

Old problem set questions

The features explained in the initial part of a problem before any possible subparts (a, b, c. . .) apply to the entire problem unless otherwise noted. The features in each subpart only apply to that subpart, unless otherwise noted.

For a solution to be acceptable it must include the explanation behind your reasoning, including calculations where applicable. A mere bottom line answer is not acceptable, unless otherwise stated.

Unless otherwise stated, you can assume that decision-makers are risk neutral and maximize the present value of their own payoffs (typically profits for firms, utility or consumer surplus for consumers). For discounting purposes, you can assume that future periodic payoffs are realized at the end of a period. In particular, period $t = 0$ refers to now immediately, period $t = 1$ to the end of first period, that is, one period from now, etc.

Even when a question asks you to find “the equilibrium” the correct answer might involve multiple equilibria or no equilibria. The singular phrasing is used for brevity only, unless otherwise noted.

In discrete type pricing problems you can assume that a customer that is indifferent between two deals will choose the one that gives the seller more profit. No need to mess around with “minus one cent” prices to handle tie-breaking.

1. The demand for chaff is $Q^D(p) = 100 - 2p$ tons per year, where the price is in €/ton. What is the price, quantity, total expenditure, consumer surplus, and producer surplus, in market equilibrium when. . .

(a) The supply is $Q^S(p) = 3p - 40$.

(b) There are 1000 suppliers, each with an individual supply of $q^S(p) = p/200 - 1/25$.

(c) The market supply comes from 100 firms, each of which will supply one ton of chaff if the price satisfies its reservation price. This reservation price is distributed uniformly between 8 and 28.

2. The demand for housing in Woebegen Heights is $Q^D(p) = 12000 - 3500p$, where the price is in thousands of €/m² and the quantity in thousands m². The housing stock in the Woebegen Heights is fixed in the short run and is currently 5 million m². Existing housing is supplied completely inelastically. Beyond the existing stock, the long-run supply of additional housing is $Q_+^S(p) = 1000(p - 2)$ if price exceeds 2000 €/m² and zero otherwise.

The Lake Woebegen region is badly hit by a pandemic. Many of the local services such as restaurants and theaters in the City of Woebegen have gone out of business. As a result, the willingness to pay for housing in the suburb of Woebegen Heights increases for some city residents, which causes the demand curve to rise to $Q^D(p) = 12000 - 2000p$.

(a) What is the long-run equilibrium of the housing market in Woebegen Heights before the pandemic hits?

- (b) What is the short-run impact of the pandemic on the housing market in Woebegon Heights?
- (c) What is the long-run impact of the pandemic on the housing market in Woebegon Heights, assuming the change in demand is persists?
- (d) Continued from part 2c. In the very long run, as city services recover and the memory of the pandemic fades, the demand for housing in Woebegon Heights returns to its pre-pandemic level. What is the new market equilibrium?

Illustrate your answers with graphs.

3. There exist only 8 copies of the novel *The Secret Mirror* that were signed by its author Herbert Quain. The valuations of the current owners are {80, 95, 100, 120, 135, 145, 200, 1000} in thousands of €. There are 7 other people with a strictly positive valuation for this rare book. Their valuations are {40, 45, 85, 90, 95, 105, 180} (1000s €). Potential buyers and sellers have until recently been unaware of each other, but now they have all joined the same marketplace for rare books.

- (a) How much total surplus would this new market generate in equilibrium?
- (b) Suppose that, instead of all buyers and sellers entering the market at once, they arrive sequentially and trade without any knowledge of potential future trading partners. How, if at all, could a fortuitous order of arrivals result in such a trading pattern that total surplus is higher than in part 3a?
- (c) Continued from part 3a. Some time after the market clears, one of the owners passes away and her copy is inherited by a son who values the book at only half the value that she did. How much additional surplus could a new trade now generate, at best?

4. Draw the demand and supply framework for your own example of two markets, each with many competitive buyers and sellers. The two markets should be connected in the sense that the goods are either substitutes or complements for the buyers (but not perfect substitutes or complements). Draw the graphs side by side. Axes should have numbered scales and labels. Illustrate a shift of either supply or demand in the market graphed on the left side. Illustrate how the market graphed on the right side responds to the change in the other market.

Write a brief 1-2 paragraph of text to explain what is in the graphs. What are the markets, their units of measurement, the time frame; what causes the shift and how does it impact the market equilibrium in the two markets? Describe also what happens to consumer and producer surplus in the two markets.

