Aalto University
School of Engineering

Ship Hulls – Modeling the perfect geometry

Rüdiger U. Franz von Bock und Polach

Content

1. Ships as complex gemetrical bodies

2. How does the geometry affect the economic success of a design?

3. How to define geometry?
   1. Lines
   2. Surfaces
   3. Fairing

4. Simulation based design / optimization

5. Conclusion

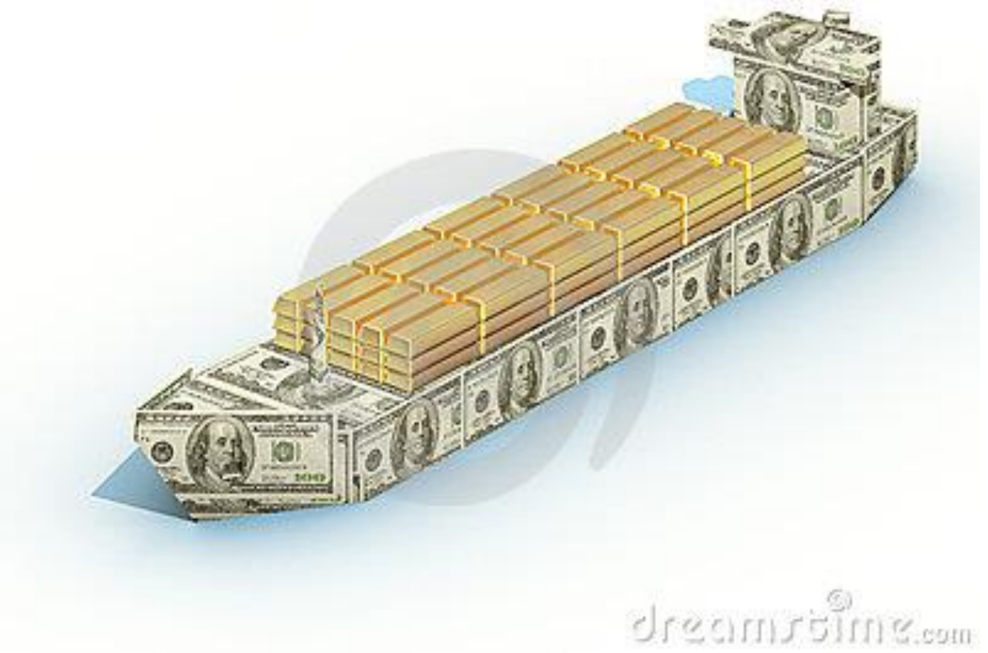
Significance of geometry

- Ships have a complecated geometry
- Accurate representation of the geometry is important for:
  - Design evaluation
  - Performance
  - Manufacturing

Why is the geometry and its representation so important?

- Ships are investments and built to create revenue

- The geometry plays a big role:
  - It defines the space for cargo
  - The performance at sea
  - The resistance



Impact of ship geometry

Transport efficiency

- Ship Merit Factor

$$SMF = \frac{k \cdot W_{PL} \cdot v_s}{AAC}$$

speed

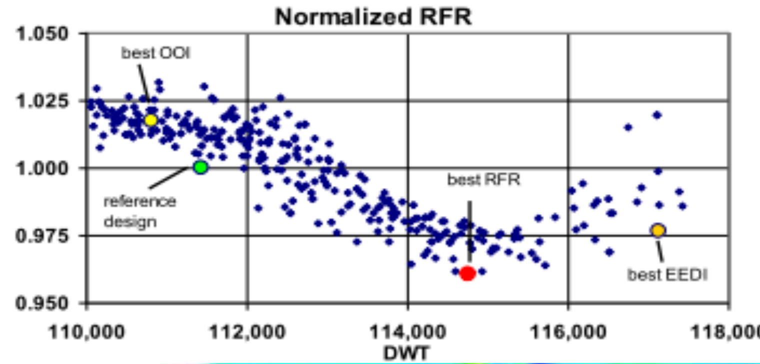
cargo

costs

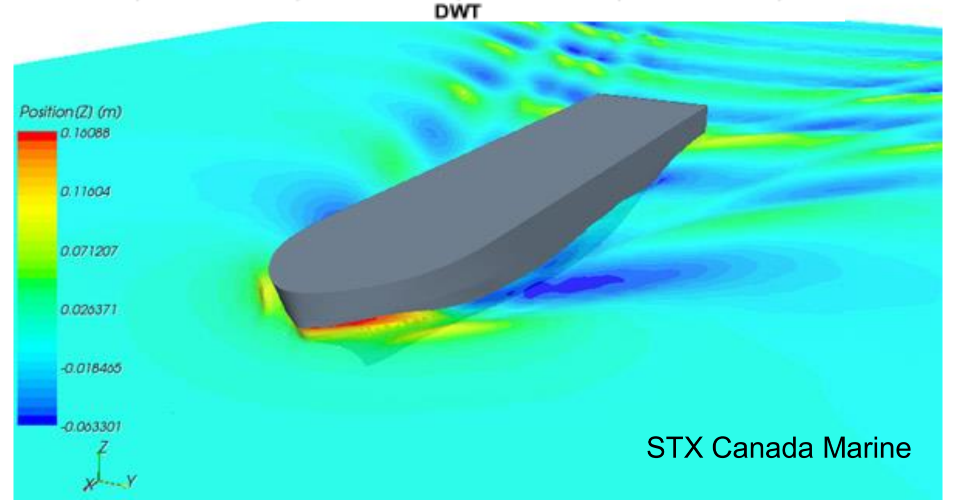
Daily fuel costs: 15t$ - 30t$

How is the best suitable geomerty in the pre-design phase determined?

- Geometry generation and parametric variation of e.g. length, width, bow section…



- Geometry analysis with:
  - CFD for open water
  - FEM for ice
  - Physical model tests

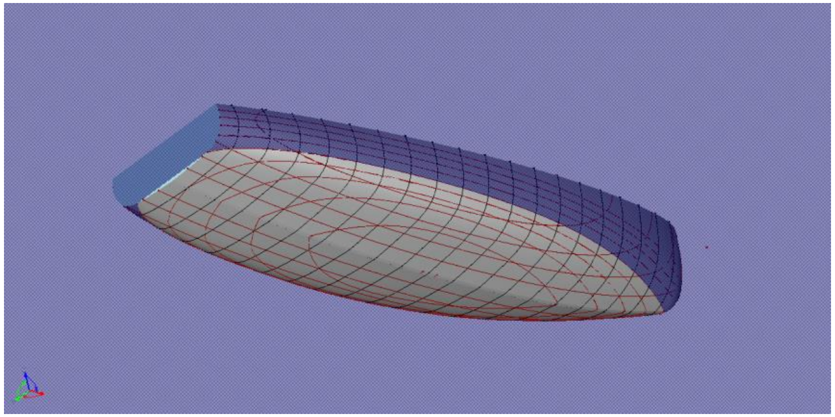


Physical model tests
The final evaluation of design

- Despite very advance numerical methods, physical testing is still performed at the end

Requirements and characteristics of ship hull

- Fair lines, fair surfaces
- Parametric description, since the hull shape model is to be used for:
  - Hydrostatic calculation (Volume, COG, etc…)
  - CFD analysis
  - Structural analysis
- Local changes should stay as local as possible



