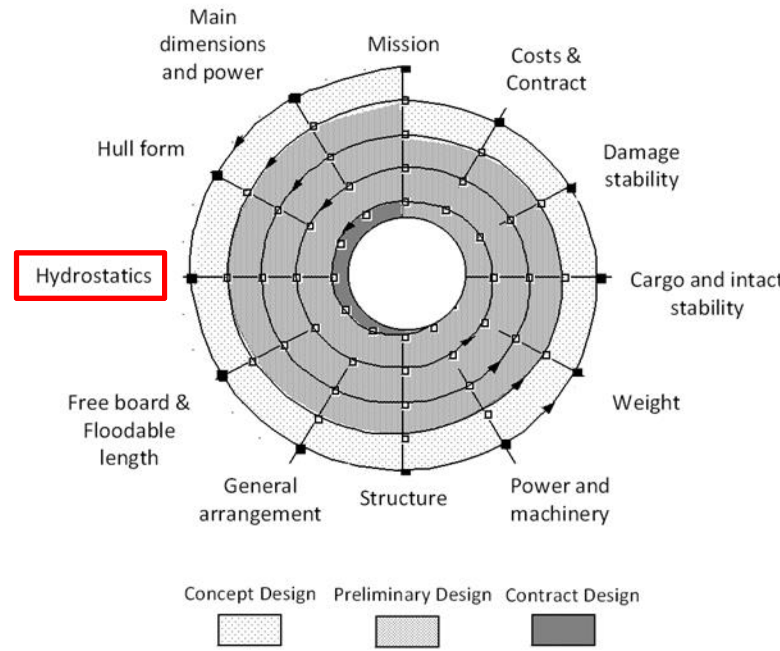
Aalto University
School of Engineering

MEC-E1004 Principles of Naval Architecture

Lecture 5 – Ship hydrostatics

Learning points !

- ❑ Explain the purpose of numerical integration in ship design
- ❑ Understand the background of hydrostatic formulas and methods
- ❑ Explain and apply the basics of transverse stability

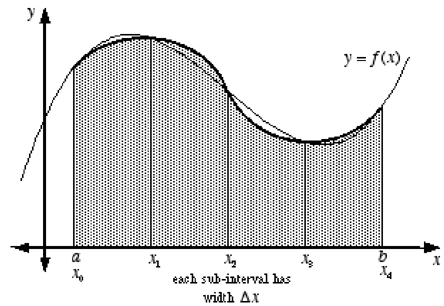


Assignment 5 – Hydrostatics

- Using the hull lines drawing of your hull, estimate the hull volume

- Verify the correctness of your calculations using numerical integration methods (e.g. Simpson Rules on frame area or hull volume)

- Review 2 research articles / book chapters with focus on basic ship hydrostatics and stability



The shaded area bounded by the parabolas (the thicker curves) is approximately equal to the area bounded by $y = f(x)$.

$$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

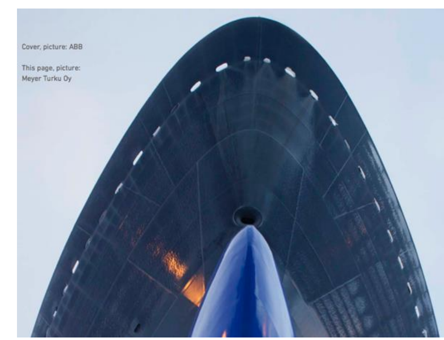


Image credit Meyer Turku

Hydrostatics

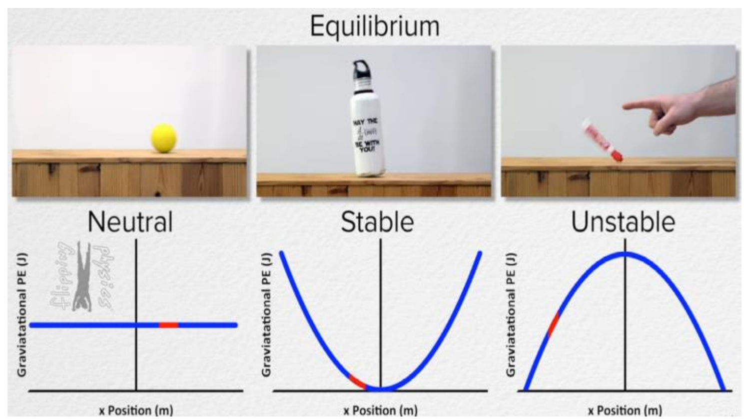
Question: What is hydrostatics and why they are important in ship design?