The example does not have to be real, but it should be reasonably realistic. A good test of “reasonably realistic” is that a fellow student unacquainted with these particular markets might think your example is based on real data.

5. (a) An unusually plentiful barley crop has caused the price of beer in Northland to drop to 15 €/liter while total consumption is 11 000 liters per month. Usually the price had been 16 €/l and total consumption 10 000 l/month. Based on these two observations, give a back-of-the-envelope estimate for the demand elasticity of beer in Northland.
- (b) Continued from part 5a. Northland's regulatory authorities are planning to impose new restrictions on the use of fertilizers in barley farming. Industry specialists anticipate this to decrease the beer production in normal times 9 500 liters/month. Using your previous estimate for the elasticity, give a back-of-the-envelope estimate for the impact of the new restrictions on beer price and on total beer revenue in normal times.
- (c) There is an integrated world market for guacamole. A new avocado pest causes a worldwide reduction in production, and as a result the world market price of guacamole increases by 4%. What happens to total consumer expenditure on guacamole in the following countries (with elasticity of demand in parenthesis). Argentina (-1.5), Belgium (-0.7) and Canada (-1.0).
- (d) Consider the demand and supply framework in 1b. What happens to equilibrium price and quantity if the number of firms doubles?
- (e) Draw your own personal supply curve for a specific type of labor. Define what kind of tasks/jobs are covered by this type of labor. Make sure to carefully label the units of measurement on both axis. Write a brief paragraph of text to explain the graph.
6. BugEye Corporation sells medical test kits to little green creatures on its home planet. Most of its demand comes from the wealthy North and the rest from the less wealthy South. The demand is $Q_N^d(p) = 24 - p/3$ in the North and $Q_S^d(p) = 18 - p/2$ in the South, where quantities are in thousands of kits and prices in Altairian dollars (Alt\$). Marginal cost is 10 Alt\$ for each kit. Until recently BugEye Corp has been able to segment the market and price the kits higher in the North. However, a new anti-gouging law in the North has made it illegal for medical suppliers to charge higher prices in the North than in the South.
- (a) What is the price and consumer surplus in each region before the anti-gouging law?
- (b) What is the impact of the anti-gouging law on prices and consumer surplus by region, as well as on the profits of BugEye Corp?
- (c) Suppose that, instead of being constant, the marginal cost of production is $MC(q) = 2 + 0.5q$. Now what are prices by region and profits before the anti-gouging law?
7. HJK Helsinki football club is planning to procure for its fan shop specially themed face masks to celebrate the club's 30th league title. To sell any masks the club has to incur a fixed marketing cost of €10 000. The club can procure the masks at a wholesale price of €5 per mask, but the minimum order size is 500 masks. The valuations of potential buyers are uniformly distributed in €[5, 35]. There are 3 000 potential buyers. (No one would buy two masks at any price above marginal cost).

- (a) What is the demand curve $P^d(q)$ for these masks?
- (b) What is the profit-maximizing price when the marginal cost is $\in x \geq 0$? What if $x = 5$?
- (c) Suppose there is a 25% probability that the Covid-19 pandemic will end before HJK clinches its 30th league title. This would cause many producers of face mask textiles to exit, causing the wholesale price of face masks to double. The face masks have to be marketed in advance, which also requires the fan shop to commit to a price before it learns the marginal cost. Increasing the previously advertised price after the title is clinched is not acceptable. However, the quantity procured can be decided later, and selling out is acceptable. Should the club incur the marketing cost? If so, what is the profit-maximizing advertised price?

8. There are 1000 households in Busytown, each with a monthly demand for tap water of $Q_i^d(p) = 10 - p$, where the price is in marks per 1000 liters and the quantity in thousands of liters. The municipal waterworks faces a marginal cost of 1 mark for each 1000l of clean water produced. The maintenance of the waterworks infrastructure costs 3000 marks per month. The waterworks can only use uniform pricing.

What is the price of tap water, monthly consumption per household, consumer surplus per household, total deadweight loss, and waterworks' profit in each of the following cases?

- (a) Waterworks maximizes its profits.
- (b) Waterworks maximizes consumer surplus.
- (c) Waterworks maximizes consumer surplus, subject to the constraint that it cannot make a loss.
- (d) Half of Busytown's households move to Duckburg. How does this population decline affect the answer to parts 8a and 8c?