Generation of ship hulls

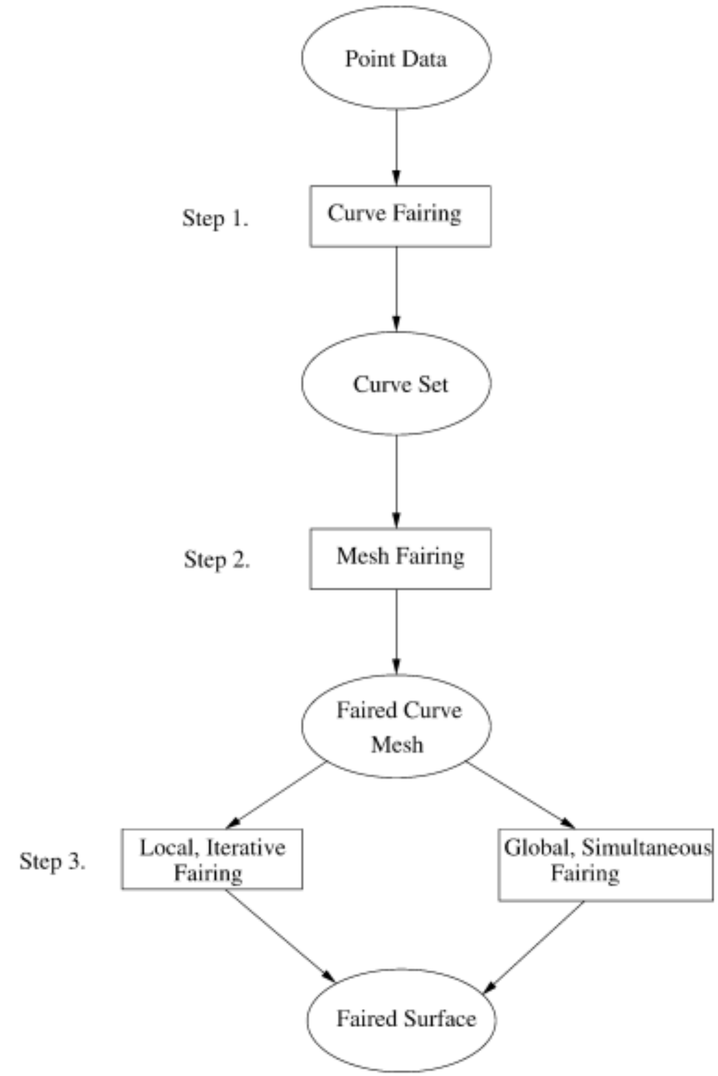
1. Set of points and basic lines
2. Definition of design sections
3. Generation of surfaces while

- Close related to the traditional process in ship building
- Definition of a few main characteristics (e.g. length, max width, draft…)
- Defined points (ducks), which are connected by lines (splines)

Hull shape development process

- Initially a set of piecewise continuous characteristic curves is generated from offset data and end constraints.
- This delivers a regular or irregular mesh, (not a lines plan)
- Lines are connected at the mesh knots.

B-splines -> B-spline surfaces
Non-uniform rational B-Splines -> NURBS surfaces



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G. Westgaard, H. Nowacki / Computer Aided Geometric Design 18 (2001) 619–638

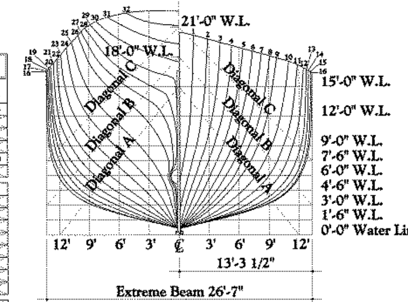
TABLE of HULL MEASUREMENTS

STATIONS 0–16

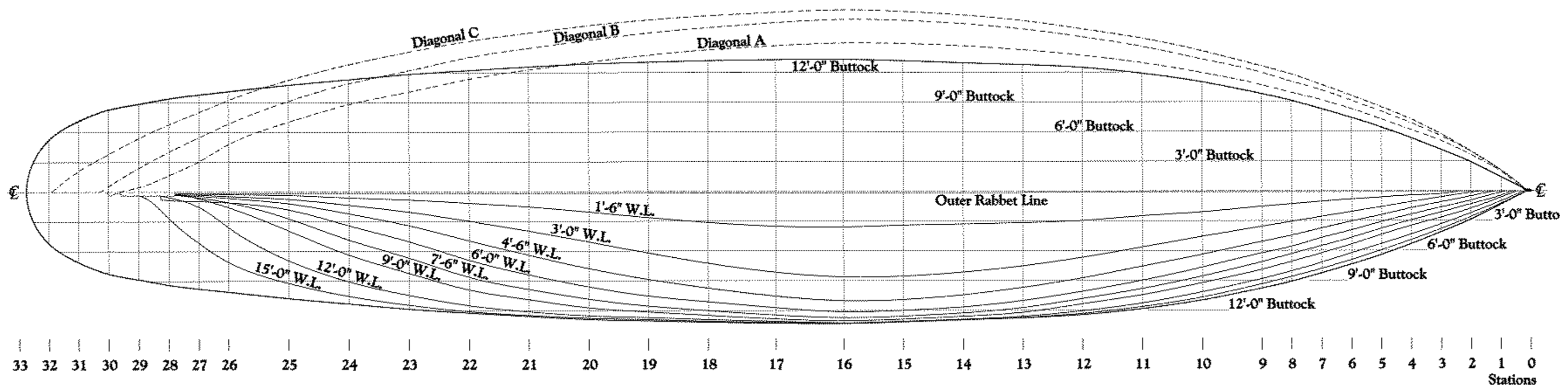
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HEIGHTS	SHEER		20-2-1	19-9-5	19-5-4	19-1-6	18-9-7	18-6-7	18-3-6	18-1-0	17-10-4	17-6-2	17-2-1	16-10-5	16-8-4	16-7-0	16-5-4	16-4-6
	12' BUTTOCK													11-6-7	8-4-3	6-8-7	5-11-6	5-8-7
	9' BUTTOCK									17-1-7	12-9-7	8-5-0	6-1-5	4-7-2	3-9-4	3-5-3	3-3-0	3-4-0
	6' BUTTOCK						15-4-1	11-0-0	8-6-5	6-8-4	5-7-6	4-1-6	3-3-3	2-7-5	2-4-7	2-2-6	2-2-3	2-1-4
	3' BUTTOCK				10-5-4	7-7-5	5-9-5	4-4-7	3-6-1	2-11-1	2-6-0	1-11-3	1-8-3	1-6-1	1-5-3	1-4-0	1-4-6	1-4-3
	RABBET		5-6-6	2-5-1	0-10-3	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0
HALF-BREADTHS	SHEER		1-5-3	2-10-4	4-1-5	5-3-3	6-5-0	7-4-5	8-3-2	9-1-3	9-9-5	10-11-7	11-10-3	12-3-4	12-9-1	12-11-4	13-1-7	13-3-6
	15'-0 W.L.		1-3-4	2-7-4	3-9-6	4-11-0	5-11-3	6-11-0	7-9-6	8-7-3	9-4-3	10-8-3	11-7-6	12-4-1	12-8-5	12-11-2	13-1-7	13-3-4
	12'-0 W.L.		1-0-4	2-2-6	3-4-0	4-4-2	5-3-6	6-3-0	7-2-3	8-0-6	8-10-0	10-2-0	11-3-3	12-0-7	12-7-2	12-10-6	13-1-2	13-3-0
	9'-0 W.L.		0-8-2	1-9-2	2-7-7	3-5-6	4-4-6	5-4-2	6-3-0	7-1-2	7-10-7	9-3-0	10-5-3	11-4-3	12-2-3	12-7-1	12-10-3	13-0-3
	7'-6" W.L.		0-5-6	1-4-0	2-1-4	2-11-3	3-9-3	4-8-4	5-6-2	6-3-7	7-2-1	8-6-7	9-9-7	10-10-0	11-8-5	12-3-1	12-7-0	12-8-5
	6'-0 W.L.		0-2-0	0-11-4	1-7-5	2-4-1	3-1-2	3-11-2	4-8-4	5-2-0	6-3-1	7-8-2	9-0-0	10-1-2	11-0-1	11-7-6	12-0-0	12-1-5
	4'-6" W.L.			0-6-3	1-1-4	1-8-4	2-4-6	3-0-5	3-9-0	4-5-5	5-1-1	6-4-6	7-6-7	8-9-1	9-8-1	10-5-1	10-10-7	11-0-2
	3'-0 W.L.			0-2-1	0-7-3	1-1-1	1-7-1	2-1-0	2-6-7	3-0-7	3-7-0	4-7-0	5-7-7	6-8-2	7-6-2	8-1-6	8-6-5	8-7-3
	1'-6" W.L.				0-3-4	0-5-1	0-7-3	0-9-7	1-0-5	1-3-5	1-6-5	2-1-3	2-6-0	2-11-6	3-2-0	3-3-5	3-5-0	3-7-1
DIAGONALS	A		1-11-0	3-7-2	5-0-4	6-1-7	7-3-0	8-3-2	9-2-0	9-11-1	10-8-3	11-11-2	12-11-1	13-8-2	14-3-1	14-8-0	14-10-7	14-11-4
	B		2-0-4	3-11-3	5-6-1	6-10-1	8-0-2	9-1-3	10-2-0	11-0-4	11-11-4	13-3-5	14-6-7	15-6-4	16-4-2	16-10-6	17-2-7	17-4-0
	C		2-0-4	3-11-4	5-8-3	7-2-4	8-5-4	9-7-2	10-8-4	11-9-1	12-7-1	14-1-4	15-4-6	16-4-1	17-3-1	17-9-2	18-1-4	18-3-4

