Problem 1: Steady-state characteristics of a DC motor

A DC motor with a separately excited field winding is considered. The rated armature voltage is $u_\mathrm{N} = 600$ V, rated torque $\tau_\mathrm{N} = 420$ Nm, rated speed $n_\mathrm{N} = 1\,600$ r/min, and maximum speed $n_\mathrm{max} = 3\,200$ r/min. The losses are omitted.

(a) The flux factor k_f is kept constant at its rated value. When the armature voltage is varied from 0 to u_N, the speed varies from 0 to n_N. Determine the rated armature current i_N.

(b) A load is to be driven in the speed range from n_N to n_max by weakening the flux factor while the armature voltage is kept constant at u_N. Determine the torque available at maximum speed, if the rated current i_N is not exceeded.

(c) Sketch the armature voltage u, flux factor k_f, torque τ_M, and mechanical power p_M as a function of the speed, when the armature current is kept at i_N.

Solution

The losses are omitted, i.e., $R = 0$ holds. Hence, the steady-state equations of the DC motor are

$$u = k_\mathrm{f}\omega_\mathrm{M} \qquad \tau_\mathrm{M} = k_\mathrm{f} i \qquad p_\mathrm{M} = \tau_\mathrm{M}\omega_\mathrm{M} = ui$$

(a) Let us first calculate the rated rotor speed in radians per second:

$$\omega_\mathrm{N} = 2\pi n_\mathrm{N} = 2\pi \cdot \frac{1\,600\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 167.6\ \mathrm{rad/s}$$

The rated flux factor is

$$k_\mathrm{fN} = \frac{u_\mathrm{N}}{\omega_\mathrm{N}} = \frac{600\ \mathrm{V}}{167.6\ \mathrm{rad/s}} = 3.58\ \mathrm{Vs}$$

The rated armature current is

$$i_\mathrm{N} = \frac{\tau_\mathrm{N}}{k_\mathrm{fN}} = \frac{420\ \mathrm{Nm}}{3.58\ \mathrm{Vs}} = 117.3\ \mathrm{A}$$

(b) The maximum rotor speed in radians per second is

$$\omega_\mathrm{max} = 2\pi n_\mathrm{max} = 2\pi \cdot \frac{3\,200\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 335.1\ \mathrm{rad/s}$$

The flux factor at the maximum speed is

$$k_\mathrm{f} = \frac{u_\mathrm{N}}{\omega_\mathrm{max}} = \frac{600\ \mathrm{V}}{335.1\ \mathrm{rad/s}} = 1.79\ \mathrm{Vs}$$

The torque at the maximum speed is

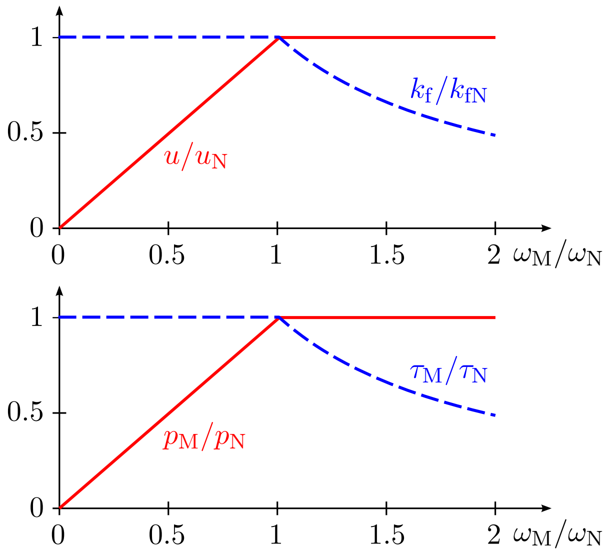
$$\tau_\mathrm{M} = k_\mathrm{f} i_\mathrm{N} = 1.79\ \mathrm{Vs} \cdot 117.3\ \mathrm{A} = 210\ \mathrm{Nm}$$

The same result could be obtained as $\tau_\mathrm{M} = (n_\mathrm{N}/n_\mathrm{max})\tau_\mathrm{N}$, i.e., the torque reduces inversely proportionally to the speed in the field-weakening region.

(c) The requested characteristics are shown in the figure below.

The armature voltage $u = k_\mathrm{f}\omega_\mathrm{M}$ increases linearly with the rotor speed until the rated (maximum) voltage u_N is reached at the rated speed. In order to reach higher speeds, the flux factor k_f has to be reduced inversely proportionally to the speed.

Since $i = i_\mathrm{N}$ is constant, the torque $\tau_\mathrm{M} = k_\mathrm{f} i$ follows the characteristics of the flux factor k_f. The mechanical power $p_\mathrm{M} = \tau_\mathrm{M}\omega_\mathrm{M}$ increases linearly with the speed until the rated speed and remains constant at speeds higher than the rated speed. It is important to notice the same mechanical power is obtained also using the electrical quantities, $p_\mathrm{M} = ui$, since the losses are omitted.