What is Hydrostatics ?

- Hydrostatics is a branch of physics that deals with the characteristics of fluids at rest. The fluid can be **gas or liquid** exerting pressure on an immersed body.

- Through hydrostatics we investigate a ship's floating position

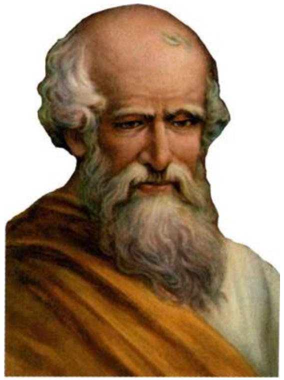
- In turn principles of hydrostatics are applied to assess a ship's floatation and stability



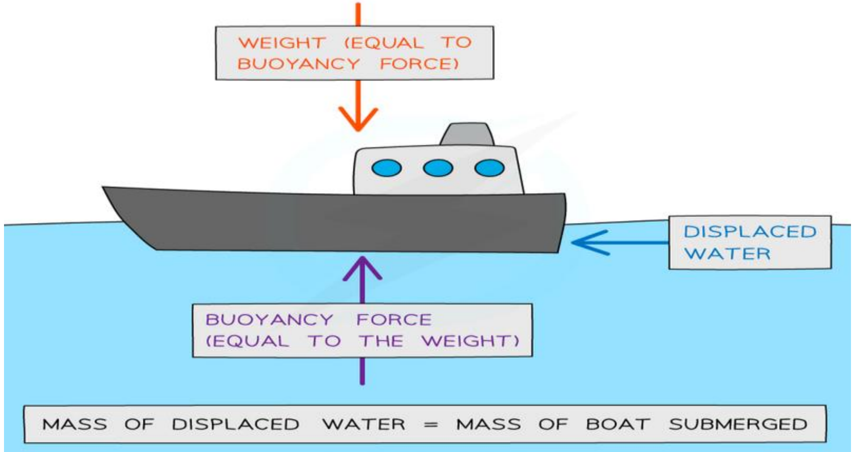
Why do ships float ?

<u>Archimedes principle:</u>*A body floating or submerged in a fluid is buoyed up by a force equal to the weight of the water it displaces.*

$$W_{ship} = \rho_w \times g \times \nabla$$



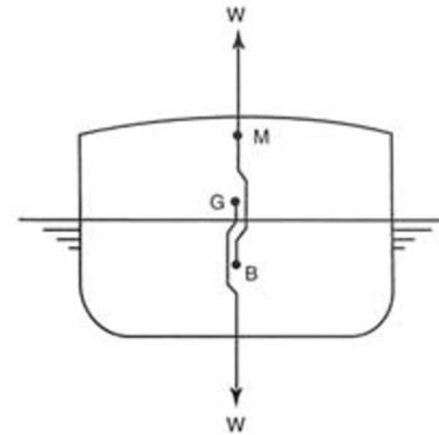
https://science4fun.info/archimedes/



"as the weight of displaced liquid increases the force acting upwards because of water pressure increases…so the ship floats"

Ship transverse stability basics (I)

- A floating ship displaces her own weight when afloat

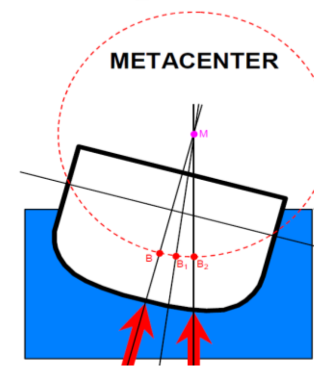


Ship weight W = Buoyancy B

- ✓ ***Weight*** *(w) acts downward through the center of gravity.*

- ✓ ***Buoyancy forces*** *act upward as one force through underwater geometric center. B is the geometric center of the volume displaced by the ship*

- ✓ ***Metacenter (M)*** *lies at the intersection of the vector of buoyancy and the centerline for* 5^o - 10^o heeling angle