9. Residents of Northland fall into three clans, each with different valuations for Northland's common defense. The defense is provided by fighter planes, each of which costs 25 million euros. The aggregate valuation of each clan for the common defense is summarized by the following demand functions: $Q_A(p) = 60 - 6p$ for the Abelian clan, $Q_B(p) = 80 - 5p$ for the Babelian clan, and $Q_C(p) = 50 - 2p$ for the Cainian clan, where the quantity of defense is measured in the number of fighter planes and the value in millions of euros.

Each clan is represented by a chief at national level. The chiefs' objective is to maximize the surplus of their own clan. According to Northland's constitution, the burden of defense spending is divided equally between the clans.

- (a) What is the efficient amount of defense spending for Northland?
- (b) If total spending is efficient, then what is the total surplus for each of the three clans?

- (c) The defense budget is decided by a majority vote in the council of clan chiefs. Each chief first proposes its most favored amount of spending, and then the chiefs vote on the proposals in the order determined by the chairman. How much would you expect Northland to spend on defense? How does the answer depend on which of the three chiefs holds the chairmanship?
10. Give your own example of each of i) private good, ii) club good, iii) common good, and iv) public good. For each example explain briefly (1-2 sentences) why it satisfies the definition.
11. Acme Ltd produces two types of gadgets, A and B. Their production requires specialized equipment which it leases from another company at 500k euros per year. The same equipment can be used for both gadget types, and year is the shortest possible rental period. The marginal cost of both types of gadgets is 100 euros a piece.
- (a) If Acme is facing only a single order of 1000 gadgets of type A, then what should be Acme's reservation price per gadget?
- (b) Suppose Acme expects that there is a 50% chance that it will get another order later during the year, and that this order would be for 2000 type B gadgets at a take-it-or-leave-it price of 200 euros a piece. How would this affect the answer to part 11a?
- (c) Acme has a production capacity of 1000 gadgets of type A and 2000 gadgets of type B. It is a price taker in the world gadget markets. At which combinations of world market prices $\{p_A, p_B\}$ should Acme produce both types of gadgets? Or A only, or B only?
12. Sirius Cybernetics Corporation is considering the launch of "Plastic Pal", a new model of human-friendly robots. However, before a model can be certified as human-friendly, it has to pass the Voight-Kampff test. This requires building a prototype, which in turn requires first developing the blueprints for the model. The cost of developing blueprints for the new model is \$20 million. The cost of building a high quality prototype is \$35 million. The probability of passing the Voight-Kampff test is 60%, if the prototype is prepared with high quality. A cheaper way to produce a prototype at a cost of \$15 million results in only 20% chance of passing the test. The test can be retaken, but the result for the same prototype quality is always the same. The high-quality prototype is always more successful: if it does not pass the test, then neither would the low-quality prototype. The demand for a certified human-friendly robot is $Q^d(p) = 1000(100 - 2p)$, where p is in \$thousands. Any number of mass-produced non-human-friendly robots can be sold, but only in the competitive world market for military robots, at a price of \$20,000. Mass production would require building a dedicated production line at a cost of \$200 million. A robot production line has a capacity to produce 400,000 robots, at a marginal cost of \$25,000.
- (a) Illustrate the situation facing Sirius Cybernetics with a decision tree. What is the optimal plan of action for Sirius and its expected value?

