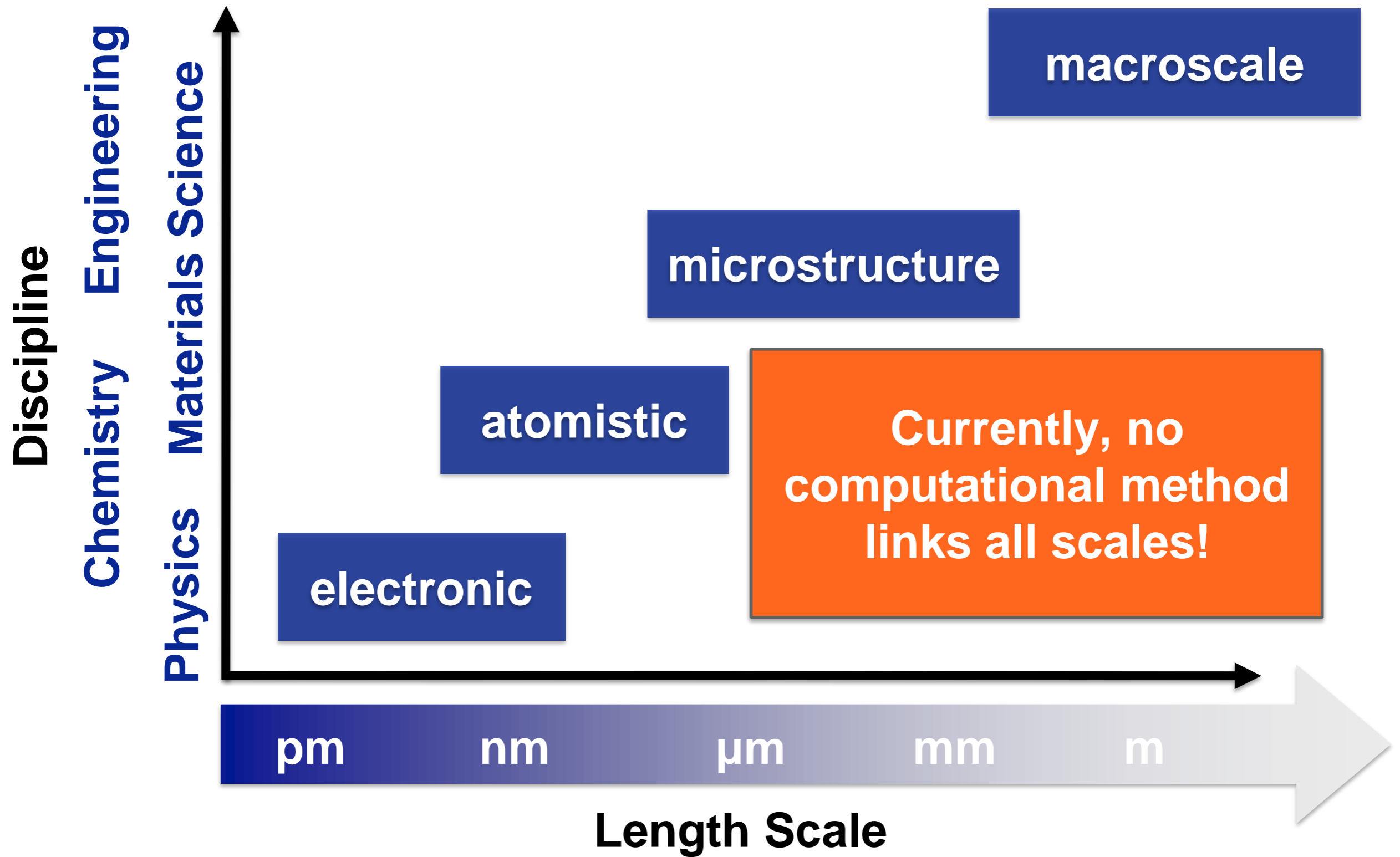
What is materials modelling?

- What can be modelled? (What is modelled in science/industry?)
- What materials modelling approaches do you already know?
- What are the strength and weaknesses of modelling, e.g. compared to experimentally characterising a material?

