Problem 1: Zero sequence and the space vector

Consider the phase voltages

$$u_\mathrm{a} = u'_\mathrm{a} + u_0 \qquad u_\mathrm{b} = u'_\mathrm{b} + u_0 \qquad u_\mathrm{c} = u'_\mathrm{c} + u_0$$

where $u'_\mathrm{a} + u'_\mathrm{b} + u'_\mathrm{c} = 0$ holds and u_0 is the zero-sequence component. Show that the zero sequence disappears in the space-vector transformation.

Problem 2: Synchronous machine model in rotor coordinates

(a) Equations for the stator voltage and stator flux linkage in stator coordinates are

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}^\mathrm{s}}{\mathrm{d}t} \qquad \boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \psi_\mathrm{f}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$$

Express these equations in rotor coordinates.

(b) Express the previous equations in rotor coordinates in steady state.

(c) Starting from

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\}$$

derive the torque expression in rotor coordinates as a function of i_d and i_q.

Problem 3: Operating points of a permanent-magnet synchronous motor

The datasheet values for a three-phase permanent-magnet synchronous motor are:

maximum continuous torque	15 Nm @ 2 400 r/min
voltage constant	0.159 V/(r/min)
number of pole pairs	$n_\mathrm{p} = 4$
stator inductance	$L_\mathrm{s} = 4.86$ mH
stator resistance	$R_\mathrm{s} = 0.46\ \Omega$

(a) The motor rotates at the speed of 2 400 r/min. Calculate the mechanical angular speed, electrical angular speed, and supply frequency.

(b) Calculate the peak-valued phase-to-neutral back-emf induced by the permanent magnets, when the motor rotates at 2 400 r/min. Calculate also the permanent-magnet flux constant ψ_f.

(c) The torque is 15 Nm. Calculate the output power of the motor at the speed of 2 400 r/min and at zero speed.

(d) The control principle $i_\mathrm{d} = 0$ is used. Calculate the stator current $\boldsymbol{i}_\mathrm{s}$ and the stator voltage $\boldsymbol{u}_\mathrm{s}$ in the following operating points: 1) torque is 15 Nm at 2 400 r/min; 2) torque is 15 Nm at zero speed; and 3) no load at 2 400 r/min.