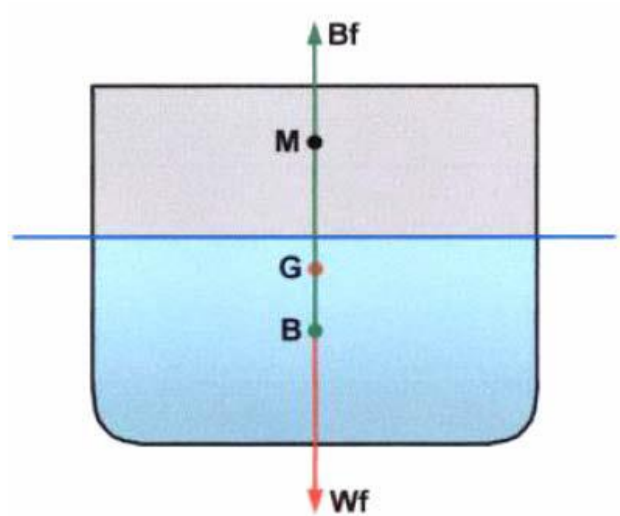


- ✓ *All of the above terms are supported by the "Archimedes Principle" which….remember…it practically states that the actual "all up weight" of a ship and her contents is equal to the weight of water displaced by the hull.*

Ship transverse stability basics (II)

- The centers of buoyancy (B) and gravity (G) strive at all times to remain vertically aligned.

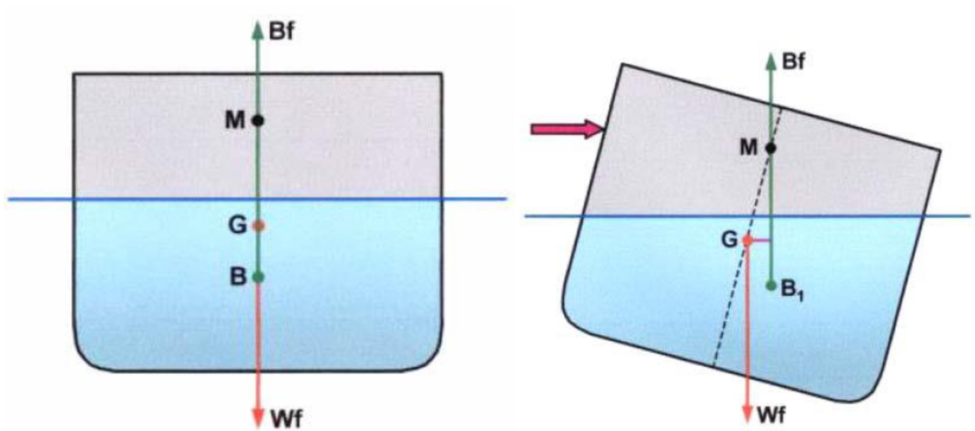
- The measure of a ship's initial stability, when she is upright or nearly upright, is indicated by the height of the metacenter (M) above (G), which is referred to as the **metacentric height (GM) and should be +ve for stable ships**



Ship transverse stability basics (III)

- When a stable ship is caused to heel by an external force, such as wind, wave, or turning motion (not weight shift), the consequent change in underwater hull shape will result in (B) moving to one side (B1) while (G) does not move.

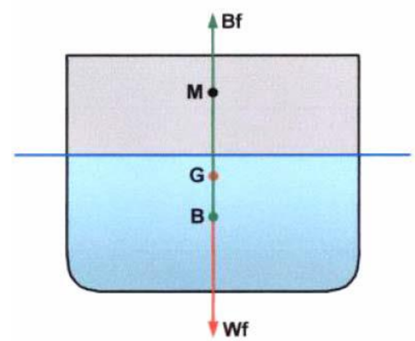
- The horizontal separation between B and G is called righting level GZ. The resulting righting moment, (w x GZ), will cause the vessel to oscillate from side to side as it attempts to realign (B) and (G)



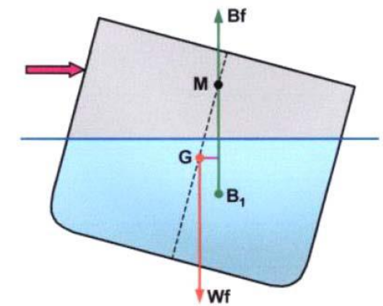
Ship transverse stability basics (IV)

- **Stable condition**
  - *The ship returns to its upright condition after removing the external load.*
  - *KM –KG = GM (+ve value)*
  - *GZ righting lever*
- **Neutral condition**
  - *The ship will have angle of loll after removing the external load.*
  - *KM –KG = 0 and GM = 0*
  - *GZ = 0*
- **Unstable condition**
  - *The ship will continue to heel further after removing the external load.*
  - *KM –KG = GM (-ve value)*
  - *GZ is a capsizing lever*