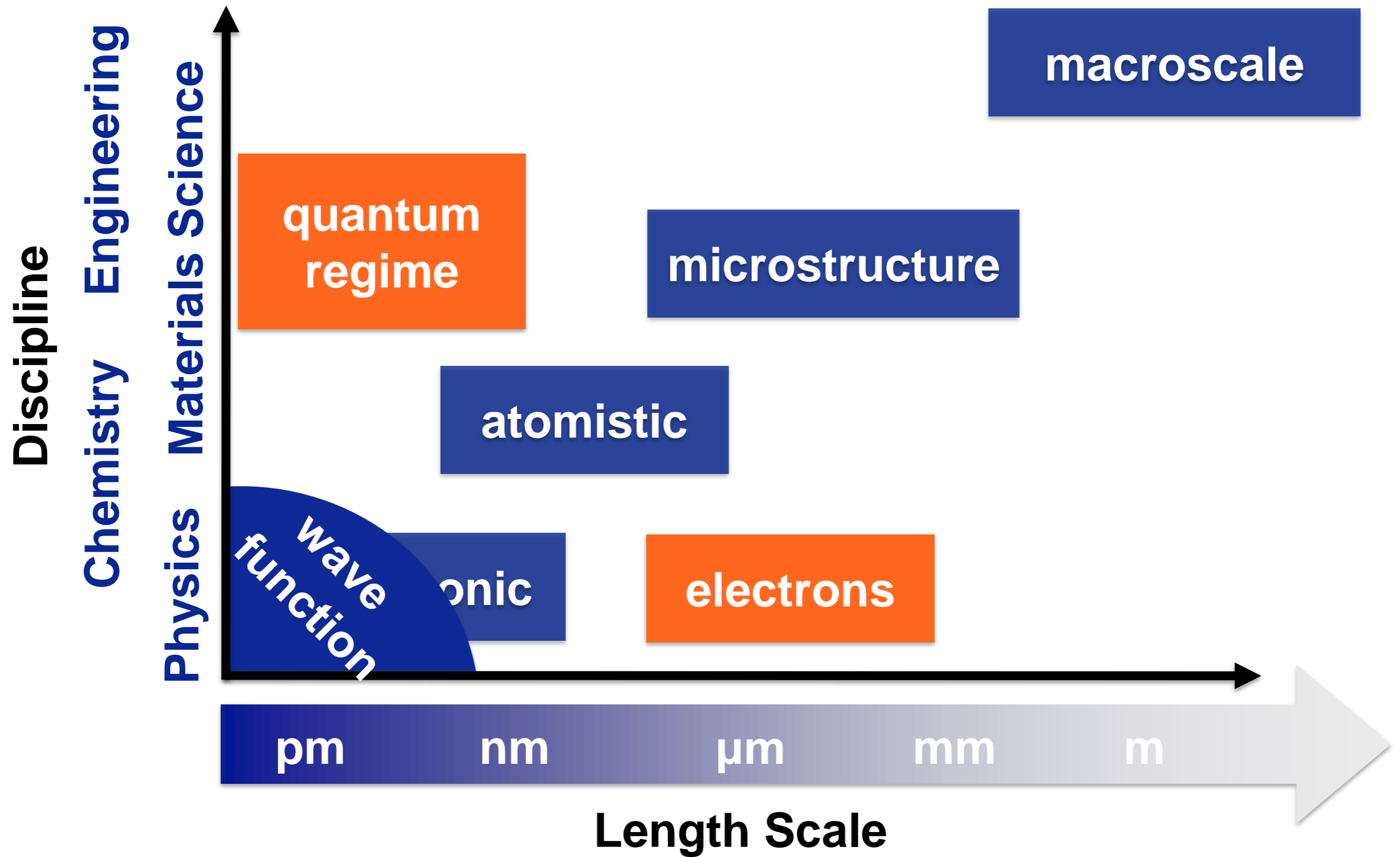
Materials modelling - Length scales



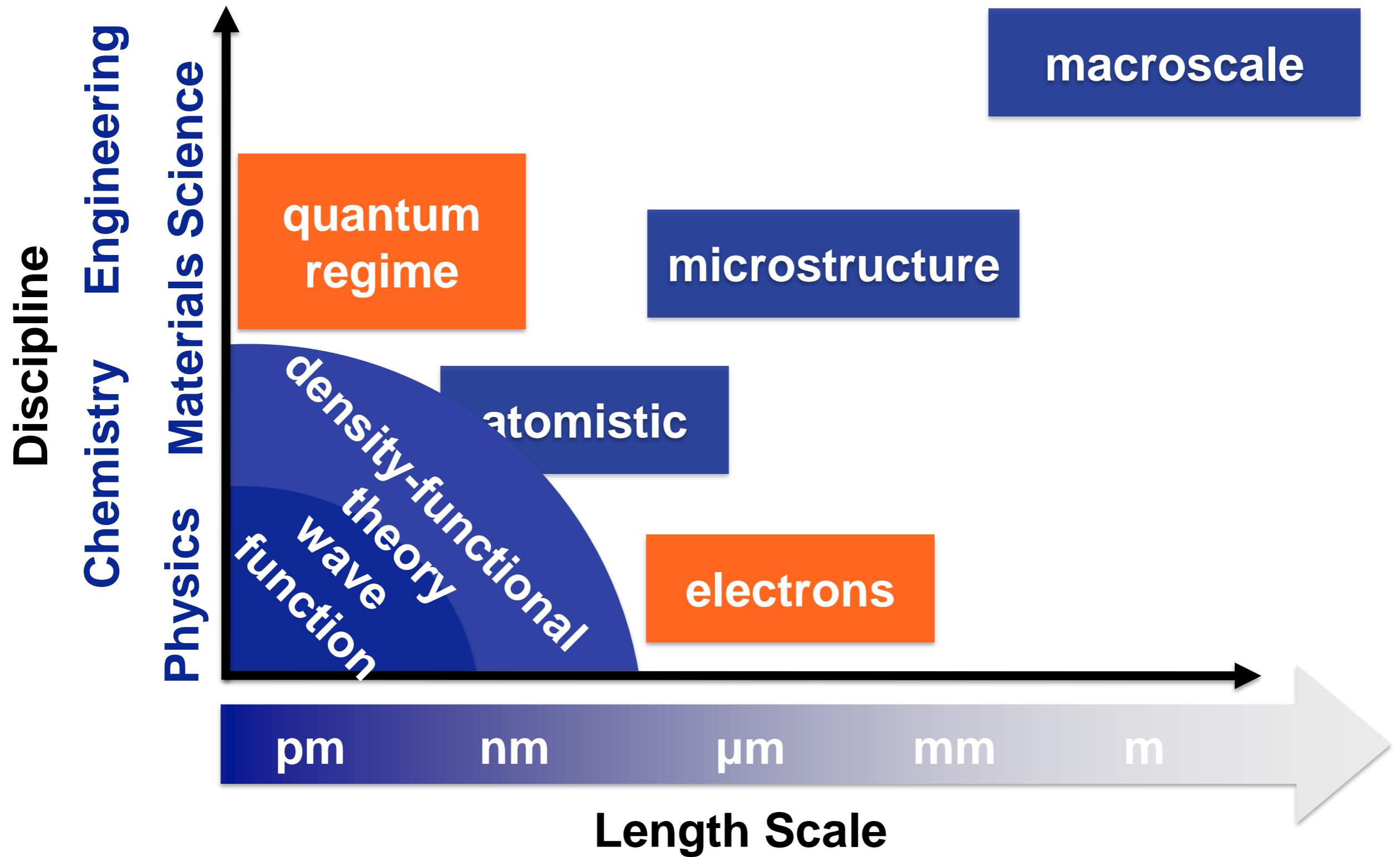
Multi-scale modelling of materials



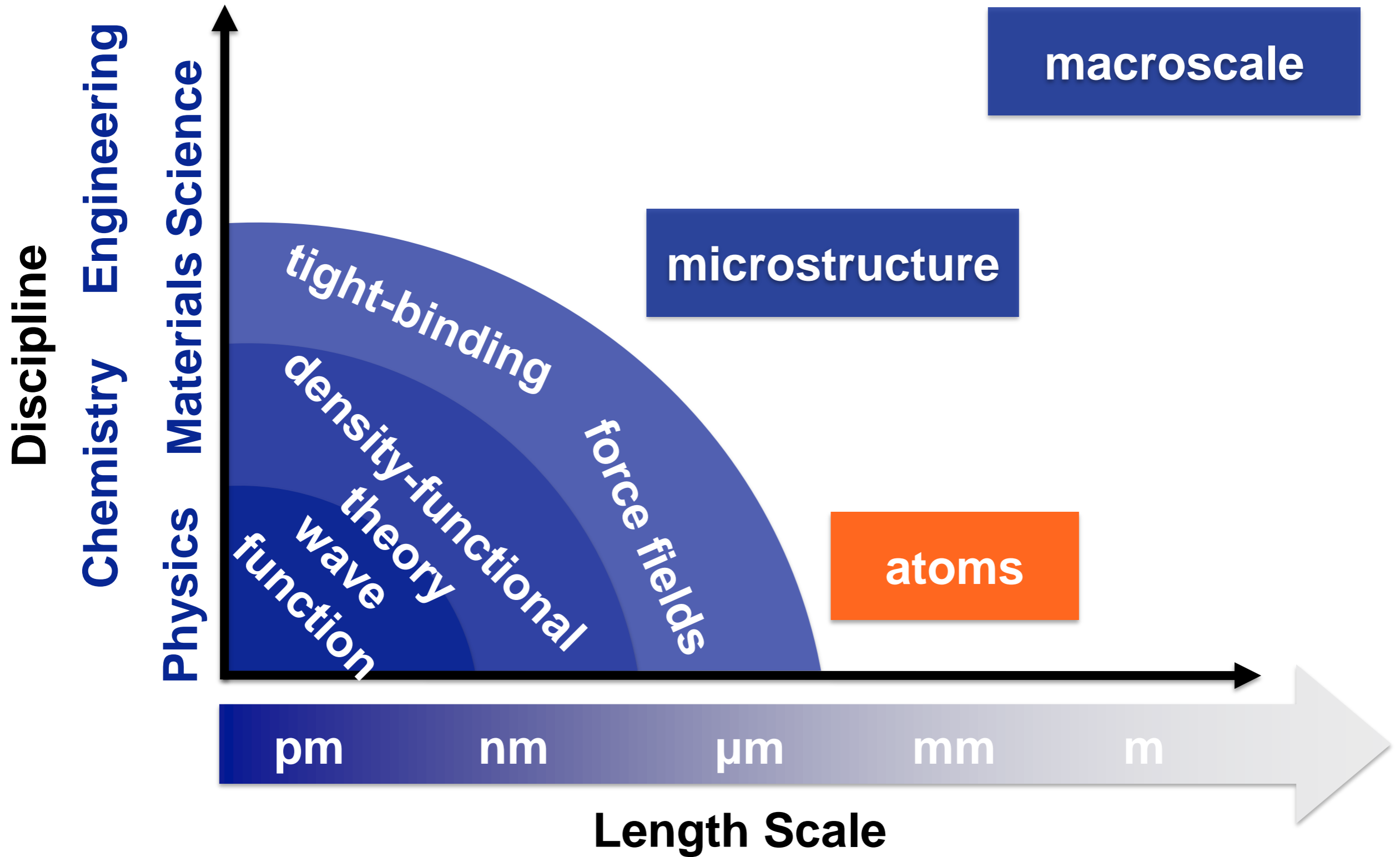
Multi-scale modelling of materials



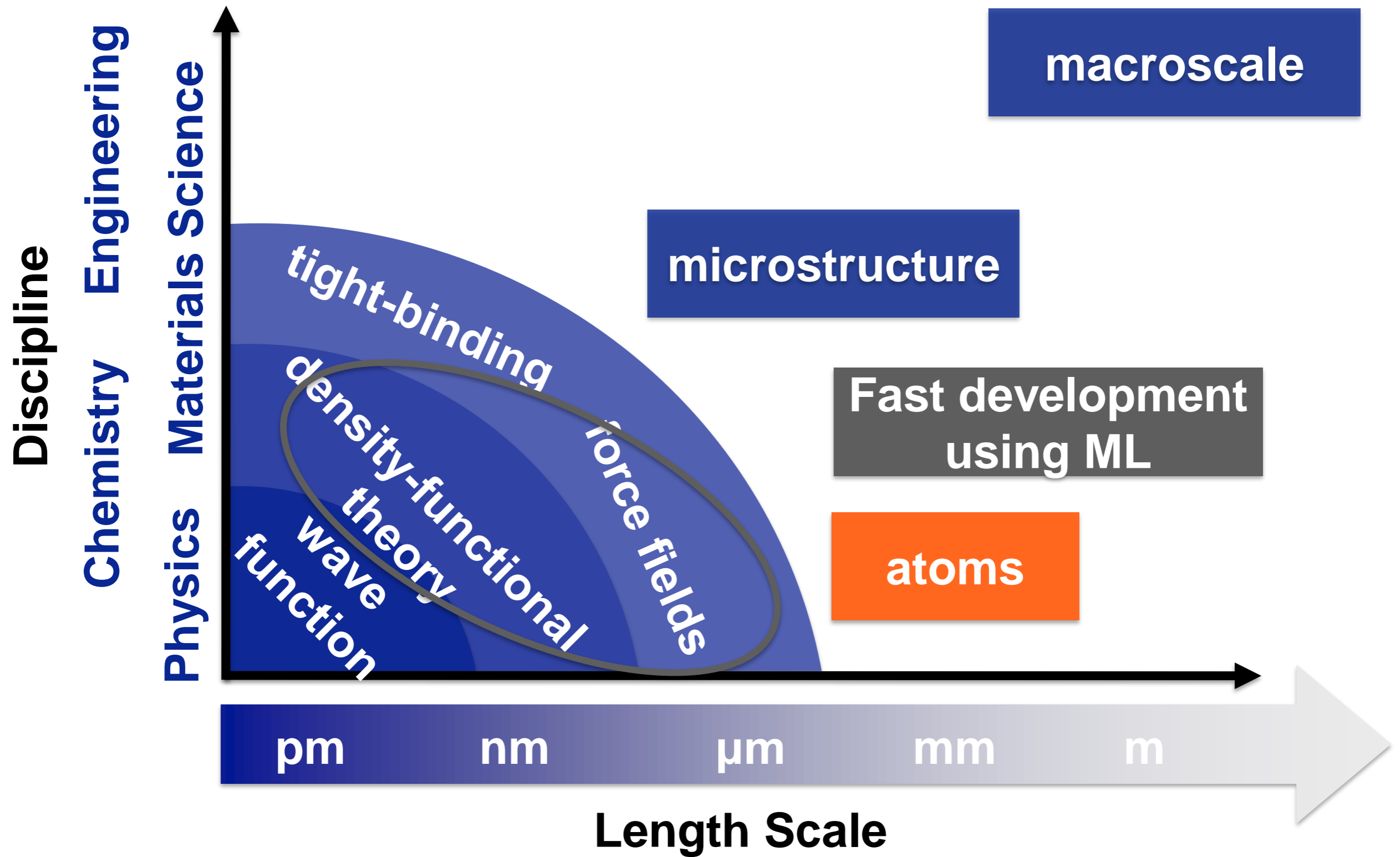
Multi-scale modelling of materials



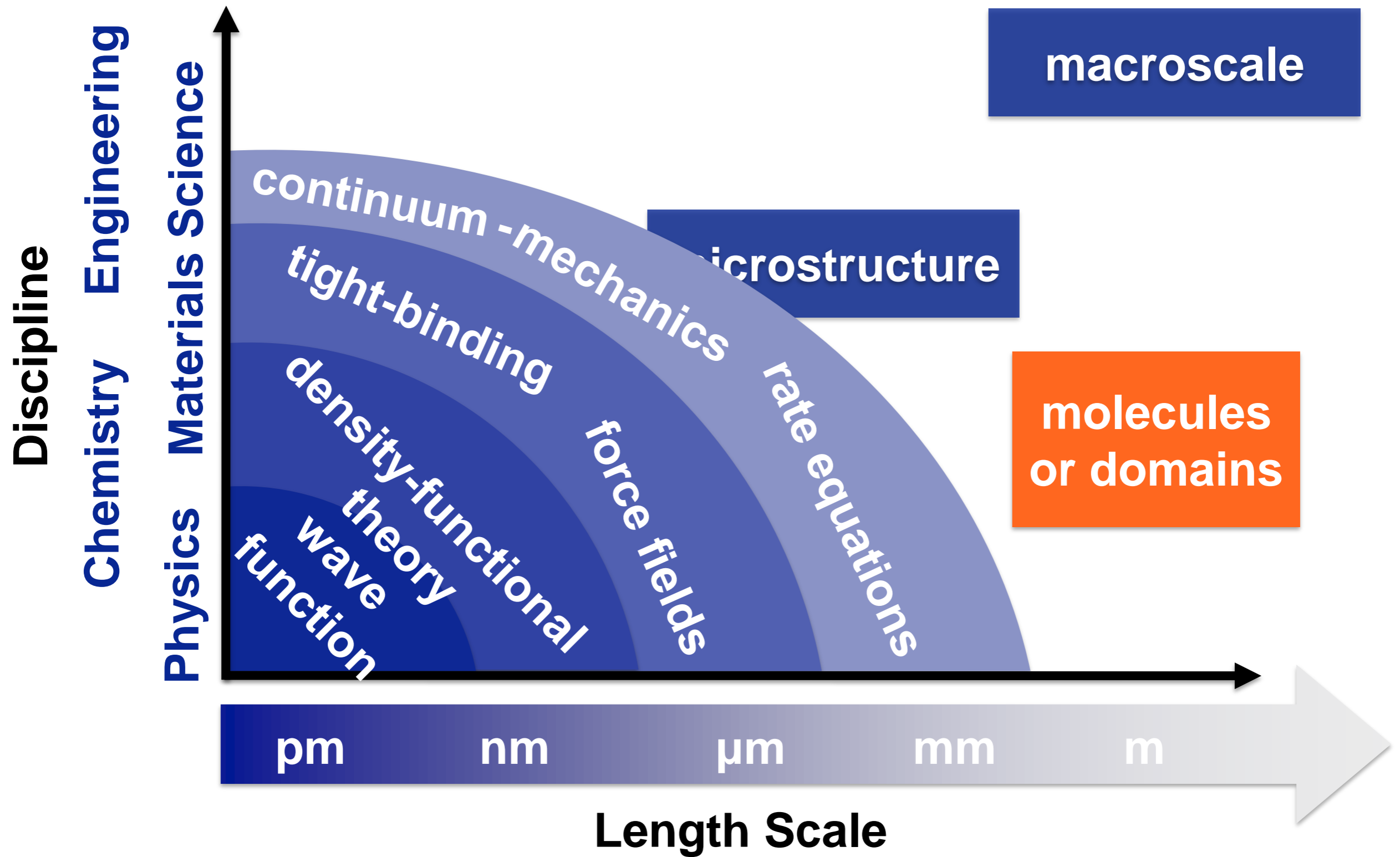
Multi-scale modelling of materials



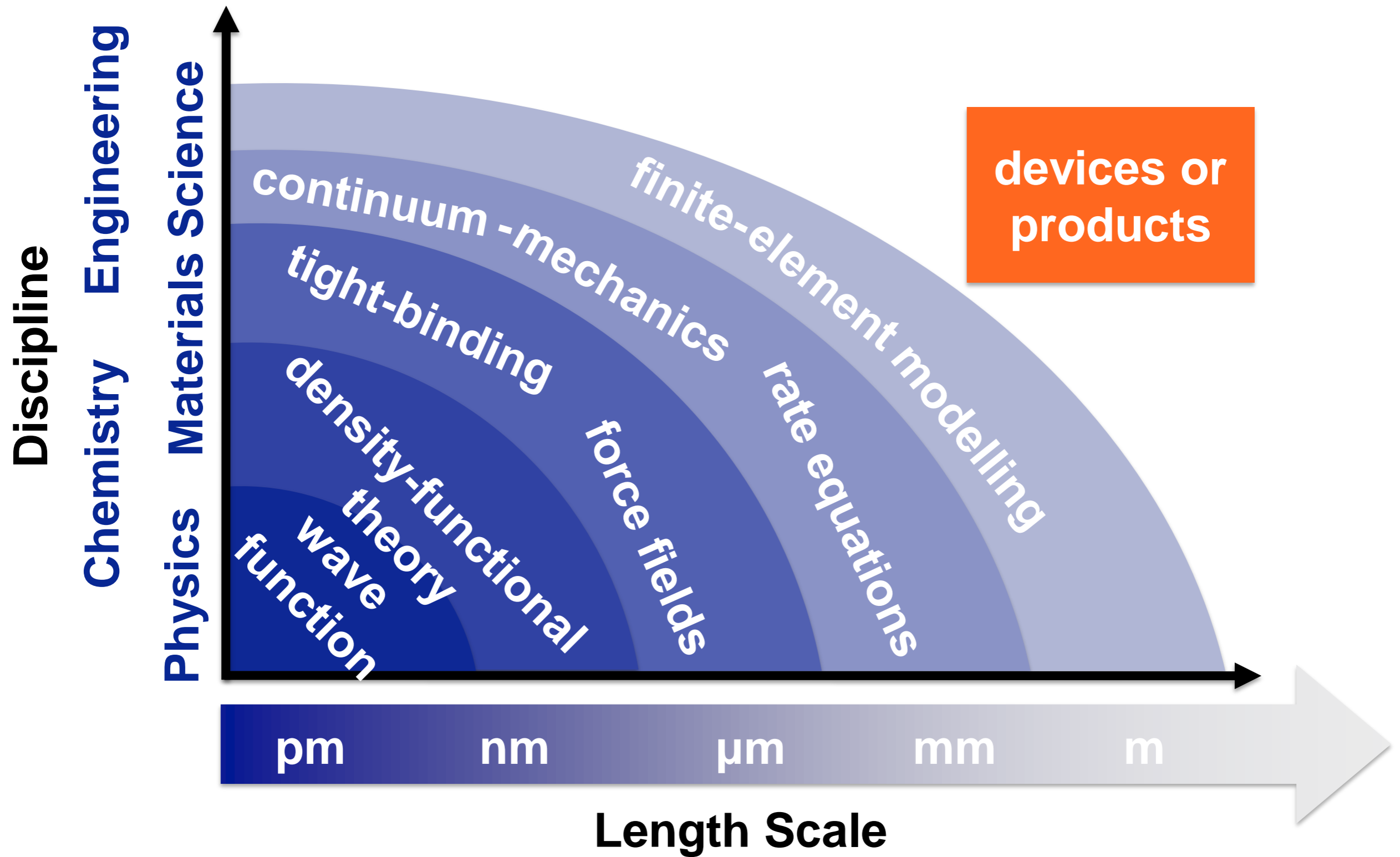