- (b) Marvin Consulting Inc has the ability to predict the outcome of the Voight-Kampff test just based on the blueprints. How much, at most, should Sirius be willing to pay for its services? Consider only a deal, where it gives its prediction for both prototypes.
- (c) How sensitive is the optimal plan of action in 12a to the number of potential buyers N , which is associated with the demand $Q^d(p|N) = N(100 - 2p)$? I.e., at what level of N does the decision change, and how?
- (d) How, if at all, does the answer to 12a change if the test results with high-quality and low-quality prototypes are independent? I.e., without the assumption that a failure with high-quality implies failure with low-quality?
13. Give your own example for the cost function of a consumer good that is currently available for purchase in Helsinki, both in equation and graphical form. While the good has to be real the numbers can be fictional, however they should be “realistic enough” so that your randomly selected classmate would not recognize whether the numbers are based on real data or your own best guesstimates. The example should include positive fixed and marginal costs, and a capacity constraint. Discuss what kind of sunk costs may be associated with this context. At what price(s) is this good currently sold?
14. O.y. Bonk A.b. is considering an expansion of its lucrative anchovy oil business to the Baltic countries. Entry to a new country requires setting up a distribution network and an initial marketing campaign. This cost of entry is €600,000 per country, although, if a distribution network is wound down, then all but the marketing cost of €20,000 can be recovered. Bonk faces an opportunity cost of capital at 10 %. The yearly demand for Bonk’s anchovy oil in each of the Baltic countries is known to be $P(q) = 125 - 0.05q$, where quantity is measured in barrels and price in €/barrel. Bonk can produce enough anchovy oil at a marginal cost of 10 €/barrel to more than satisfy any realistic level of demand. The marginal cost of distribution is known to be 12 €/barrel in Estonia. However, in Latvia, due to a more uncertain regulatory environment, the marginal cost of distribution is deemed to be equally likely either 2 or 22 €/barrel. The actual marginal cost would be found after a year of sales. Anchovy oil is banned in Lithuania, so entry there is out of question.¹
- (a) What is the expected present value of entry in each country?
- (b) How much should Bonk be willing to pay to find out the actual distribution costs in Latvia prior to committing to a market campaign there?
15. Wally has come up with a cost-saving idea, described in the following document:
dilbert.com/strip/2009-01-11.
- (a) What is the present value of Wally’s idea as a function of the discount rate $r > 0$?
What if $r = 0.03$?

¹Additional irrelevant information is available here: <http://bonkcentre.fi/en/businessjananchovy>.

- (b) What is the smallest integer number of years T , at which it holds true that even if Wally's idea only delivers its cost benefit for the first T years, it still provides at least 99% of the present value of an infinitely lasting idea? Assume a discount rate of 3%.
16. The most popular game show in Lintukoto consists of a series of true-or-false questions. The prize for getting the first answer right is €10. After getting an answer right the contestant can either quit and leave with the current prize money, or enter another round where the total prize money for the night is doubled. However, if at any point a contestant gives an incorrect answer then they will go home with zero prize money. The maximum number of questions for a contestant is 12, after which the show would end.
- (a) Give a mathematical description of the "lottery" implied by the game show, for a contestant who never voluntarily quits and whose probability of getting the right answer is the same $p \in (0, 1)$ for any question.
- (b) Ukko has starting wealth €100,000 and a utility function over wealth $u(w) = \sqrt{w}$. He is clueless about the questions and believes that his probability of getting any single answer correct is 50%. If he were planning to quit after two questions then what would be Ukko's certainty equivalent for the game? What if he were planning to never quit?
- (c) Continued from 16b. After how many correctly answered questions should Ukko choose to quit the game?
- (d) Akka has the same wealth and preferences as Ukko in part 16b, but a 75% chance of answering any single answer correctly. What should be her reservation value to get to play the game?

It makes sense to use a computer to do some of the calculations, e.g., with Excel or Python/NumPy. If you do, show the formulae you used and explain the reasoning behind.

17. A consumer has the utility function $u(a, b) = a^{\frac{1}{4}}b^{\frac{3}{4}}$, where a is apples and b the baskets of all other goods. The internal structure of the basket b is not affected by the levels of consumption. The price of b is normalized at $p_b = 1$, in other words, it is the *numeraire* in terms of which other prices are measured. The price of apples is simply denoted by p .
- (a) If the consumer has a budget of $M = 100$ and $p = 0.5$, then which consumption bundle would maximize her utility?
- (b) In general, if this consumer has a budget of $M > 0$ and the price of apples is $p > 0$ then what is her (Marshallian) demand for apples?
- (c) What is the expenditure share of apples in this consumer's total spending?
18. Give your own example of an indifference map for a pair of private goods, where the prices of all other goods (those not in the graph) are held constant, and the two goods shown are...
- (a) imperfect substitutes or complements,

- (b) perfect complements,
- (c) perfect substitutes.

Make sure to describe the units and the time period for each example. Give a one-sentence explanation why this pair of goods fits the required example in your consumption preferences. A map should include at least 5 indifference curves.