STATIONS 17–32

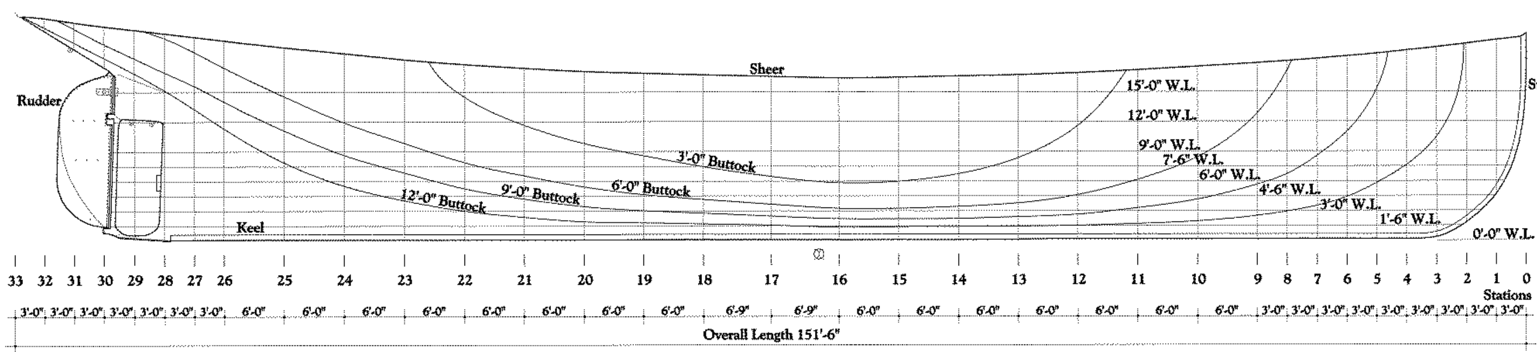
		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
HEIGHTS	SHEER	16-6-5	16-8-3	16-10-4	17-0-6	17-4-5	17-9-3	18-3-0	18-10-4	19-5-5	20-2-0	20-6-3	20-10-5	21-3-0	21-7-1	22-0-0	22-5-3
	12' BUTTOCK	6-5-4	7-5-3	8-6-6	9-9-5	11-7-2	14-6-0										
	9' BUTTOCK	3-8-0	4-1-6	4-7-6	5-5-2	6-6-6	7-10-7	9-10-0	11-10-7	14-6-5	17-5-2	19-1-1	20-4-3				
	6' BUTTOCK	2-5-6	2-9-2	3-0-3	3-5-4	4-2-6	5-3-3	6-10-2	8-7-4	11-1-2	14-0-2	15-7-1	16-11-4	18-4-5	19-9-6	21-4-4	
	3' BUTTOCK	1-6-6	1-8-2	1-9-6	1-11-2	2-4-6	3-0-1	4-0-6	5-6-2	7-10-4	11-3-5	13-6-7	15-1-7	16-9-6	18-5-1	20-1-5	21-6-0
	RABBET	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-8-0	0-9-2	0-9-2	1-6-6				
HALF-BREADTHS	SHEER	13-3-3	13-2-1	13-0-3	12-10-4	12-6-7	12-2-5	11-9-7	11-3-4	10-9-0	10-1-1	9-9-3	9-4-6	8-10-2	8-3-1	7-1-3	5-2-6
	15'-0 W.L.	13-2-3	13-1-1	12-10-7	12-9-0	12-5-5	12-0-7	11-5-1	10-6-3	9-2-6	7-1-2	5-2-1	2-8-3	0-3-7			
	12'-0 W.L.	13-2-1	12-11-0	12-8-4	12-6-1	12-1-1	11-5-5	10-5-2	9-0-7	6-11-3	3-7-1	1-6-4	0-6-5				
	9'-0 W.L.	12-10-2	12-6-0	12-1-6	11-8-4	10-10-4	9-10-5	8-2-6	6-4-1	3-11-2	1-7-0	0-8-2	0-2-5				
	7'-6" W.L.	12-5-1	12-0-2	11-6-1	10-10-0	9-10-1	8-7-3	6-8-2	4-10-3	2-8-5	1-5-0	0-7-0	0-3-6				
	6'-0 W.L.	11-8-6	11-1-5	10-6-4	9-7-2	8-4-5	6-10-7	5-0-4	3-5-1	1-9-7	1-1-1	0-11-3	0-10-0				
	4'-6" W.L.	10-6-7	9-9-0	8-10-3	7-9-1	6-4-7	4-11-3	3-5-1	2-2-5	1-2-3	0-7-6	0-5-0	0-3-5				
	3'-0 W.L.	8-1-7	7-3-0	6-2-5	5-1-0	3-11-6	2-11-5	2-0-5	1-4-2	0-9-0	0-5-3	0-3-4	0-2-3				
	1'-6" W.L.	3-6-2	3-1-5	2-7-5	2-0-6	1-6-3	1-1-7	0-9-7	0-7-2	0-4-2	0-3-0	0-2-5					
DIAGONALS	A	14-8-2	14-3-2	13-8-6	13-0-7	12-3-3	11-4-1	10-0-6	8-9-5	7-0-3	4-9-2	3-0-7	1-8-0	0-5-5			
	B	17-0-1	16-5-3	15-11-5	15-3-1	14-5-3	13-6-3	12-2-6	10-10-6	9-1-5	6-10-0	5-4-7	4-0-3	2-4-7	0-6-5		
	C	18-0-5	17-7-2	17-2-2	16-8-6	15-11-6	15-2-0	14-0-7	12-9-3	11-2-7	9-2-6	8-0-0	6-9-4	5-7-6	3-9-4	2-1-0	



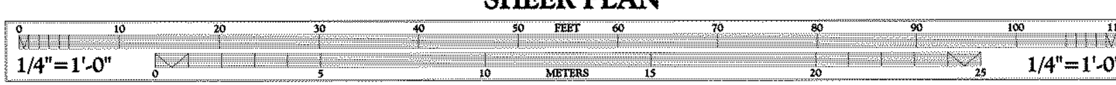
BODY PLAN



HALF-BREADTH PLAN



SHEER PLAN



DRAWN BY: DANA LOCKETT, 1997

HERCULES RECORDING PROJECT, 1997
NATIONAL PARK SERVICE
UNITED STATES DEPARTMENT OF THE INTERIOR

STEAM TUG HERCULES
SAN FRANCISCO MARITIME NATIONAL HISTORICAL PARK, HYDE STREET PIER
SAN FRANCISCO COUNTY

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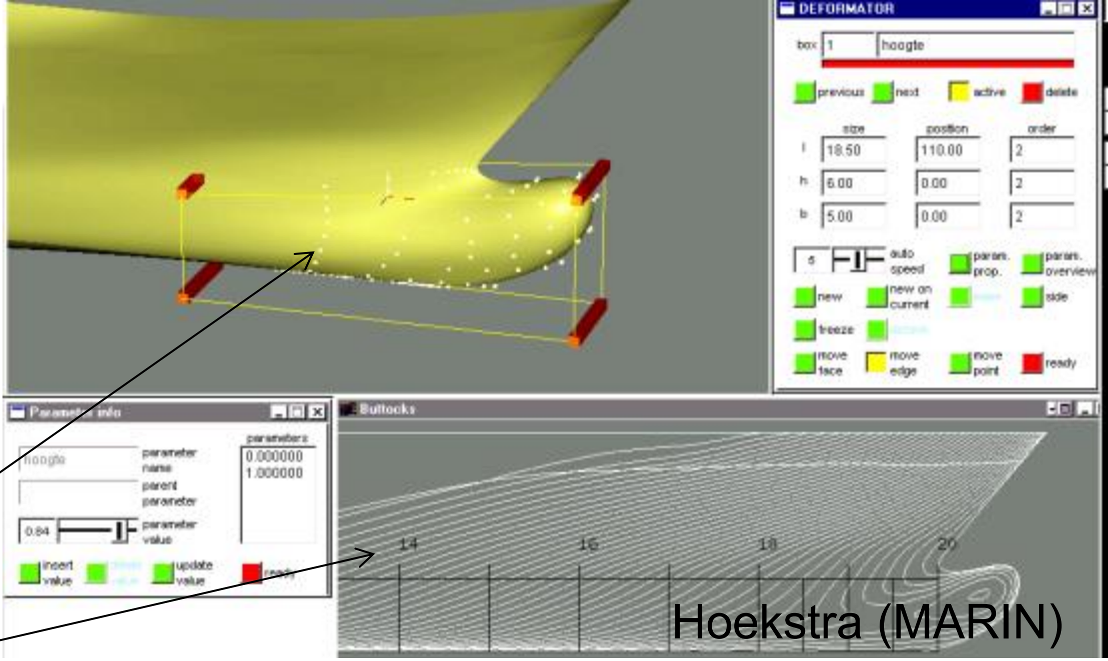
SHEET 5 OF 7 SHEETS

HISTORIC AMERICAN ENGINEERING RECORD
CA-62

B-Splines
Basic definition



$$s(u) = \sum_{i=0}^{n} d_i N_i^k(u) \qquad n \geq k\text{-}1 \qquad (4$$

$s(u)$ = Points along the curve as a function of parameter u

d_i = control points also known as the weight or the point coefficients.