Multi-scale modelling of materials



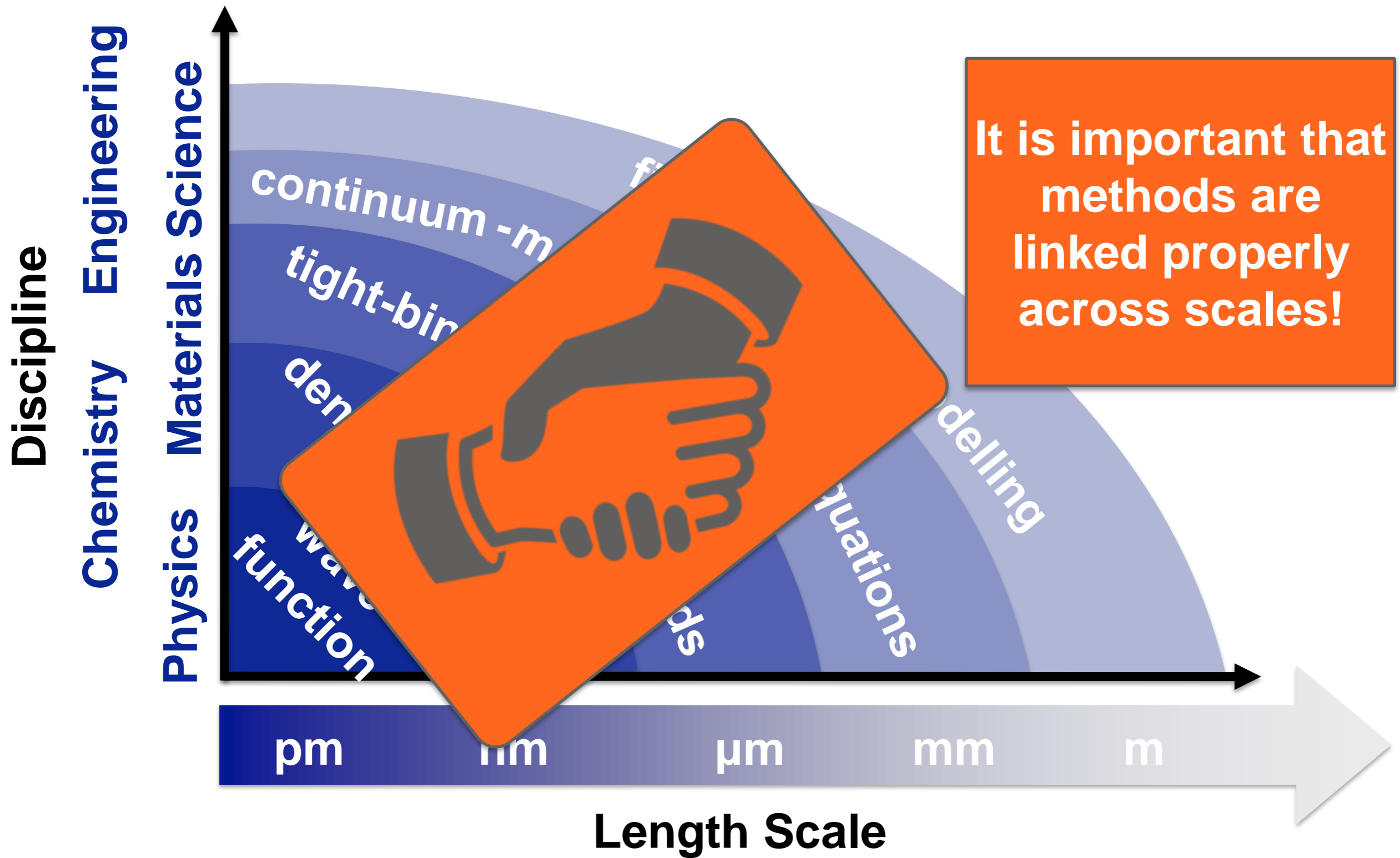
Multi-scale modelling of materials



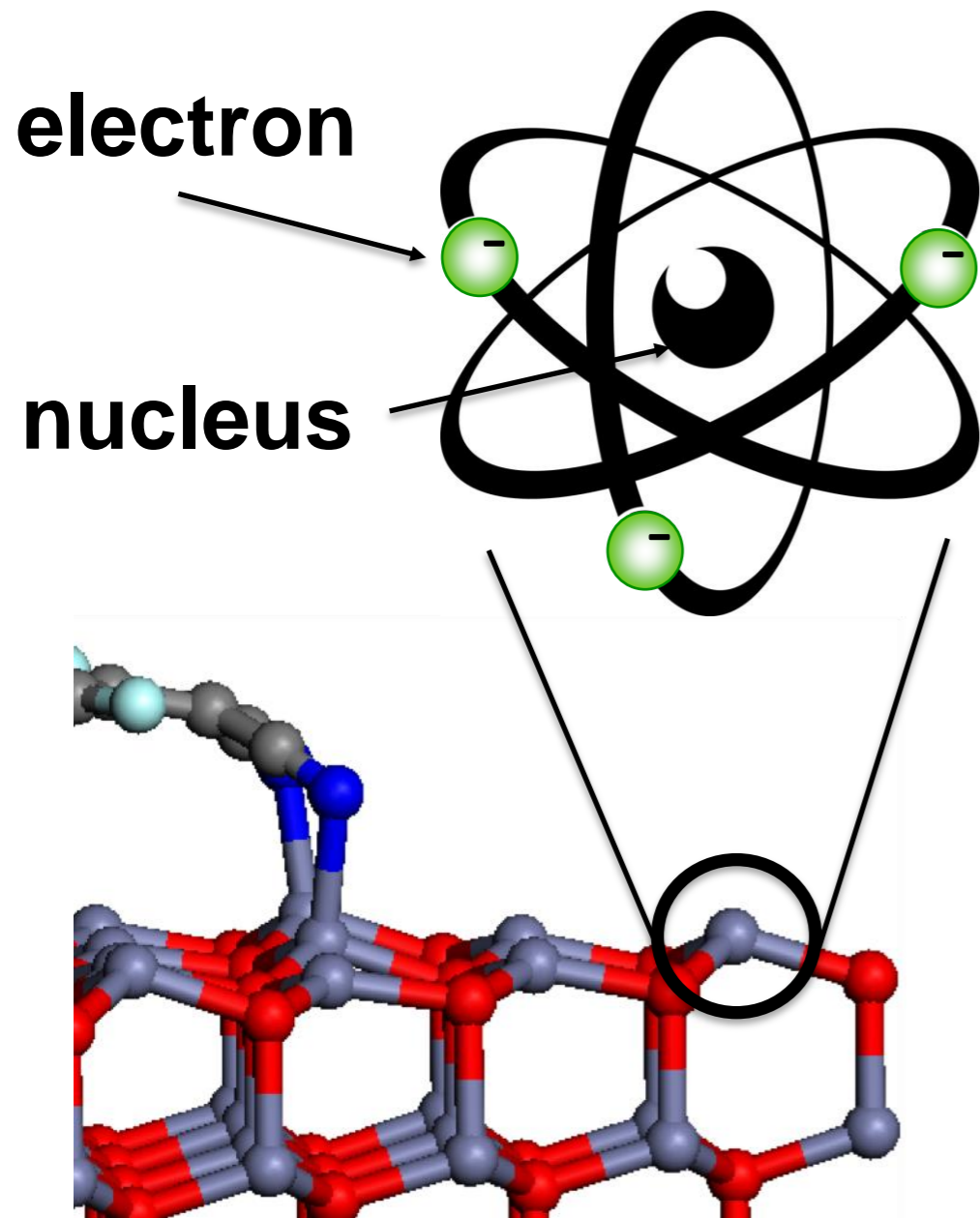
Multi-scale modelling of materials



Multi-scale modelling of materials



Non-relativistic quantum mechanics



Quantum mechanics:

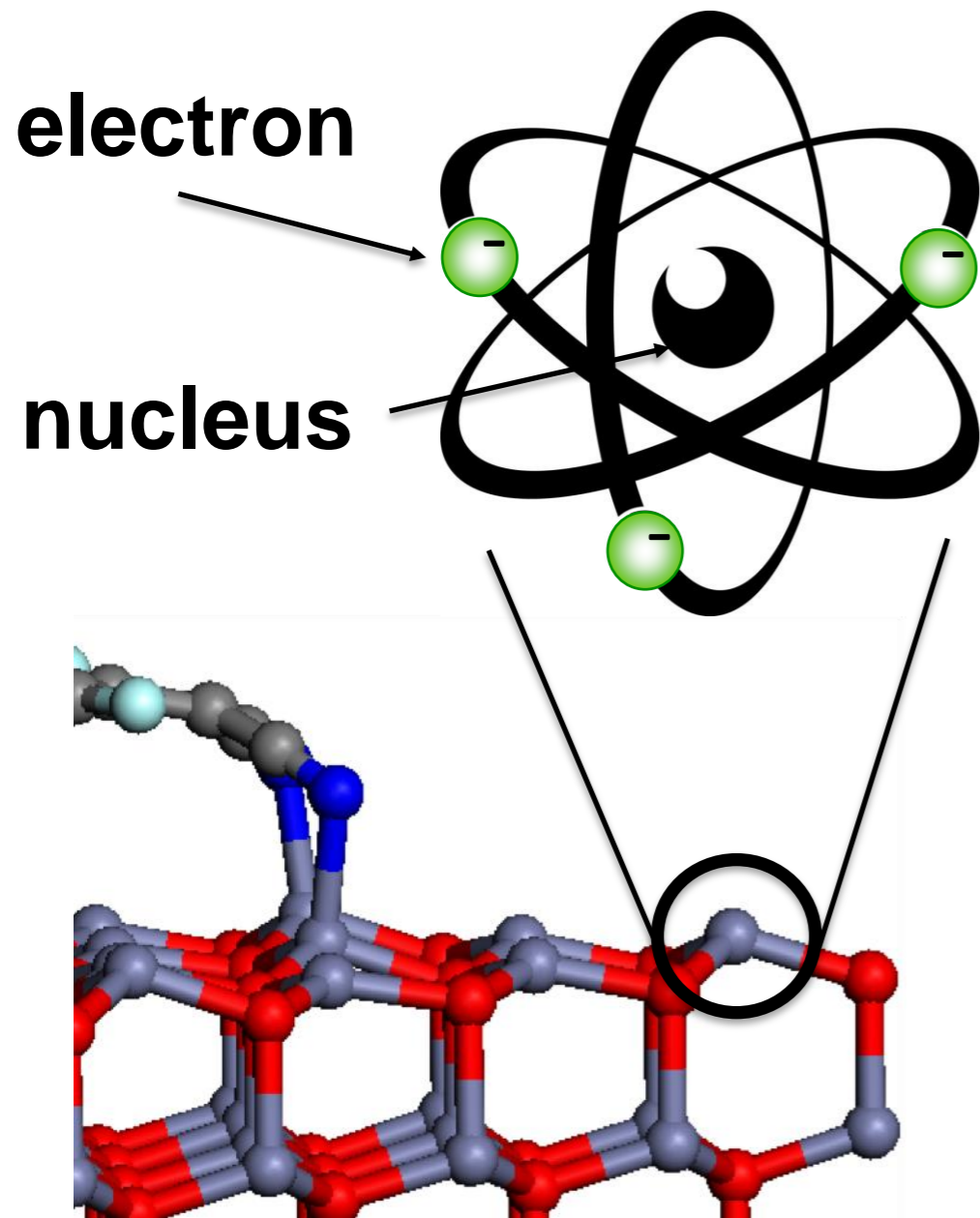
- Schrödinger equation

$$H\Psi = E\Psi$$

↑ wave function ↑ energy

Hamilton operator
(brings in system specific information)

Non-relativistic quantum mechanics



Quantum mechanics:

- Schrödinger equation

$$H\Psi = E\Psi$$

$$H \left(\begin{array}{cccc} \text{grey} & \text{red} & \text{grey} & \dots \end{array} \right)$$
$$\Psi \left(\begin{array}{cccc} \text{grey} & \text{red} & \text{grey} & \dots \end{array} \right)$$

The equations above show the application of the Schrödinger equation to a molecular system. The Hamiltonian operator H and the wavefunction Ψ are represented as vectors of particles. The first row of each vector contains three particles: a grey sphere, a red sphere, and another grey sphere, followed by an ellipsis. The second row contains three green spheres, each with a minus sign, followed by an ellipsis. This represents the interaction of the Hamiltonian with the wavefunction across different particle coordinates.