19. A firm that produces stuff is a price-taker in the world market for stuff. It has a fixed cost of 10 monetary units (MUs) and a marginal cost of $2 + 0.2q$ MUs.
- (a) What is the firm's supply curve?
 - (b) What are the firm's profits as a function of p , the price of stuff.
 - (c) The world is full of potential producers that could produce stuff with this same cost structure. More of them will enter the market if profits are available, while some will exit if it is not possible to earn at least zero profits. In equilibrium, all firms make zero profits and no firm can gain by increasing or decreasing its level of output. How much stuff does each firm produce in equilibrium?
20. An electric scooter company in the town of Podunk offers three types of rentals for its customers. It is possible to buy any amount of travel time at 0.25 €/minute. It is also possible to buy a package of 30 minutes at a price of €6 or a package of 60 minutes at €9. Finally, it is also possible to buy the right to an unlimited travel time for one week at €50.
- (a) Describe the cost function of scooter time for one week for one consumer, who might consume anything between 0 and 15 hours of scooter time. Plot the cost function on a graph.
 - (b) Describe the budget set of a consumer, who is allocating a weekly mobility budget of €100 between scooter rentals and taxi travel. Taxi travel costs 1 €/minute. Plot the budget set in consumption space with taxi minutes on horizontal and scooter minutes on vertical axes.
 - (c) While there is much variety in their exact shapes, all Podunkian consumers have "smooth preferences" over scooter rentals and taxi trips. That is, their indifference curves are smoothly curved in consumption space. Some particular levels of weekly scooter minutes consumption are observed to be more common than other nearby levels. What would you expect these common consumption levels to be, and why?

You can assume that any paid-for but unused minutes expire at the end of the week.

21. The only input of production in the insular nation of Lilliput is labor, of which there is a total endowment of 1 million worker-years. It can be used to produce either health care or other goods. Lilliputians only value health care inasmuch it affects their life expectancy

- x . If h million worker-years are used in the production of health care, then life expectancy is $X(h) = 20 + 100\sqrt{h}$. Labor can also be used to produce other goods y , with a linear technology where every h million worker-years contributes into one unit of y (measured in units of GDP per capita). Lilliputians share a common utility function $U(x, y) = (x-20)^{\frac{1}{2}}y^{\frac{1}{2}}$.
- (a) Describe the production possibilities available for Lilliput. You can show a graph or a formula.
- (b) What is Lilliputian GDP per capita and life expectancy in their social optimum?²
22. The yearly demand for horse milk in Berserkistan is $Q^D(p) = 20 - 0.05p$ while the supply is $Q^S(p) = 0.2p - 40$, where prices are in dollars and quantities in kilotons.
- (a) The government of Berserkistan has long been beholden to its powerful horse milk producers, and has been paying them a subsidy of \$100 for each kiloton (kt) produced. What are the welfare effects of this subsidy?
- (b) To quell a revolt by urban taxpayers the government ends the production subsidy for horse milk. Instead the government begins to pay the buyers of horse milk a consumption subsidy of 100 \$/kt. What are the welfare effects of this subsidy?
- (c) Berserkistan transitions into a two-party democracy. Half the time the power is held by the Farmers' party and the other half it is held by the Urban party. In years when the Farmers' party is in power the production subsidy is raised to 200 \$/kt, whereas when the Urban party is in power it is not paid at all. What are the welfare effects of this unstable policy on average across the years?
- (d) A coalition government comes to power in Berserkistan. A grand bargain is struck between farmers and urban dwellers: all subsidies are eliminated, and producers are promised the same level of welfare as they used to have when they received the 100 \$/kt subsidy (in part 22a). The government achieves this by buying horse milk from the market and then selling its purchases in the world market at a price 40 \$/kt. Importation of horse milk is banned in Berserkistan. What are the welfare effects of this new policy?
23. Alice and Bernard are neighbors and the small patch of forest between their cottages is home to delicious chanterelles. Both value the consumption of freshly picked chanterelles at 16 €/liter, and both have an effort cost of picking of x^2 €/h where x is the picking speed at l/h. A total of 24 liters of chanterelles grows in the patch. Both start picking at the same time, choose their picking speed simultaneously and continue at the same speed until the patch is empty of chanterelles. (A player that does no picking gets nothing).

²Partial credit (7p) available for showing graphically how the answer could be obtained.