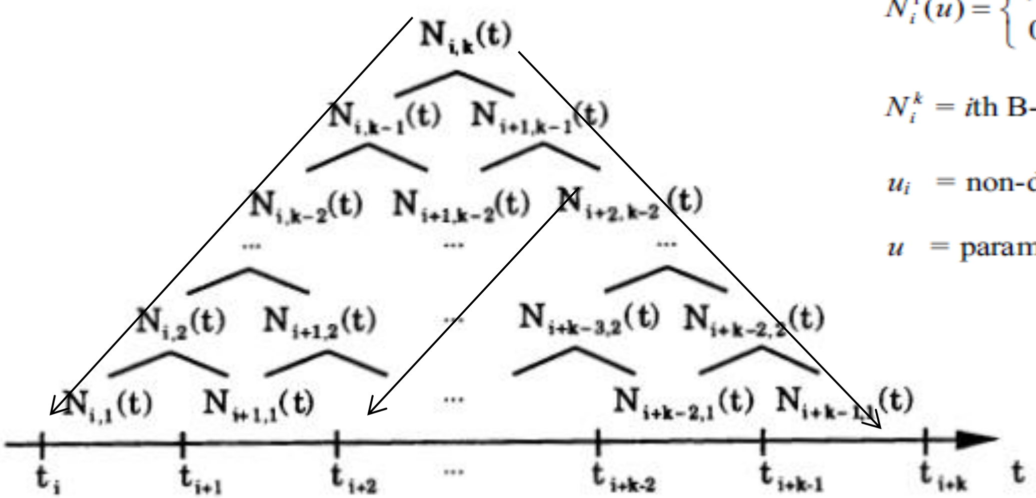
N_i^k = ith B-spline basis function of order k.

B-Splines
B(asis)-splines

Basis functions (not directly visible, but very important)

$$\sum_{i=0}^{k} N_{i,k}(t) = 1.$$

(here: t = u)



$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u)$$

$$N_i^1(u) = \begin{cases} 1, u_i \le u < u_{i+1} \\ 0, \text{otherwise} \end{cases}$$

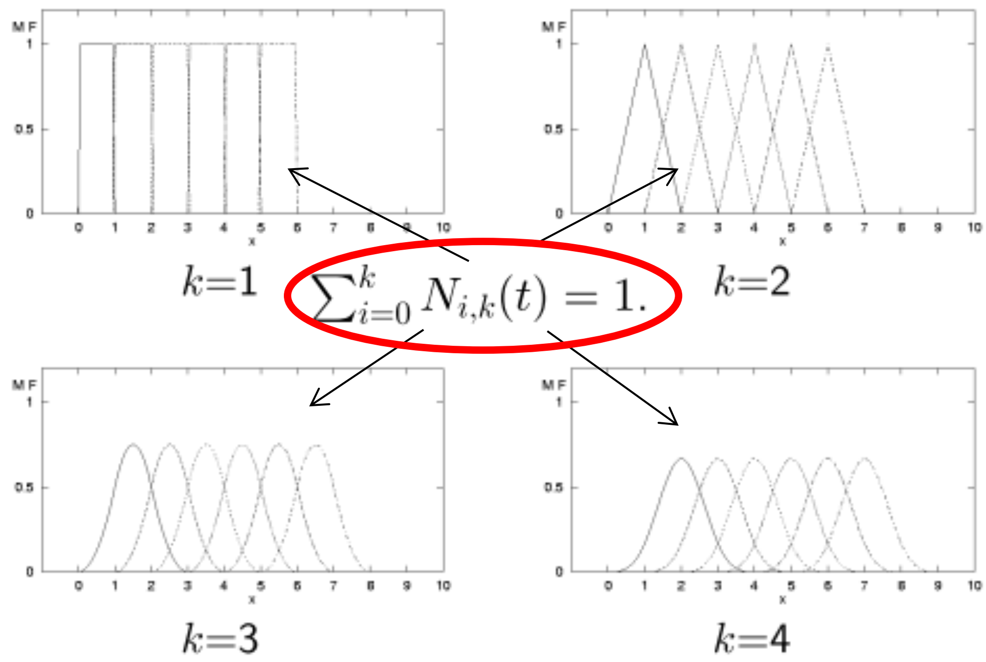
N_i^k = ith B-spline basis function of order k.

u_i = non-decreasing set of real numbers also called as the knot sequence.

u = parameter variable.

B-Spline
Basis functions N

- Examples of Basis functions of different order k
- N(i,k) is k-2 times continously differentiable



B-splines
The interval points t

- Uniformly spaced
- Chord length method

$$t_0 = 0$$

$$t_k = \frac{\sum_{i=1}^{k} |\mathbf{D}_i - \mathbf{D}_{i-1}|}{L} \quad \text{for } k = 1, \cdots, n-1$$

$$t_n = 1$$

- Centripetal method

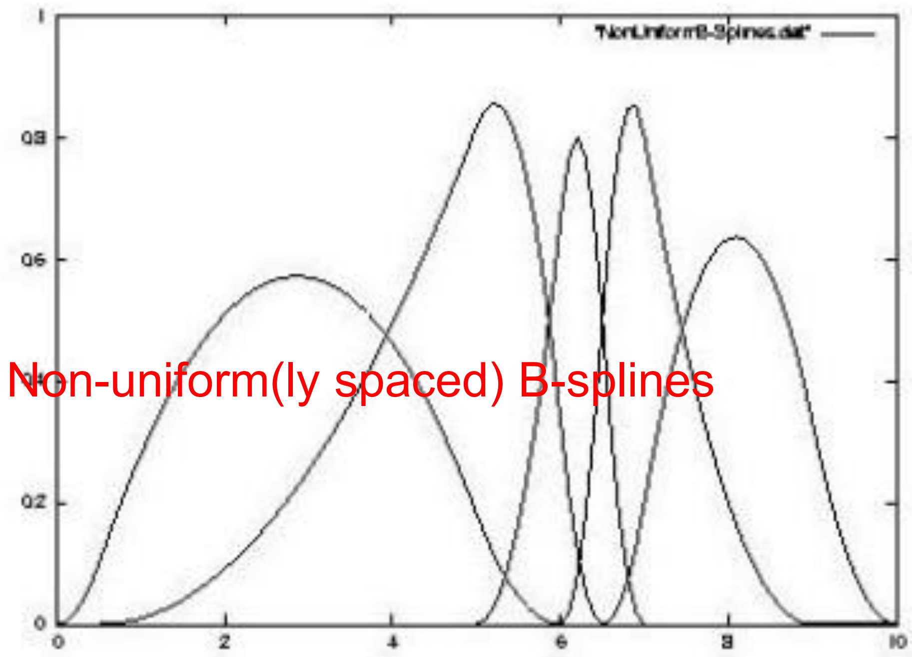
The parameters on $[0,1]$:

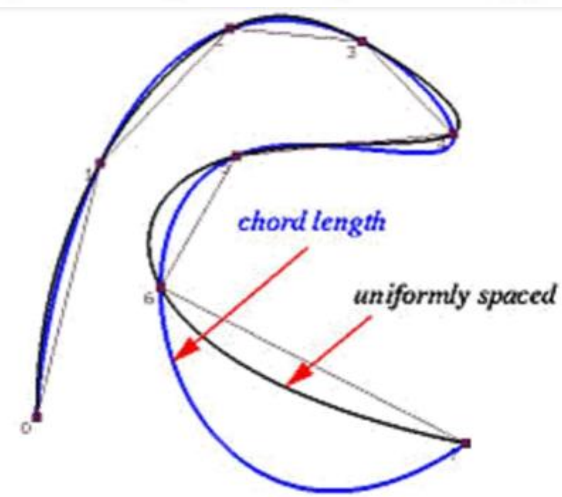
$$t_0 = 0$$

$$t_k = \frac{\sum_{i=1}^{k} |\mathbf{D}_i - \mathbf{D}_{i-1}|^{\alpha}}{L} \quad \text{for } k = 1, 2, \cdots, n-1$$

$$t_n = 1$$

- Gauss approximation
- Arc length method, etc…





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B-spline curves
The interval points t II

Copy from Parametric Spline Curves

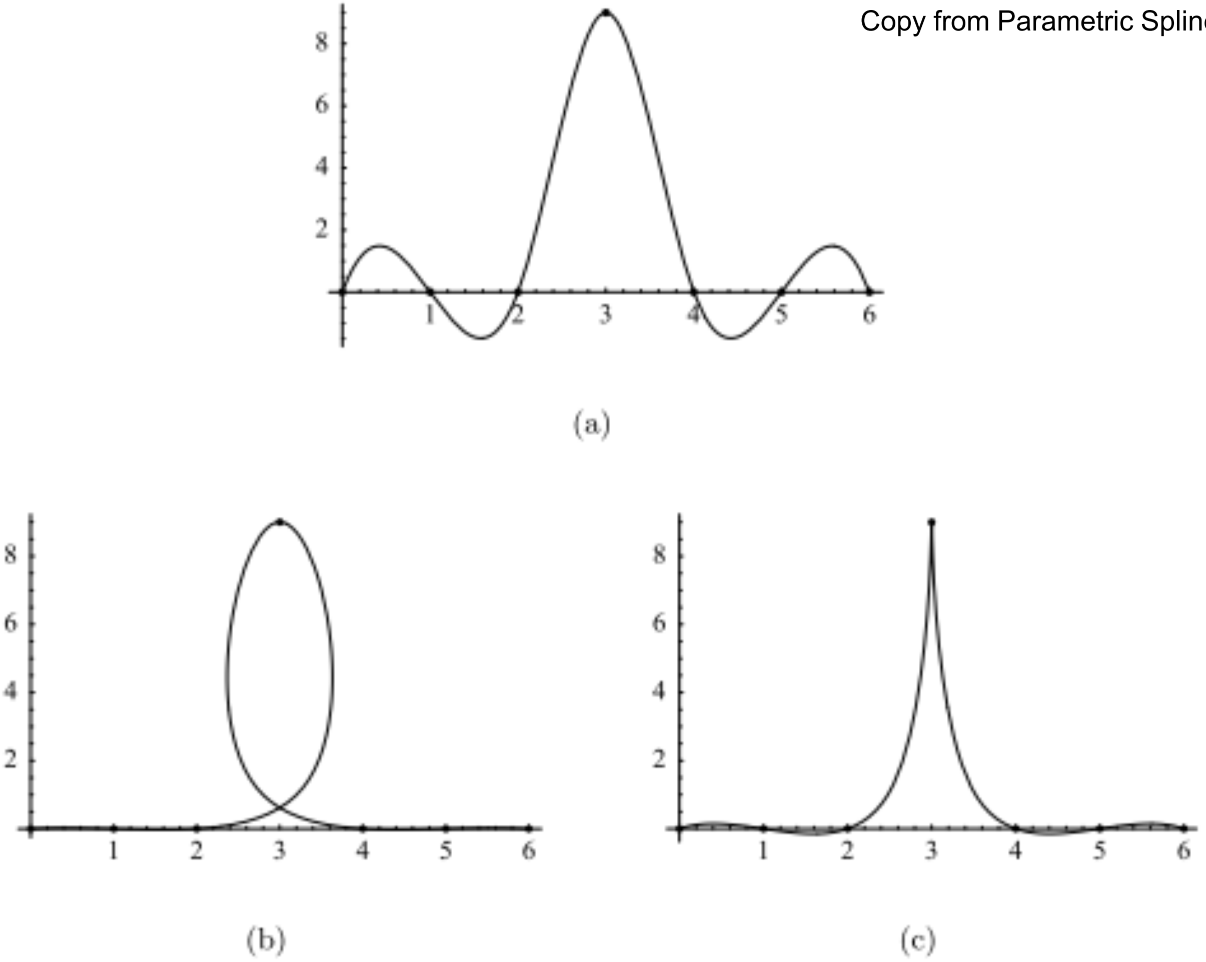
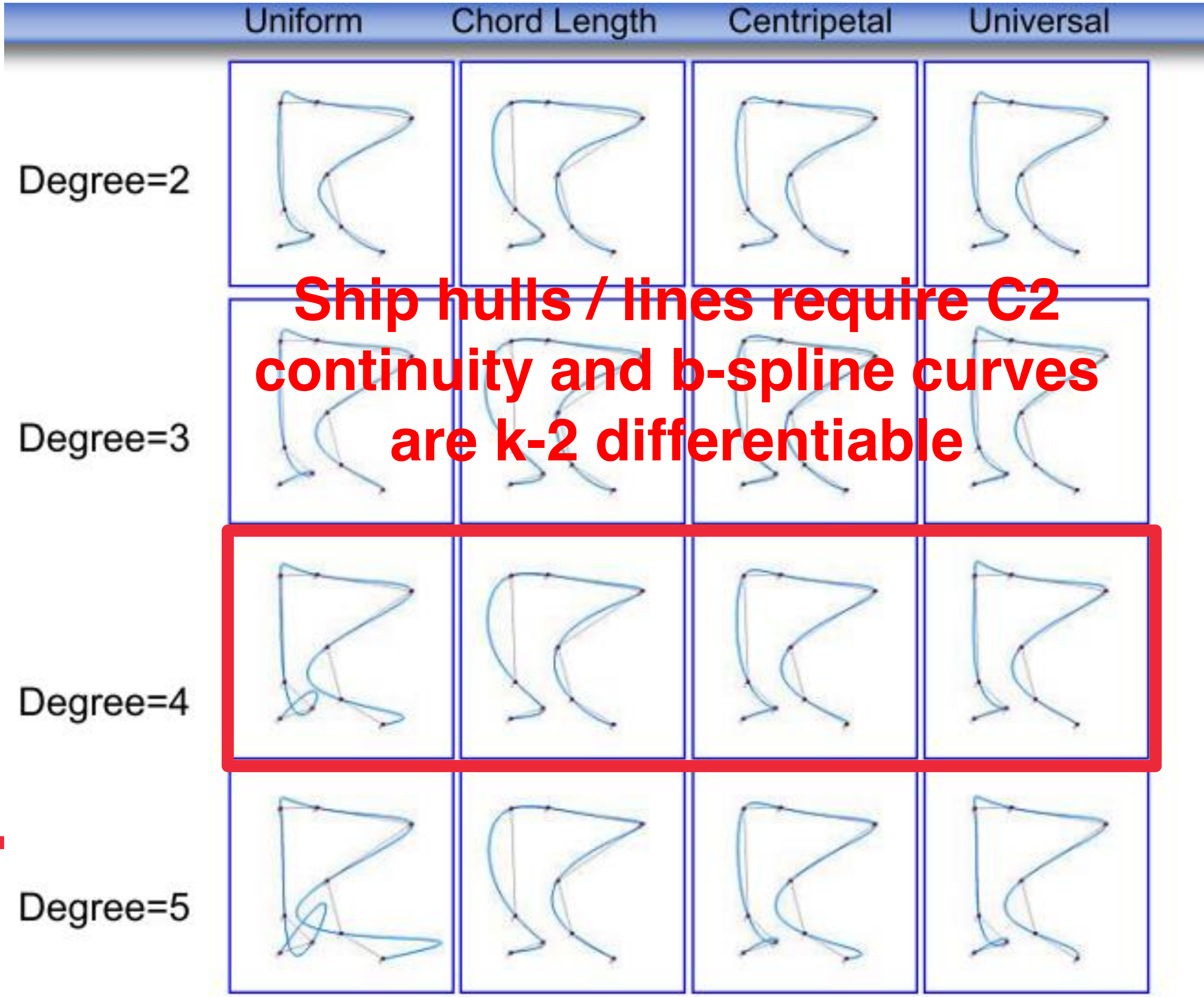


Figure 6.4. Parametric, cubic spline interpolation with uniform parametrisation (a), cord length parametrisation (b), and centripetal parametrization (c).

