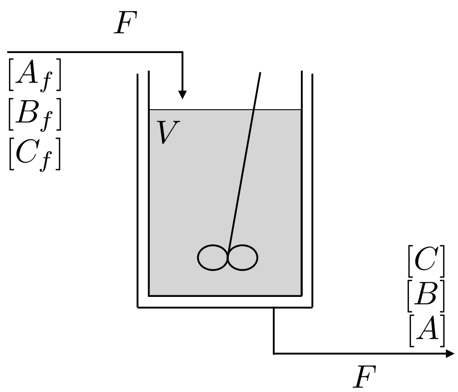
[October 25, 2019] **CHEM-E7140 (Exam)**

Exercise 01 (50%). Consider the isothermal continuous stirred tank reactor in the figure. Suppose that the following irreversible reactions take place



$$A \xrightarrow{k_1} B \xrightarrow{k_3} C$$

Reaction rates: $r_1 = k_1[A]$, $r_2 = k_3[B]$

- k_1 and k_3 are constants

$A \to B$ is order 2, $B \to C$ is order 1/2

It is required to control $[A]$ and $[B]$ by adjusting the feed concentrations $[A_f]$ and $[B_f]$.

Let V be the volume of the reactor and F the flow-rate. The modelling equations are

$$V\frac{\mathrm{d}[A]}{\mathrm{d}t} = F\left([A] - [A]_f\right) - Vk_1[A]^2 \tag{1a}$$

$$V\frac{\mathrm{d}[B]}{\mathrm{d}t} = F\left([B] - [B]_f\right) - V\left(k_1[A]^2 - k_3[B]^{1/2}\right) \tag{1b}$$

- (40%) Linearise the modelling equations around a steady-state operating point (SS) and make use of the deviation variables $x_1 = [A] - [A]_{SS}$ and $x_2 = [B] - [B]_{SS}$, and $u_1 = [A_f] - [A_f]_{SS}$ and $u_2 = [B_f] - [B_f]_{SS}$ to write the state-space model of the process;
- (10%) Characterise the resulting linear state-space model in terms of its elements (vectors and matrices), under the assumption that all state variables can be measured.

Exercise 02 (50%). You are given the linear and time-invariant model of a process

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t) \tag{2a}$$

$$y(t) = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix} x(t) \tag{2b}$$

You are asked to study its dynamics and design a controller

- (10%) Explain all the terms in the model (tell what they are and what is their size);
- (10%) Determine and discuss the stability of the model and its modes;
- (10%) Determine and discuss the controllability of the model;
- (10%) Suggest a strategy to tune a state feedback controller;
- (10%) Determine and discuss the model's observability.