- (a) Write down the payoff matrix of a game where both choose their picking speed from the discrete set $\{0, 2, 4, 6\}$ l/h. What is the Nash equilibrium of this game? What would be socially efficient?
- (b) Suppose both Alice and Bernard put some weight on each other's welfare. In particular, their payoffs are now $u(v_i, v_j) = (1 - \beta)v_i + \beta v_j$, where v_i is one's own and v_j the neighbor's "selfish" payoff, and $\beta = 0.25$ is a parameter capturing the strength of social preferences. Now what is the Nash equilibrium?
- (c) Suppose Alice is a faster picker, incurring only the effort cost $0.5x^2$ €/h. Assuming that speeds are selected from the same set as in part 23a, now what is the Nash equilibrium? What would be socially efficient?
- (d) Suppose Bernard publicly commits to a picking speed before Alice makes her choice. Assuming that speeds are selected from the same set as in part 23a, now what is the equilibrium of the game?
24. Alpha Inc and Beta Corp are the only companies capable of mining unobtainium, which is found in the asteroid belt. The cost of building an asteroid mining ship is 4 trillion euros. A single-use mining ship can bring down to earth an asteroid with q tons of unobtainium at a further cost of 2 trillion euros per ton. The demand for unobtainium is $Q(p) = 60 - 12p$ tons, where p is in trillions of euros.
- (a) The companies decide on the size of their mining operation before finding out each others' choices. What are their profits in equilibrium?
- (b) Beta Corp has the capability of building a more advanced mining ship, but it would add 2 trillion to the cost. The benefit would be that it would bring down the marginal cost to 1.5 trillion €/ton. Beta would have the option of hiding this investment (and the fact that it even had the option) from Alpha. Should Beta make this investment?
- (c) As in part 24a, but Alpha gets to launch its ship first. Beta sees the capacity of Alpha's ship before deciding on its own.
- (d) Suppose Beta made the investment in part 24b. Now one company launches its ship first. The other company sees its capacity before deciding on its own capacity. What are equilibrium profits and how do they depend on which company launches first?
25. Two electric scooter companies are making a decision between staying in and exiting from Northland. If there are n firms in the market then each makes a yearly revenue of $R(n) = 100/n$ €million at a fixed cost of 60 €million. Firms commit to paying the fixed cost once a year before finding out each others' decisions. Not paying the fixed cost even in one year results in irreversible exit. All firms use the discount rate $r = 0.05$.
- (a) Suppose the market is about to enter its final year, after which scooters are known to become obsolete. What is the Nash equilibrium?

- (b) What would the highest feasible present value of profits for the industry?
- (c) What is the symmetric equilibrium of the repeated game, and the associated expected present value of profits for the industry?
26. Use game theory to model your own example of a strategic economic decision-making situation under symmetric information and sequential choices. Describe the players, the actions, the payoffs. Describe the optimal strategies and the equilibrium.
27. Consider the one-time interaction between Alice and Bernard in problem 6, with a choice of picking speed from 0, 2, and 4 l/h. The same situation is repeated every autumn. Both neighbors have a yearly discount rate $r = 0.1$.
- (a) Both Alice and Bernard expect to remain neighbors for five more years. What is the subgame perfect Nash equilibrium?
- (b) Both Alice and Bernard expect to remain neighbors forever. How (if at all) can they achieve a higher payoff, every period, in an equilibrium of this repeated game than what they received in the Nash equilibrium of the one-time interaction?
- (c) Suppose that, every autumn, the chanterelles only appear with a 50% probability. Picking only commences if there are chanterelles. How does this change the answers to parts 27a and 27b?
- (d) Suppose Bernard is less patient than Alice, with $r > 0.1$. How impatient does Bernard have to be for the answer to part 27b to change?
28. Abholos and Bokrug are the only ice cream vendors on Shell Beach, which is a straight 1000 meter long stretch of shoreline. It is uniformly populated by 1000 sunbathers, all of whom have unit demand for ice cream at reservation value €5 and who also experience a shopping cost of €0.50 per 100 meters of distance from their beach location to the vendor. Marginal cost of ice cream is €1. Prices are posted in increments of €0.01 and possible vendor locations are 1 meter apart.
- (a) Abholos is located 300 meters from the western end of Shell Beach while Bokrug is located at 400 meters from the eastern end. If both charge €2.00 for ice cream, what are their profits? Use a graph to describe the total consumer surplus.
- (b) Continued from 28a. Suppose Abholos can relocate, while knowing that Bokrug cannot. Where should it move?
- (c) Continued from 28a. Suppose Bokrug can change its price, while knowing that Abholos cannot. How should it price its ice cream?³

³Hint: pay attention to two special prices. At one price level B will start losing “captive” customers at the eastern end of the beach. At another threshold level it will suddenly lose all remaining customers to A.