Problem 2: Transfer functions

(a) A DC motor is considered. Derive the transfer function from the terminal voltage $u(s)$ to the terminal current $i(s)$.

(b) A lumped thermal capacity model is considered:

$$p_\mathrm{d}(t) = \frac{1}{R_\mathrm{th}}\theta(t) + C_\mathrm{th}\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}$$

Derive the transfer function from the power loss $p_\mathrm{d}(s)$ to the temperature rise $\theta(s)$.

Solution

(a) The terminal voltage is

$$u(t) = Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + e(t)$$

where $e(t) = k_\mathrm{f}\omega_\mathrm{M}(t)$ is the induced voltage. The Laplace transformation gives

$$u(s) = Ri(s) + sLi(s) + e(s)$$

from which the current $i(s)$ can be solved as

$$i(s) = \frac{1}{R + sL}[u(s) - e(s)]$$

The inputs $u(s)$ and $e(s)$ can be considered separately based on the superposition principle. Hence, the transfer function from $u(s)$ to $i(s)$ is

$$Y(s) = \frac{1}{R + sL} = \frac{1/R}{1 + Ts}$$

where $T = L/R$ is the time constant of the armature winding. It is worth noticing that this transfer function can be interpreted as the input admittance of the motor; this is also the reason why we chose the notation $Y(s)$ here.

(b) The Laplace transformation gives

$$p_\mathrm{d}(s) = \frac{1}{R_\mathrm{th}}\theta(s) + sC_\mathrm{th}\theta(s)$$

The transfer function from the power loss $p_\mathrm{d}(s)$ to the temperature rise $\theta(s)$ becomes

$$Z_\mathrm{th}(s) = \frac{R_\mathrm{th}}{1 + T_\mathrm{th}s}$$

where $T_\mathrm{th} = R_\mathrm{th}C_\mathrm{th}$ is the thermal time constant. This transfer function can be interpreted as the thermal impedance, which is the reason for our notation.

Problem 3: Properties of first-order systems

Consider a first-order system

$$G(s) = \frac{K}{1 + sT}$$

(a) What is the steady-state gain of the system?

(b) Derive the rise time from 10% to 90% for a step input.

(c) What is the 3-dB bandwidth α of the system?

Solution

(a) The steady-state gain is obtained by substituting $s = 0$ in the transfer function, giving $G(s) = K$.

(b) The transfer function $G(s)$ is multiplied with the unit-step input $u(s)$

$$y(s) = G(s)u(s) = \frac{K}{1 + Ts}\frac{1}{s}$$

By using the inverse Laplace transform, the unit-step response of the system in the time domain can be obtained:

$$y(t) = K\left(1 - \mathrm{e}^{-t/T}\right)$$

Let us solve the time as a function of y:

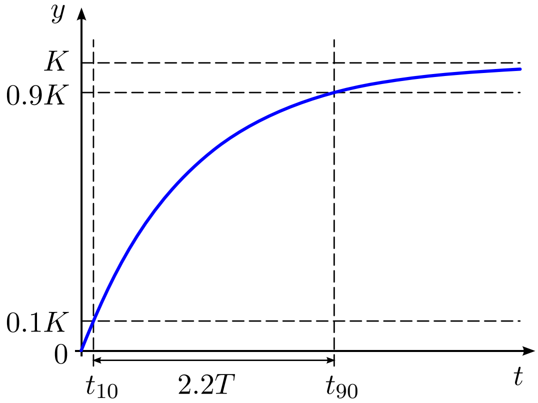
$$e^{-t/T} = 1 - y(t)/K \quad \text{or} \quad t = -T\ln[1 - y(t)/K]$$

The time instants t_{10} and t_{90}, at which $y(t_{10}) = 0.1K$ and $y(t_{90}) = 0.9K$, respectively, are

$$t_{10} = -T\ln(0.9) \quad \text{and} \quad t_{90} = -T\ln(0.1)$$

The rise time is the difference

$$t_r = t_{90} - t_{10} = T\,[\ln(0.9) - \ln(0.1)] = T\ln 9 \approx 2.2T$$



(c) The frequency response of the system is

$$G(j\omega) = \frac{K}{1 + j\omega T}$$

whose magnitude is

$$|G(j\omega)| = \frac{K}{\sqrt{1 + \omega^2 T^2}}$$

The 3-dB bandwidth α refers to the frequency at which the magnitude has dropped to $1/\sqrt{2} \approx 0.71$ of the steady state gain, i.e., $|G(j\alpha)| = K/\sqrt{2}$. Hence,

$$\sqrt{1 + \alpha^2 T^2} = \sqrt{2} \quad \Rightarrow \quad T = 1/\alpha$$

Remark: It is also common to represent first-order systems in the form

$$G(s) = \frac{K\alpha}{s + \alpha}$$

where the bandwidth is readily visible. The figure below shows the magnitude of the frequency response. The frequency scale is logarithmic.