

CS–E4500 Advanced Course in Algorithms

Week 01 – Tutorial

- Suppose that we independently roll two standard six-sided dice. Let X_1 be the number that shows on the first die, X_2 the number on the second die, and X the sum of the numbers on the two dice.
 - What is $E[X \mid X_1 \text{ is even}]$?
 - What is $E[X \mid X_1 = X_2]$?
 - What is $E[X_1 \mid X = 9]$?
 - What is $E[X_1 - X_2 \mid X = k]$ for k in the range $[2, 12]$?
- We flip a fair coin ten times. Find the probability of the following events.
 - The number of heads and the number of tails are equal.
 - There are more heads than tails.
 - The i th flip and the $(11 - i)$ th flip are the same for $i = 1, \dots, 5$.
 - We flip at least four consecutive heads.
- The following problem is known as the Monty Hall problem, after the host of the game show “Let’s Make a Deal”. There are three curtains. Behind one curtain is a new car, and behind the other two are goats. The game is played as follows. The contestant chooses the curtain that she thinks the car is behind. Monty then opens one of the other curtains to show a goat. (Monty may have more than one goat to choose from; in this case, assume he chooses which goat to show uniformly at random.) The contestant can then stay with the curtain she originally chose or switch to the other unopened curtain. After that, the location of the car is revealed, and the contestant wins the car or the remaining goat. Should the contestant switch curtains or not, or does it make no difference?