29. Describe an actual mixed bundling scheme currently available to buyers in the Helsinki metropolitan area. What are the goods or services on sale, what are the “savings” from buying bundles as opposed to individual goods? (If the number of goods and bundles is vast, pick a subset of interest). Are there additional fees and complications (e.g. hassle costs) not covered by the simple classroom examples? Is the bundling scheme combined with other types of price differentiation strategies?
30. A buffet restaurant has two types of customers, regulars and gluttons, with respective monthly individual demands $Q_r^d(p) = 12 - 0.5p$ and $Q_g^d(p) = 24 - p$, where q is the number of restaurant visits and p in euros. There are 300 regulars and 200 gluttons. The cost of serving one buffet visit is €4. The restaurant also has a fixed cost of 40 k€/month.
- What is the profit maximizing simple price for the buffet?
 - The restaurant is able to also charge a monthly subscription fee, using a customer loyalty card. Design the profit maximizing pricing scheme for the buffet. Compare consumer surplus by individual customer type with simple pricing.
 - Suppose the cost of serving gluttons is €6/visit. How does this affect the optimal pricing scheme in part 30b?
31. Acme Ltd produces breakfast gruel. The cost of producing one portion of gruel is €2. It can be mixed with water to produce thin gruel at half the marginal cost. Potential customers are either health-conscious (H) or low-income (L), both equally numerous. Either would buy at most one portion of gruel at a time. Customer valuations are ...

	Gruel	
€	Thin	Thick
L	1.50	2.90
H	1.20	2.30

- Describe the profit-maximizing pricing scheme.
 - The share of health-conscious customers begins to increase. How much higher can it get before the optimal pricing scheme from part 31a changes?
32. Fish plc sells jellied eels to two types of potential customers. There are 1000 households each with demand $Q_1(p) = 60 - 12p$, and 2000 industrial customers each with demand $Q_2(p) = 100 - 20p$, where quantities are in kg and values in £. Production incurs a fixed cost of £ 250k, while marginal cost is 0.50 £/kg.
- Design the profit-maximizing pricing scheme and package sizes.
 - Fish plc is worried that middlemen might emerge and start buying large packages of jellied eel in order to take them apart and sell them as small packages. How might such

a plan work, and how much at most could the middlemen earn from such a scheme if Fish were using the pricing scheme from 32a?

33. Acme food truck is known for two specialties, grilled pineapples and grouse stew. It has three types of potential customers, 100 of each type. Their valuations for Acme's servings are...

€	Grouse	Pineapple
Bourgeois	15	13
Students	6	11
Workers	14	9

The marginal cost of portions is €5 and €3 for grouse and pineapple respectively. The goods are neither substitutes nor complements to the consumers. What is the profit-maximizing pricing scheme and resulting profits if Acme were to use...

- (a) basic pricing.
- (b) pure bundling.
- (c) mixed bundling.

34. Bluefin tuna is a popular delicacy in the nations around the Great Sea. Most of the sea lies outside territorial waters, so there are no fishing restrictions. The cost of sending a tuna fishing vessel to the open sea is 20k monetary units (MUs). The yearly catch of any single fishing boat x in tons is decreasing in the number of boats n , so that $x(n) = 80 - 0.2n$. The market price of tuna is determined by its more abundant substitutes, and is 2k MU/ton.

- (a) What is the efficient number of tuna fishing boats, the associated total yearly catch, and the resulting total profits of the tuna industry?
- (b) In the absence of any restrictions on the entry of tuna fishing boats, how many will enter? Compare the profits and the tuna catch, both in total and as the average per-boat, with their efficient levels.
- (c) The Great Sea nations decide to take control of the open sea in order to prevent over-fishing. They decide to auction off fishing licenses, so that a boat is only allowed to fish tuna if it buys a yearly license. In order to maximize total welfare, how many licenses should be sold? What would be their market price?

35. There are two alternative routes between Easton and Weston, and every day 10 000 drivers travel between them. Travel time on the Expressway is $T_1(n) = 30 + (n - 5000)/50$ minutes if $n \geq 5000$ drivers choose the Expressway, and 30 minutes otherwise. On the Highway travel time is $T_2(n) = 45 + (n - 500)/100$ minutes if $n \geq 500$ drivers choose the Highway, and 45 minutes otherwise. Roads are owned by the government. Driver utility is decreasing in travel time: drivers value each saved minute on the road by €0.20.