B-spline curves
The impact of the degree

- Degree and t-vector can hardly be treated seperately
- Increasing degree leads to wiggling, loops and oscilations (as for other polynomials)
- Keeping the degree as low as possible
- What is the robustness of these methods?



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B-spline surfaces

- Definition analogous to B-splines curves

$$Q(u,w) = \sum_{i=1}^{n} \sum_{j=1}^{m} B_{i,j} N_{i,k}(u)\, M_{j,\ell}(w)$$

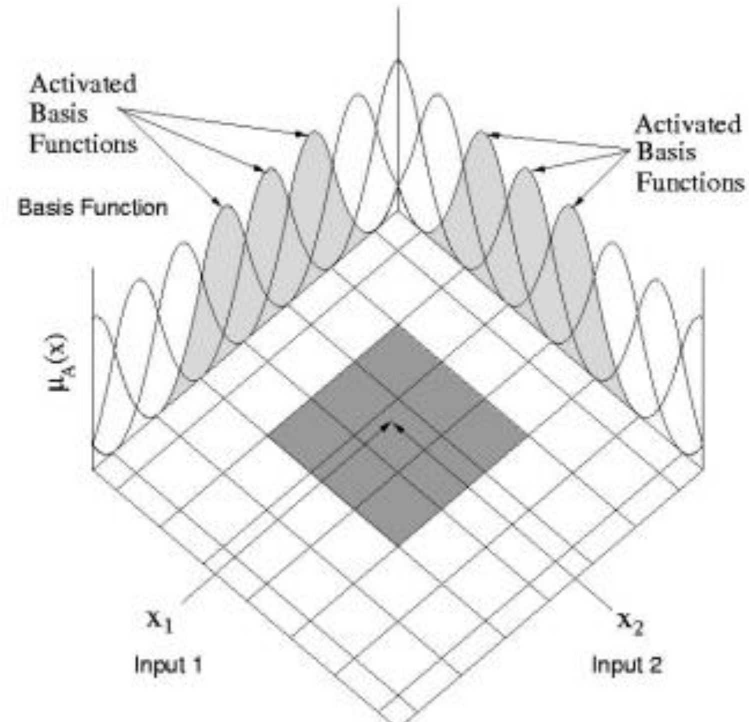
where

$$N_{i,1}(u) = \begin{cases} 1 \text{ if } x_i \le u < x_{i+1} \\ 0 \text{ otherwise} \end{cases}$$

$$N_{i,k}(u) = \frac{(u - x_i) N_{i,k-1}(u)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - u) N_{i+1,k-1}(u)}{x_{i+k} - x_{i+1}}$$

$$M_{j,1}(w) = \begin{cases} 1 \text{ if } y_j \le w < y_{j+1} \\ 0 \text{ otherwise} \end{cases}$$

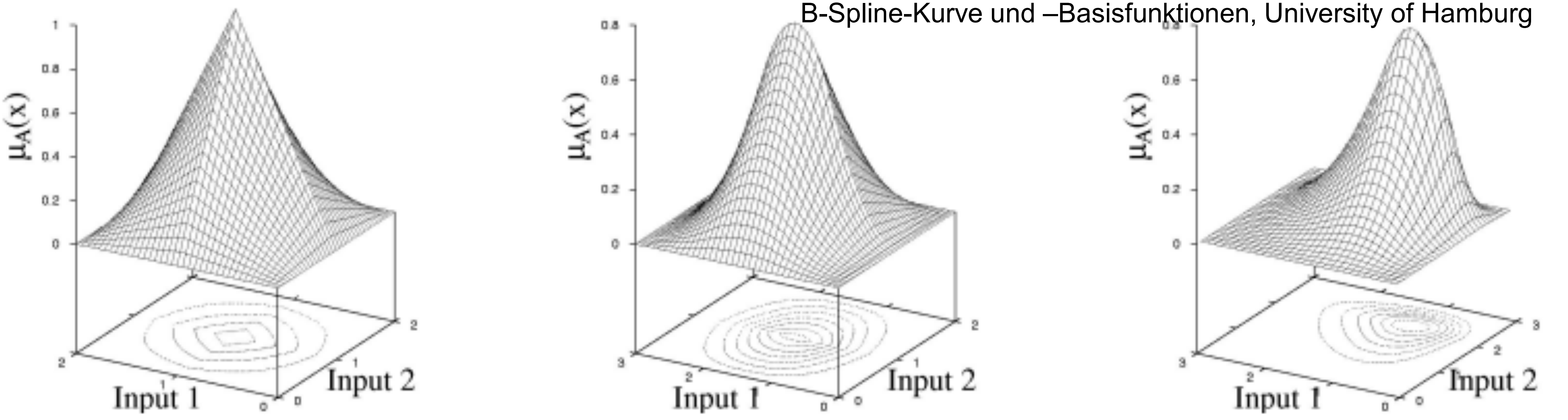
$$M_{j,\ell}(w) = \frac{(w - y_j) M_{j,\ell-1}(w)}{y_{j+\ell-1} - y_j} + \frac{(y_{j+\ell} - w) M_{j+1,\ell-1}(w)}{y_{j+\ell} - y_{j+1}}$$



B-Spline-Kurve und –Basisfunktionen, University of Hamburg

B-spline surfaces

B-Spline-Kurve und –Basisfunktionen, University of Hamburg



(a) Tensor product of two, order 2 univariate B-splines.

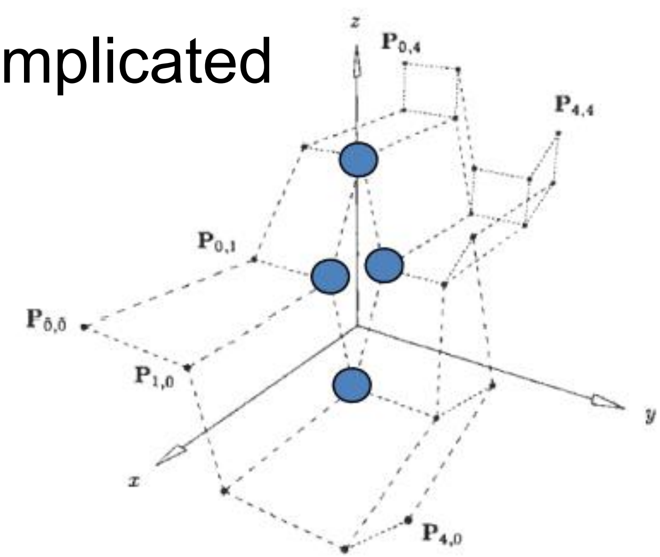
(b) Tensor product of one order 3 and one order 2 univariate B-splines.

(c) Tensor product of two univariate B-splines of order 3.

NURBS

- NURBS have additional weight functions for additional control
- Useful for complicated shapes

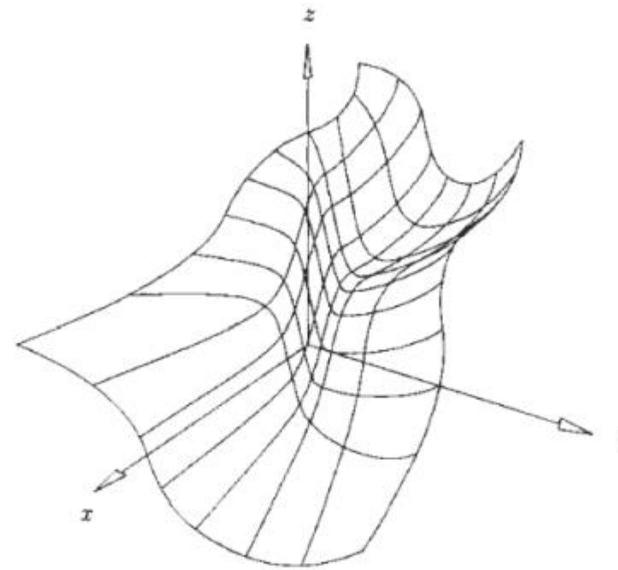
$$\sum_{i=0}^{n}\sum_{i=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{ij} \boldsymbol{P}_{ij}$$



