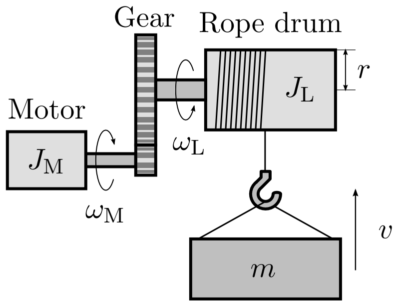
Problem 1: Gears

Consider a hoist drive shown in the figure. The motor is coupled to the rope drum through a gear mechanism, whose gear ratio is $i = \omega_\mathrm{M}/\omega_\mathrm{L} = 9.6$. The load mass is $m = 500$ kg, the motor inertia is $J_\mathrm{M} = 0.5$ kgm^2, and the rope drum inertia is $J_\mathrm{L} = 48.5$ kgm^2. The radius of the rope drum is $r = 0.25$ m. The rope mass, gear inertias, and the mechanical losses are omitted. Calculate the equivalent total inertia at the motor side and the equivalent load torque at the motor side.



Solution

The electromagnetic torque produced by the motor is

$$\tau_\mathrm{M} = J\frac{\mathrm{d}\omega_\mathrm{M}}{\mathrm{d}t} + \tau_\mathrm{L}$$

where J is the equivalent total inertia at the motor side and τ_L is the equivalent load torque also at the motor side. The problem here is to determine J and τ_L.

The inertia of the mass m at the load side is

$$J_m = \int_0^m r^2 \mathrm{d}m = mr^2 = 500 \text{ kg} \cdot (0.25 \text{ m})^2 = 31.25 \text{ kgm}^2$$

It can be seen that the load-side inertia $J_m + J_\mathrm{L} = 79.8$ kgm^2 is very large (approximately 160 times the motor inertia J_M). However, the total equivalent inertia seen by the motor is only

$$J = J_\mathrm{M} + \frac{J_\mathrm{L} + J_m}{i^2} = 0.5 \text{ kgm}^2 + \frac{48.5 + 31.25}{9.6^2} \text{ kgm}^2 = 1.36 \text{ kgm}^2$$

due to the gear ratio.

The mass m causes the torque mgr on the rope drum. Hence, the load torque seen by the motor is

$$\tau_\mathrm{L} = \frac{mgr}{i} = \frac{500 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 0.25 \text{ m}}{9.6} = 127.7 \text{ Nm}$$

It is important to notice that the mass m affects both the equivalent total inertia J and the equivalent load torque τ_L.

Problem 2: Electromagnetic torque vs. shaft torque

A torque sensor is connected between the motor shaft and the load shaft. The load torque is constant $\tau_L = 150$ Nm and the load inertia is $J_L = 1.0$ kgm^2. The motor inertia is $J_M = 0.6$ kgm^2. The speed is increased from zero to $\omega_M = 100$ rad/s in 0.5 s with a constant angular acceleration.

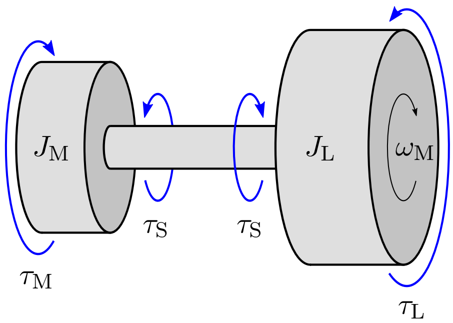
(a) What is the electromagnetic torque during acceleration? What about the measured torque?

(b) What is the electromagnetic torque at constant speed? What about the measured torque?

Solution

The system is illustrated in the figure below. The shaft can be assumed to be rigid. The total inertia is $J = J_M + J_L = 1.6$ kgm^2. The angular acceleration is

$$\alpha_M = \frac{d\omega_M}{dt} = \frac{100 \text{ rad/s}}{0.5 \text{ s}} = 200.0 \text{ rad/s}^2$$



(a) During the acceleration, the motor produces the electromagnetic torque

$$\tau_M = J\alpha_M + \tau_L = (1.6 \cdot 200 + 150) \text{ Nm} = 470 \text{ Nm}$$

On the other hand, the torque sensor measures the shaft torque, which is

$$\tau_S = \tau_M - J_M\alpha_M = (470 - 0.6 \cdot 200) \text{ Nm} = 350 \text{ Nm}$$

Alternatively, the shaft torque can be calculated from the load side as

$$\tau_S = \tau_L + J_L\alpha_M = (150 + 1.0 \cdot 200) \text{ Nm} = 350 \text{ Nm}$$

(b) The shaft torque at constant speed equals the electromagnetic torque:

$$\tau_S = \tau_M = \tau_L = 150 \text{ Nm}$$

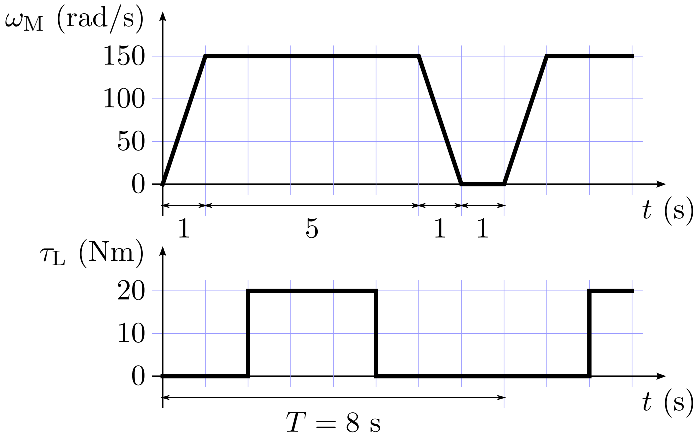
Problem 3: Torque and power

In periodic duty, the mechanical angular speed ω_M and load torque τ_L vary as shown in the figure. The total equivalent inertia is 0.04 kgm^2. The cycle duration is $T = 8$ s.