The full Hamiltonian

$$H(\text{●} \text{●} \text{●} \dots \text{●}^- \text{●}^- \text{●}^- \dots)$$

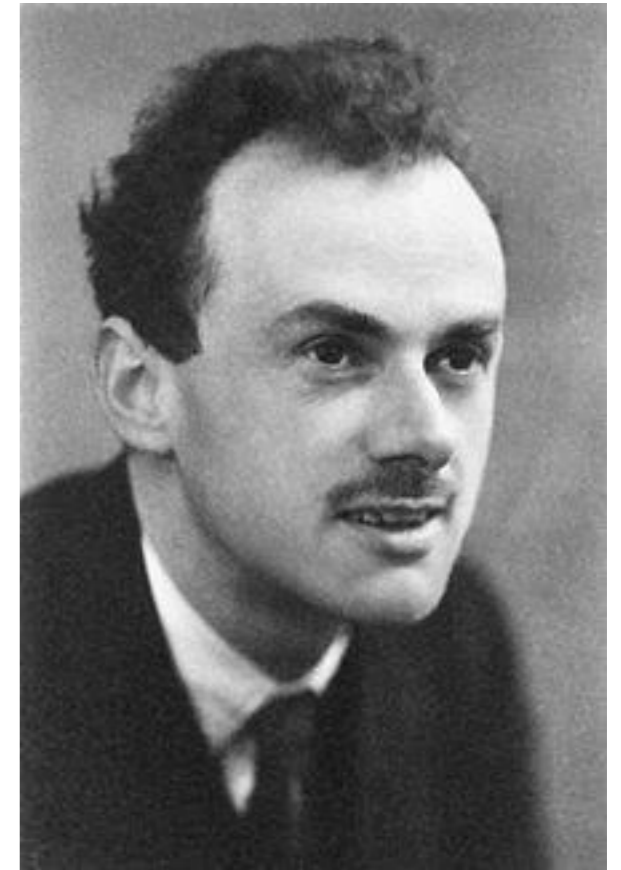
$$\mathcal{H} = \underbrace{-\sum_{I=1}^{N_n} \frac{\nabla_{\mathbf{R}_I}^2}{2M_I}}_{T^n} + \underbrace{\sum_{I=1}^{N_n} \sum_{J>I}^{N_n} \frac{Z_I Z_J}{|\mathbf{R}_I - \mathbf{R}_J|}}_{V^{nn}}$$

$$\underbrace{-\sum_{i=1}^{N_e} \frac{\nabla_{\mathbf{r}_i}^2}{2}}_{T^e} + \underbrace{\sum_{i=1}^{N_e} \sum_{j>i}^{N_e} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}}_{V^{ee}} + \underbrace{\sum_{i=1}^{N_e} \sum_{J=1}^{N_n} \frac{-Z_J}{|\mathbf{r}_i - \mathbf{R}_J|}}_{V^{ne}}$$

Dirac's challenge (1929)

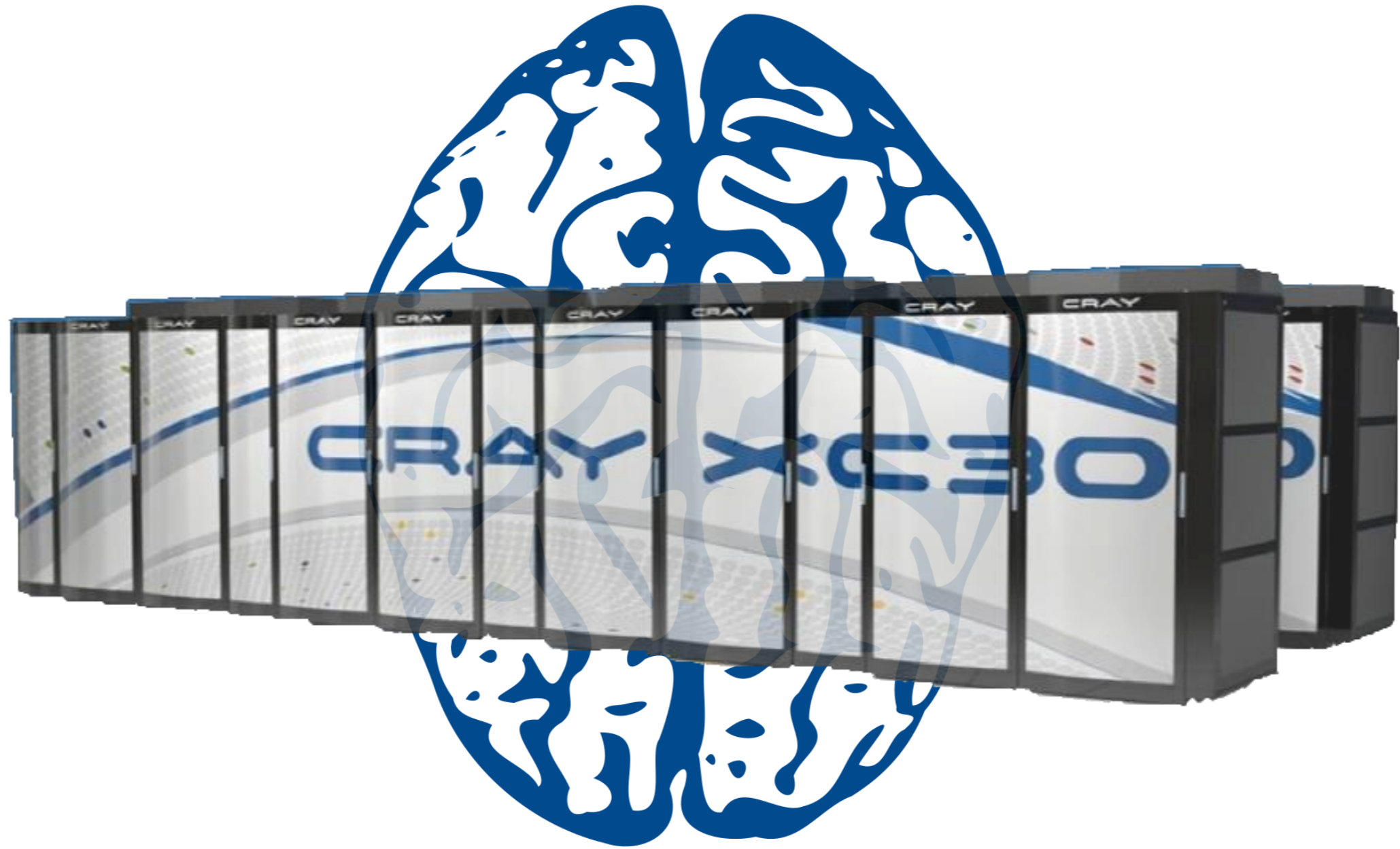
The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus **completely known**, and the difficulty is only that the exact application of these laws leads to equations much **too complicated** to be soluble.

It therefore becomes desirable that **approximate practical methods** of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems without too much computation.



Paul Dirac

What Dirac didn't know...