Control net

$$U = V = \{0,0,0,1/3,2/3,1,1,1\}$$

$$w_{ij}(\bullet) = 10,\ w_{ij}(\cdot) = 1$$

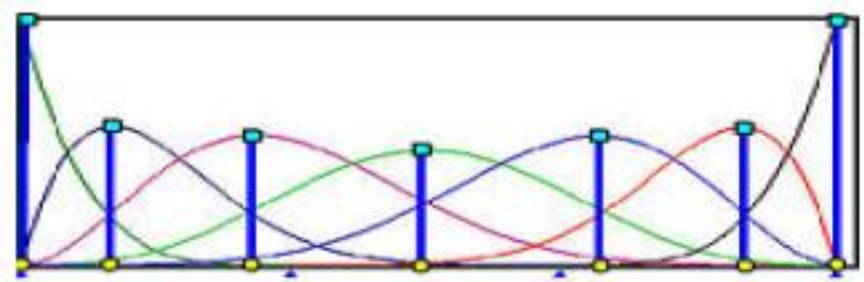


NURBS Surface

Mirela Ben-Chen, B-Spline and Subdivision Surfaces

B- spline surfaces

- B-spline surface patch utilizes open knot vectors. The open knot vectors cause each surface edge to coincide with the B-spline curve defined by the edge control vertices
- The defined hull must be ”water-tight” – all patches perfectly sealed / connected



Basis functions

$$\begin{bmatrix} B_0^p(t_0) & B_1^p(t_0) & \cdots & B_N^p(t_0) \\ B_0^p(t_1) & B_1^p(t_1) & \cdots & B_N^p(t_1) \\ \vdots & \vdots & \vdots & \vdots \\ B_0^p(t_{np}) & B_1^p(t_{np}) & \cdots & B_N^p(t_{np}) \\ B_0'^p(t_0) & B_1'^p(t_0) & \cdots & B_N'^p(t_0) \\ B_0'^p(t_{np}) & B_1'^p(t_{np}) & \cdots & B_N'^p(t_{np}) \end{bmatrix} \cdot \begin{vmatrix} P_0 \\ P_1 \\ \vdots \\ P_N \end{vmatrix} = \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{np} \\ tg(\beta) \\ tg(\delta) \end{bmatrix}$$

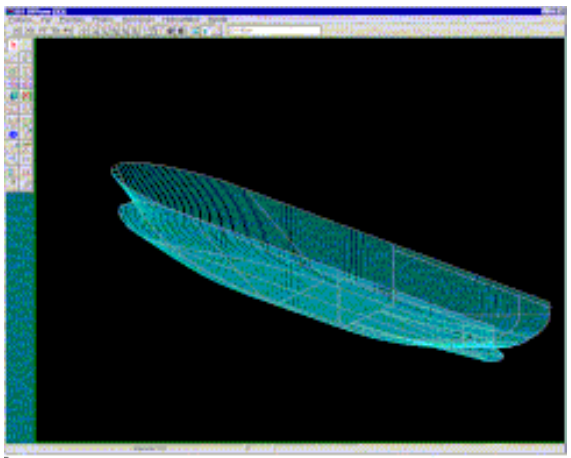
Data points

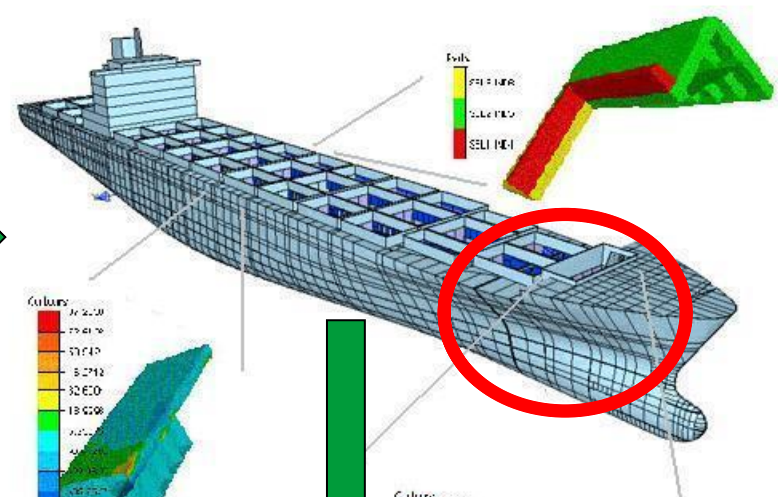
$$[M] \cdot [P] = [Q] \Rightarrow [M]^T \cdot [M] \cdot [P] = [M]^T \cdot [Q] .$$

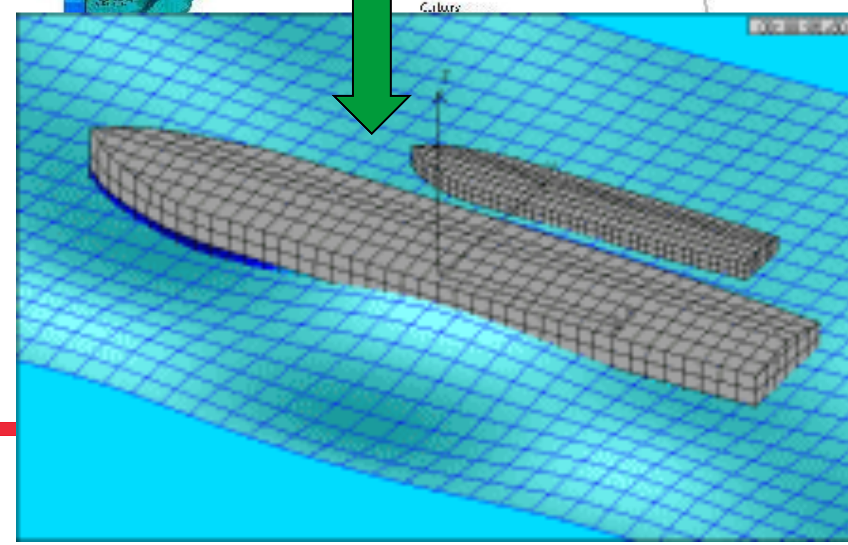
Control points

Excursion: practical problem: models not ”water-tight” after transfer

- CAD – FEM – Panel solver





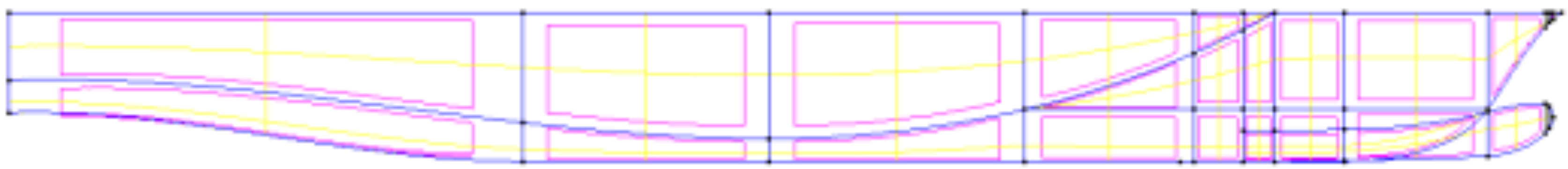


- Some FE tools with problems representing curved area

Fairing – minimization of curvature

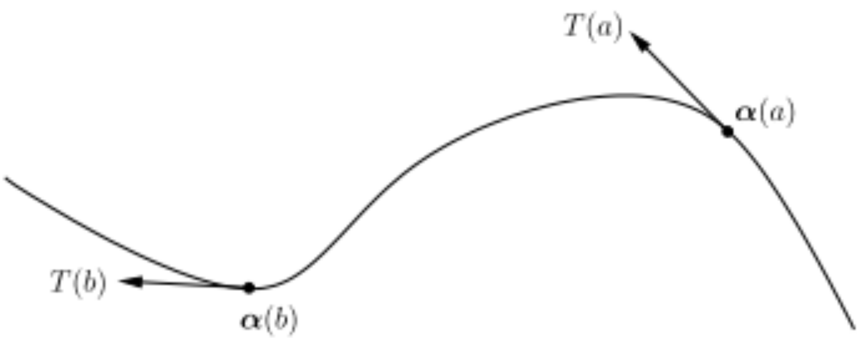
- In naval architecture a fair line has a different meaning as in mathematical context

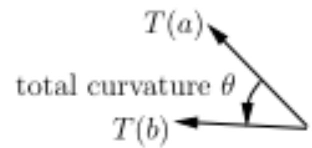
Abt et al. 2001

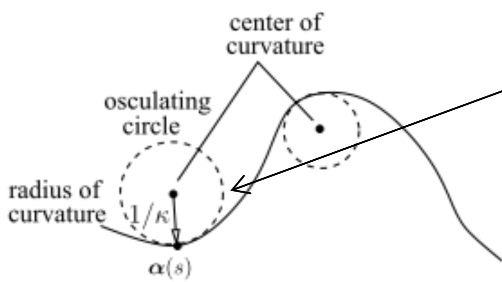


- fairness is important (sometimes manual post-processing) -> surface must be re-faired
- fairness "optimization" is already used to compute the curves
- Minimization of spline-deformation energy, curvature…

Fairing of geometry
what is curvature?