(a) Draw the conceptual waveforms of the electromagnetic torque τ_M and mechanical power p_M for one cycle.

(b) Calculate the rms value of the electromagnetic torque.

(c) A permanent-magnet DC motor is applied in this periodic duty. The rated torque and rated armature current of the motor are $\tau_N = 14.3$ Nm and $i_N = 33$ A, respectively. What is the maximum armature current during the period?



Solution

(a) The required electromagnetic torque is

$$\tau_M = J\frac{d\omega_M}{dt} + \tau_L$$

where $J = 0.04$ kgm^2 is the total equivalent inertia. The mechanical power is

$$p_M = \tau_M \omega_M$$

During acceleration at $t = 0 \ldots 1$ s, the electromagnetic torque is

$$\tau_M = J\frac{\Delta\omega_M}{\Delta t} = 0.04 \text{ kgm}^2 \cdot \frac{150 \text{ rad/s}}{1 \text{ s}} = 6 \text{ Nm}$$

During the loading phase at $t = 2 \ldots 5$ s, the electromagnetic torque has to be $\tau_M = \tau_L = 20$ Nm since the speed is constant. The braking torque at $t = 6 \ldots 7$ s is

$$\tau_M = J\frac{\Delta\omega_M}{\Delta t} = 0.04 \text{ kgm}^2 \cdot \frac{-150 \text{ rad/s}}{1 \text{ s}} = -6 \text{ Nm}$$

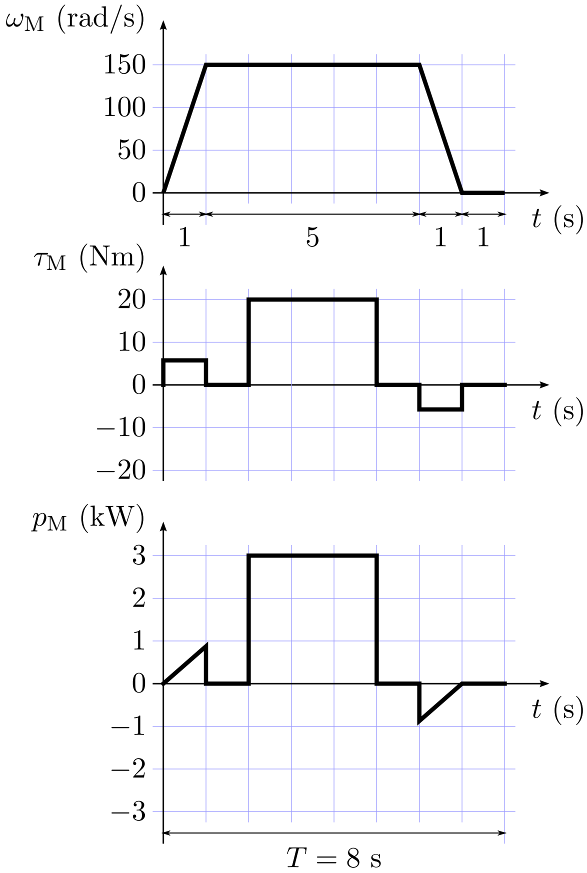
The value of the mechanical power at $t = 1$ s is

$$p_M = 6 \text{ Nm} \cdot 150 \text{ rad/s} = 0.9 \text{ kW}$$

and at $t = 2 \ldots 5$ s it is

$$p_\mathrm{M} = 20 \text{ Nm} \cdot 150 \text{ rad/s} = 3.0 \text{ kW}$$

Based on the above calculations, we can plot the following waveforms.



(b) The rms value of the electromagnetic torque over the period T is

$$\tau_\mathrm{M,rms} = \sqrt{\frac{1}{T}\int_0^T \tau_\mathrm{M}^2 \,\mathrm{d}t} = \sqrt{\frac{(6 \text{ Nm})^2 \cdot 1 \text{ s} + (20 \text{ Nm})^2 \cdot 3 \text{ s} + (-6 \text{ Nm})^2 \cdot 1 \text{ s}}{8 \text{ s}}} = 12.6 \text{ Nm}$$

Remark: The period $T = 8$ s is much shorter than thermal time constants (several minutes or tens of minutes) of motors in this power range. Hence, the motor can be selected based on the average temperature rise, which leads to the selection criterion $\tau_\mathrm{N} > \tau_\mathrm{M,rms}$. Furthermore, the motor should be able to produce the required maximum torque $\tau_\mathrm{M,max} = 20$ Nm.

(c) The flux factor of the motor is $k_\mathrm{f} = \tau_\mathrm{N}/i_\mathrm{N} = 14.3 \text{ Nm}/(33 \text{ A}) = 0.43$ Nm/A. The maximum armature current is $i = \tau_\mathrm{M}/k_\mathrm{f} = 20 \text{ Nm}/(0.43 \text{ Nm/A}) = 46.2$ A.

Remark: Thermal time constants of power converters are much shorter (typically seconds or tens of seconds) than those of the motors. Therefore, the converter has to be typically selected based on the maximum current during the period (instead of the rms value over the period).