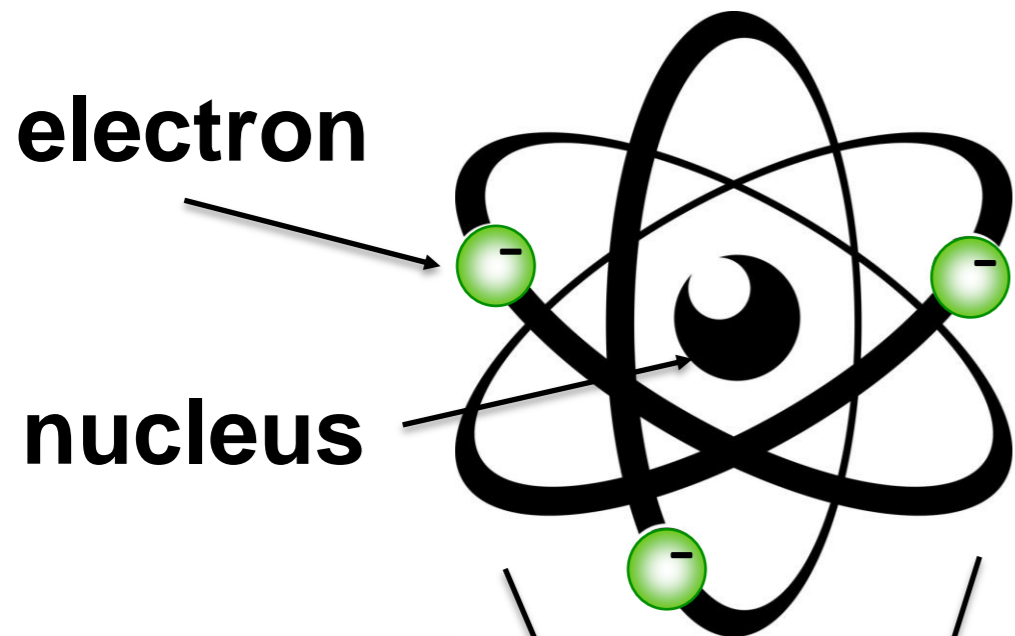
What Dirac didn't know...



New concepts!



Non-relativistic quantum mechanics

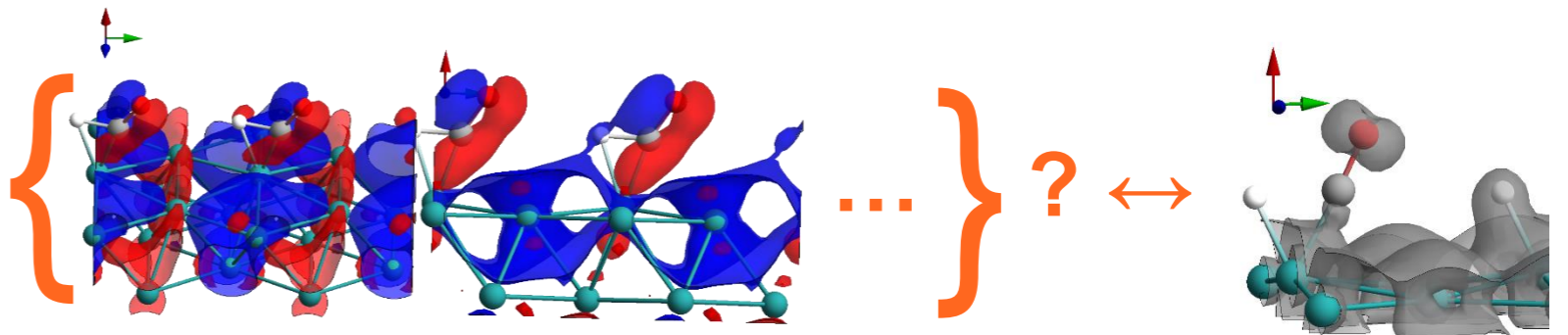
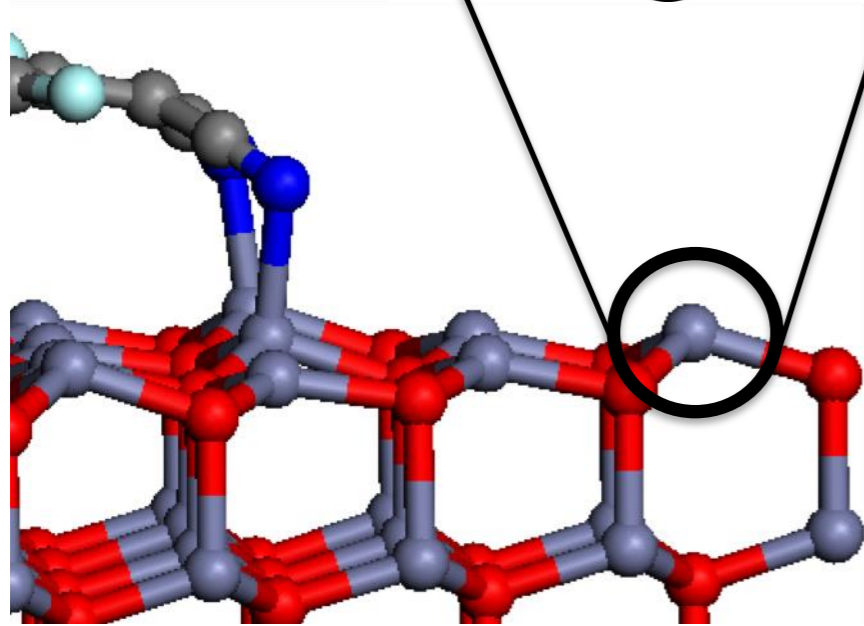


Quantum mechanics:

- Schrödinger equation

$$H\Psi = E\Psi$$

$$\Psi(\text{grey sphere, red sphere, grey sphere, } \dots, \text{green sphere}^-, \text{green sphere}^-, \text{green sphere}^-, \dots)$$



DFT - Hohenberg-Kohn theorems

Theorem 1:

The ground state density $n_0(\mathbf{r})$ uniquely determines the potential up to an arbitrary constant.



Walter Kohn

$$\begin{array}{ccc} \bullet & \bullet & \bullet \\ \color{red}\bullet & & \color{red}\bullet \\ \bullet & & \bullet \end{array} \quad \begin{array}{ccc} \ominus & \ominus & \ominus \\ \ominus & & \ominus \\ \ominus & & \ominus \end{array}$$
$$v_{\text{ext}}(\mathbf{r}) \Leftrightarrow n_0(\mathbf{r})$$

$$-\underbrace{\sum_{i=1}^{N_e} \frac{\nabla_{\mathbf{r}_i}^2}{2}}_{T^e} + \underbrace{\sum_{i=1}^{N_e} \sum_{j>i}^{N_e} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}}_{V^{ee}} + \underbrace{\sum_{i=1}^{N_e} \sum_{J=1}^{N_n} \frac{-Z_J}{|\mathbf{r}_i - \mathbf{R}_J|}}_{V^{ne}}$$