K=1/r

the maximum and minimum values are the principal curvatures, κ1, κ2.

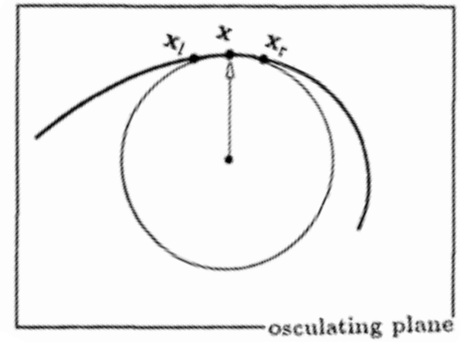
$k_1, k_2 =$	principal curvature
$K = k_1 \cdot k_2$	Gaussian curvature
$H = (k_1 + k_2)/2$	mean curvature
$A = \lvert k_1 \rvert + \lvert k_2 \rvert$	absolute curvature.

The *total curvature* over a closed interval $[a, b]$ measures the rotation of the unit tangent $T(s)$ as s changes from a to b:

$$\begin{aligned} \Phi(a,b) &= \int_a^b \kappa \, ds \\ &= \int_a^b \frac{d\phi}{ds} ds \\ &= \int_a^b d\phi \\ &= \phi(b) - \phi(a). \end{aligned}$$

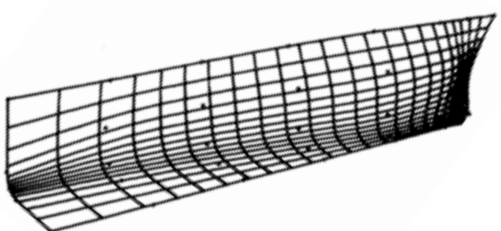
$\frac{dT}{ds} \cdot N$

Fairing of surfaces
Gaussian curvature



Farin and Sapidis 1989

- most points on most surfaces of different sections will have different curvatures;
- The Gaussian curvature is the product of the two principal curvatures K = κ1 κ2.



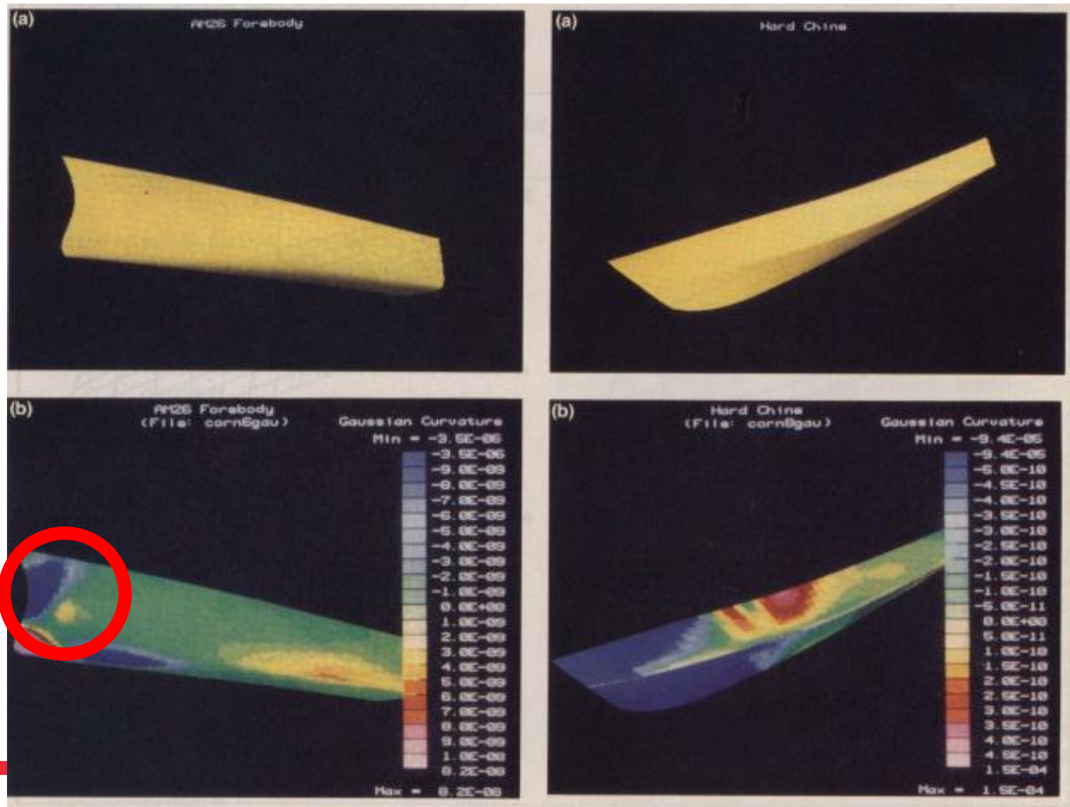


Figure 7. Forebody of US Navy ammunition ship; normally colored image (a), color-coded Gaussian curvature (b).

Figure 8. Ship with hard chine; normally colored image (a), color-coded Gaussian curvature (b).

Rogers et al. 1983

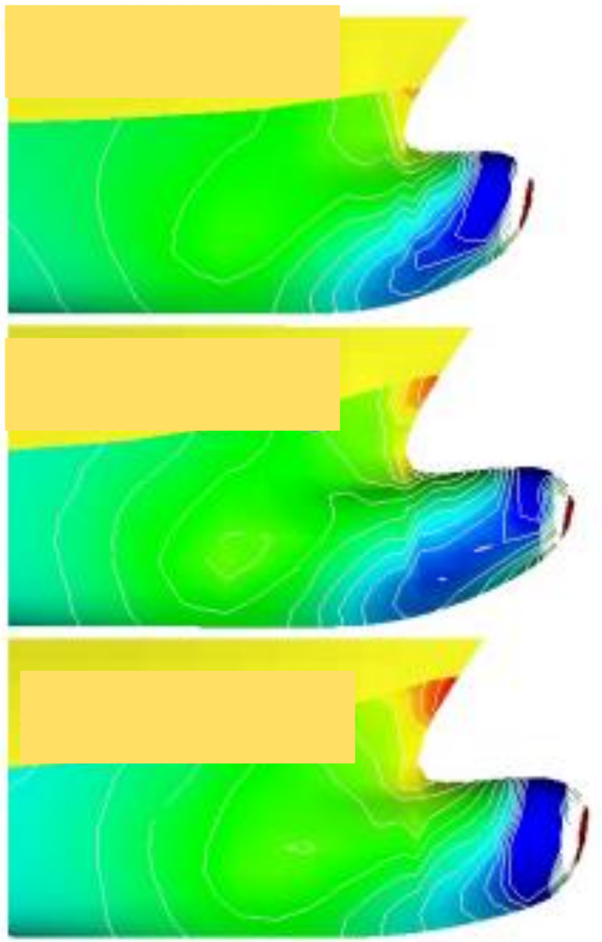
Optimization methods

- Simulation based design
- Feasible design spaces are often non-convex, due to (possible) geometric non-liniarities
- PSO (Particle Swarm Optimization):
  - iteratively improvement of candidate solutions (hull) with respect to a quality measure (CFD, resistance, SMF)

Optimization

Resistance reduction by 10%,

this could lead to fuel cost savings of up to 0.5M$ / year



Summary & Conclusion

- The design of ship-hull is complex, since it requires the generation of lines and surfaces, fairing algorithms ,optimization procedures and decision making criteria

- Each of the disciplnes above is of high complexity with significant space for future development and research

- Complex and abstract mathematical problmes are linked with economic yield

Thank you for your Attention!