- (a) How many drivers choose each road in equilibrium? What is the average travel time?
- (b) Design the welfare-maximizing road pricing scheme, in which drivers have to pay a toll on one of the roads. What are its welfare effects? How much does it decrease the average travel time between the cities?
- (c) A popular outcry against road pricing causes the government to exempt low-income drivers from the optimal fee derived in part 35b. Half of drivers are classified as low-income. Now what is the average travel time? Compare total welfare, as well as travel time and welfare for both low and high income drivers separately, with what is obtained in part 35a and in part 35b.
36. The residents of Lintukoto conduct their non-professional networking activities on the age-old platform AllCaps. All 10 000 residents have an active AllCaps account, each with a yearly valuation $v(z, n) = z\sqrt{n}$ euros for access to the network, where n is the number of active members and z a preference parameter that depends on the quality of the user interface. AllCaps has only a fixed operating cost of €200k/year.
- (a) If all residents have a quality preference $z = 1$ then what is the profit-maximizing yearly membership fee for AllCaps?
- (b) If quality valuations are distributed uniformly, with $z \in [0, 2]$, then what is the profit-maximizing yearly membership fee for AllCaps?
- (c) Continued from part 36a. A new startup FreeRant launches a competing app with a nicer user interface valued at $z = 2$. It has initially no users, but can lure away AllCaps users by giving away swag. (Users are active in at most one platform at a time.) How many users does FreeRant need to attract in order to drive AllCaps out of business?⁴
37. Give your own example of a network good and explain briefly the nature of the network externality between users. The good has to be currently available in Finland, and the users have to be paying a strictly positive price for it.
38. There are 1000 sailboats in the used boat market. These are the boats that have passed an independent certification of being at least acceptable in terms of quality. However, there remains some uncertainty over the exact quality of these boats, which only the current owner can know. All potential buyers have the same valuations, and there are more potential buyers than there are boats. The reservation values are...

⁴Hint: This means leaving AllCaps so few customers that it can no longer compete at any price.

€k	Valuations	
	Seller	Buyers
Junk	15	20
Fine	20	24
Good	25	28
Perfect	32	36

- (a) All boat types equally common. How many boats are traded? Compare total welfare to a world with symmetric information.
- (b) Continued from part 38a. Suppose a boat dealer develops a superior quality evaluation technology. In particular, by paying a cost of €2k it can verify and credibly disclose the true quality of a boat. How would this impact total welfare?
- (c) Fraction $x \in (0, 1)$ of boats are perfect quality, while other types are equally common. How high/low does x have to be for there to be either full market unraveling or no adverse selection at all?
39. There are numerous watchmakers in Ruritania and enough buyers for all the watches they can make. Buyers value durable watches at 100 ducats, but non-durable watches are only valued at 40 ducats. Buyers are not able to assess the durability of a watch nor the skills of watchmakers, but they know that only half of watchmakers are highly skilled. It costs 15 ducats to produce a watch. Watchmakers can also spend time engraving the case of a watch. The case is visible to the buyers, who don't care about the engravings but understand that the laborious process costs 75 ducats to the low-skill watchmakers, but only a third of that to the highly skilled. Neither skill level nor engraving affects a watchmaker's level of output.
- (a) Show that watchmakers can use engravings to signal their skill level.
- (b) What would be the change in total surplus (in %) if someone invented a costless way to produce engravings that are indistinguishable from hand-crafted engravings?
40. Acme Mutual offers marine insurance to shipowners. The insurance pays out in case of total loss (e.g., if the ship sinks). The most carelessly managed ships have a 20% probability of total loss during the contract period, but shipowners can reduce this probability at a cost. There are some verifiable methods such as meeting fire standards that reduce this probability to 8%; these cost €1m and are required by Acme as a prerequisite for purchasing insurance. There are also various unverifiable methods of further risk reduction. A moderate level of unverifiable spending (€1m) would reduce the loss probability to 4%, while a high level (€3m) would reduce it to 1%.
- Acme observes the true values of ships and cargo. In parts from 40a to 40c, consider two cases: a ship and cargo of "High" total value €100m and of "Low" total value €20m.
- (a) What would be the efficient level of spending on loss risk reduction?

- (b) Suppose Acme were to offer insurance with a 35% coinsurance rate. How much would risk neutral shipowners spend on risk reduction? What would then be the cost of actuarially fair insurance?
- (c) What range of coinsurance rates would incentivize a risk neutral shipowner to choose the efficient level of safety spending?
- (d) Suppose Acme is offering insurance with a 35% coinsurance rate. To prevent moral hazard it decides to only insure ships of sufficiently high value. What minimum value requirement would achieve this aim?