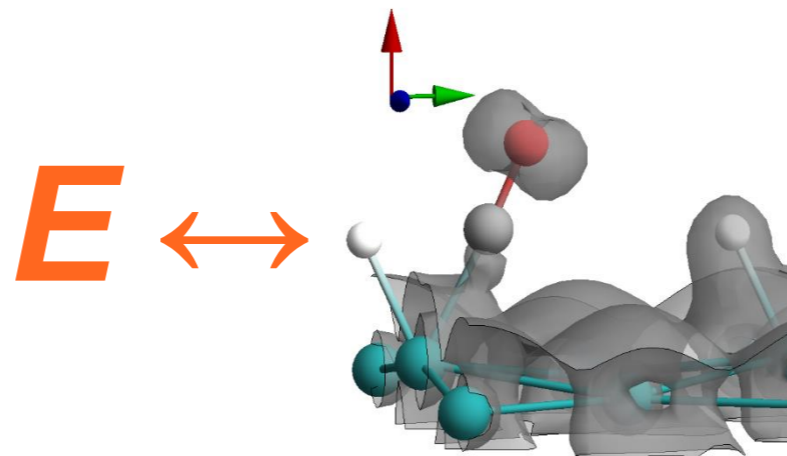
Pierre Hohenberg

DFT - Hohenberg-Kohn theorems

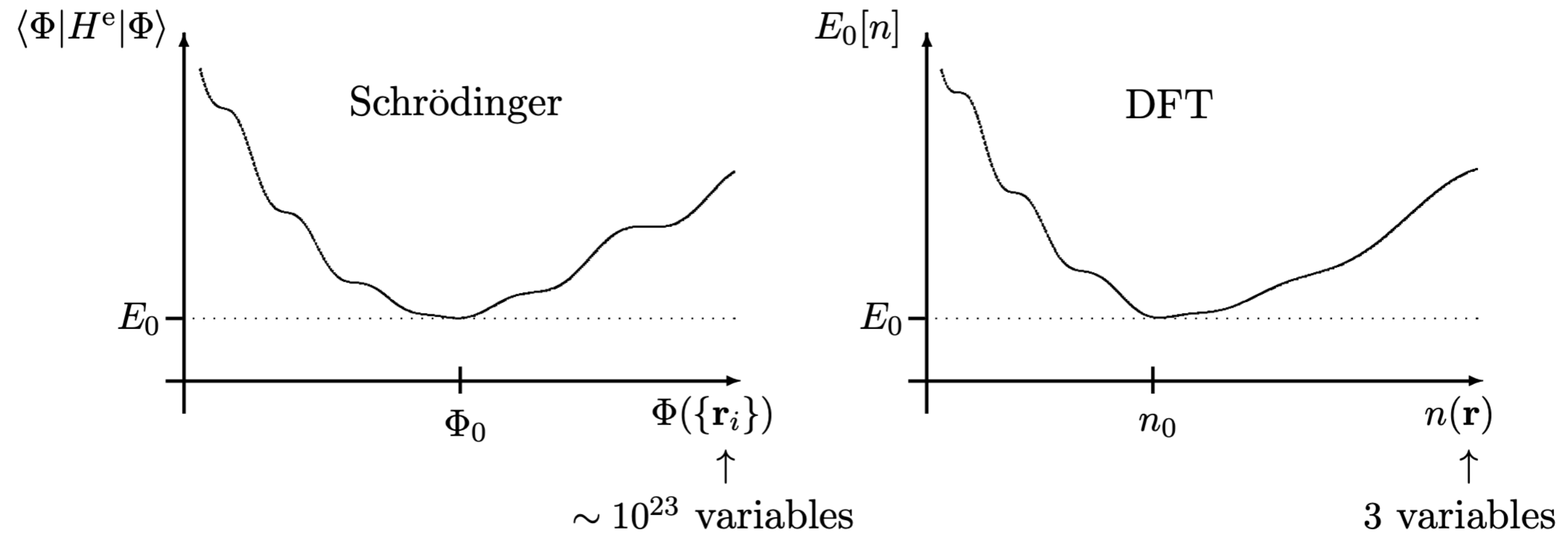
Theorem 2:

A universal functional for the energy $E[n]$ in terms of the density $n(r)$ can be defined, valid for any external potential $v_{ext}(r)$.

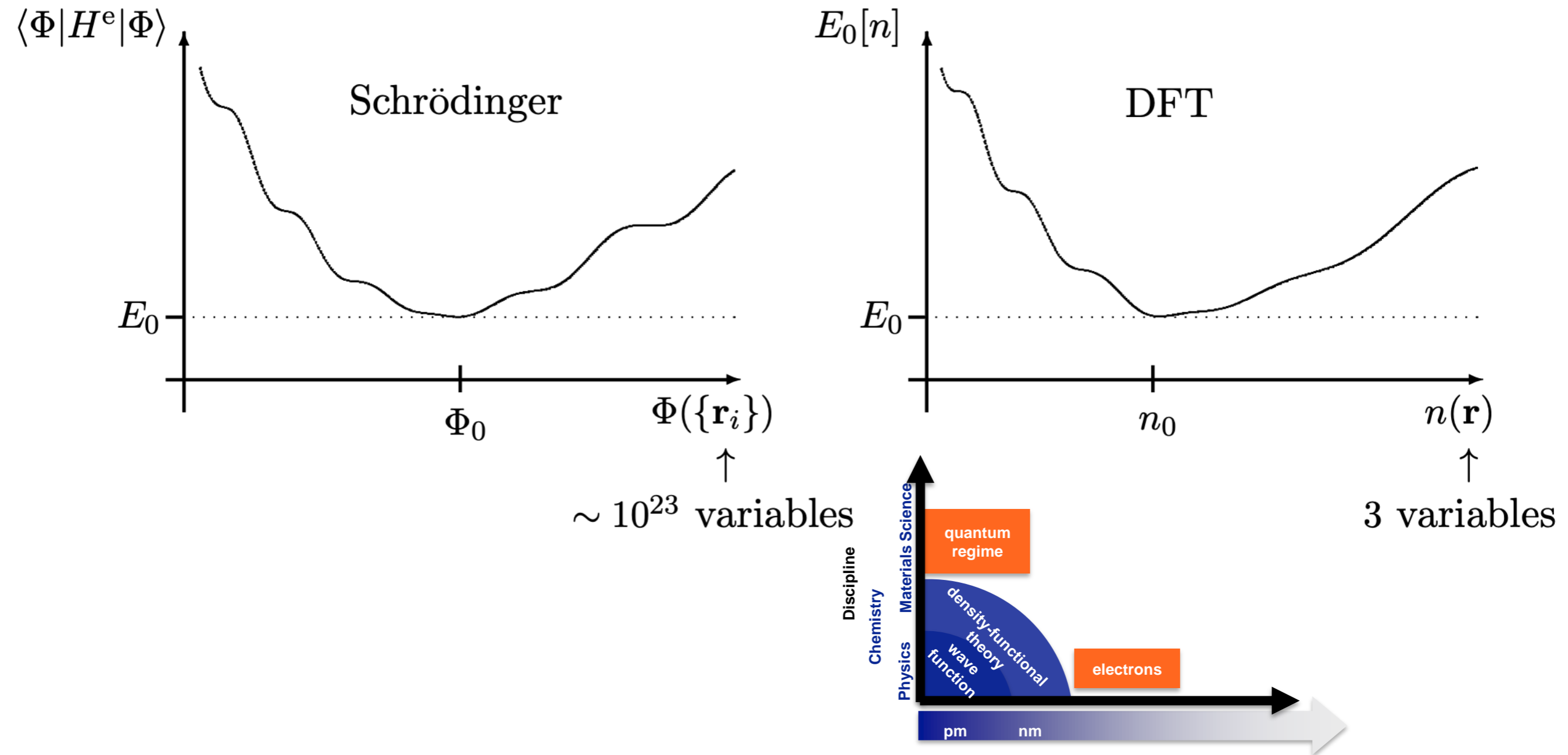
For any given $v_{ext}(r)$, the exact ground state energy of the system is the global minimum of this functional and the density that minimizes the functional is the exact ground state density.



DFT - Variational principle



DFT - Variational principle



The three *principles* of DFT

1: DFT is in principle an exact ground state theory.

**2: The energy is a functional of the density.
The ground state energy is attained at the ground state density.**

We work with this concept in this course.

3: The energy functional needs to be approximated in practice.

The approximations will be subject of the next course in fall 2023.

Questions, comments, ideas?

- Problem with study rights?
- Interesting materials modellings:
 - Quantum chem. in pharmacy:
<https://youtu.be/9DESuICWbRQ>
 - DFT in industry for Quantum Computing:
<https://quantum.hrl.com/>
 - Biology ADP-to-ATP:
<https://youtu.be/kXpzp4RDGJI>
 - FEM microcracks in metals:
<https://youtu.be/zhIKWleOjBY>