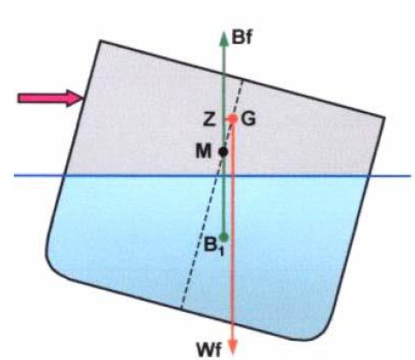
Stable condition



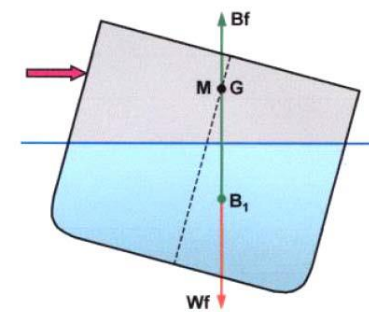
Stable condition



Unstable condition

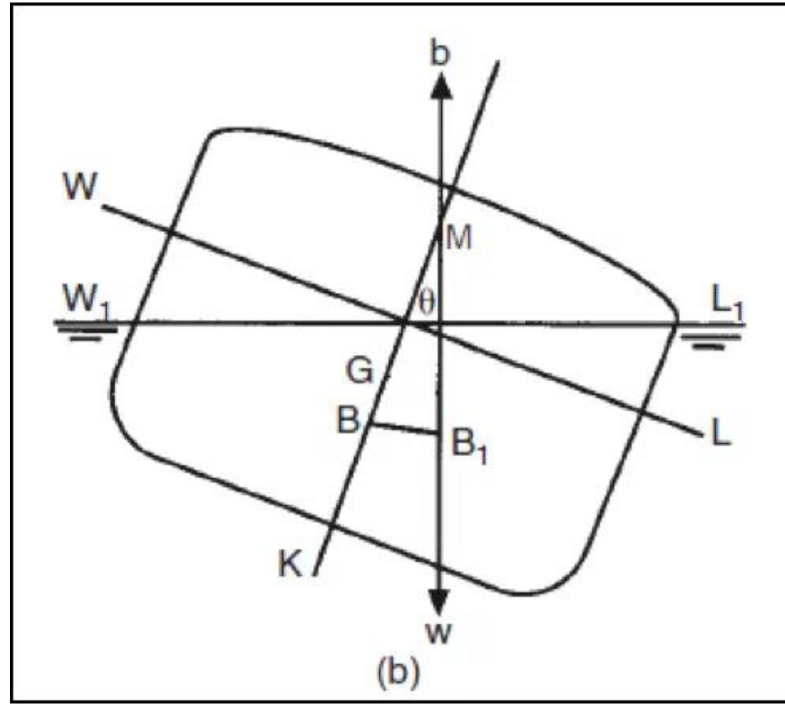


Neutral condition



LCF and Metacentric Height

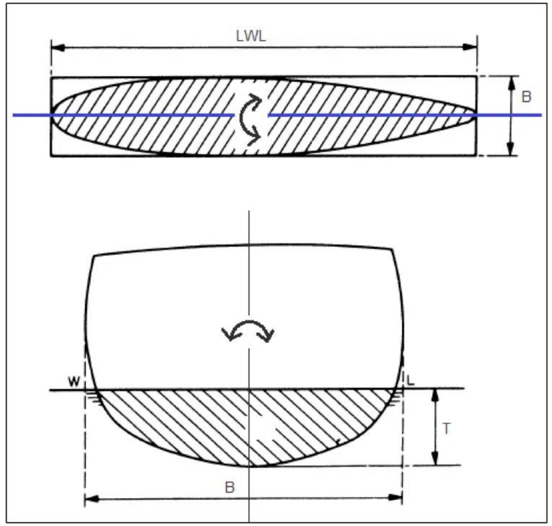
- When the ship floats at a particular draft, any trimming moment acting on the ship would act about a particular point on the water plane. This point is the centroid of the area of the water plane, and is called the center of the floatation.
- The metacentric radius of a ship is the vertical distance between its center of buoyancy and metacenter.
- This parameter can be visualized as the length of the string of a swinging pendulum of the center of gravity of the pendulum coincides the center of buoyancy of the ship. In other words, the ship behaves as a pendulum swinging about its metacenter.



Transverse moment of inertia

$$\textit{Transverse Metacentric height } (BM) = \frac{\textit{Transverse moment of Inertia of Waterplane}}{\textit{Volume Displacement of ship}}$$

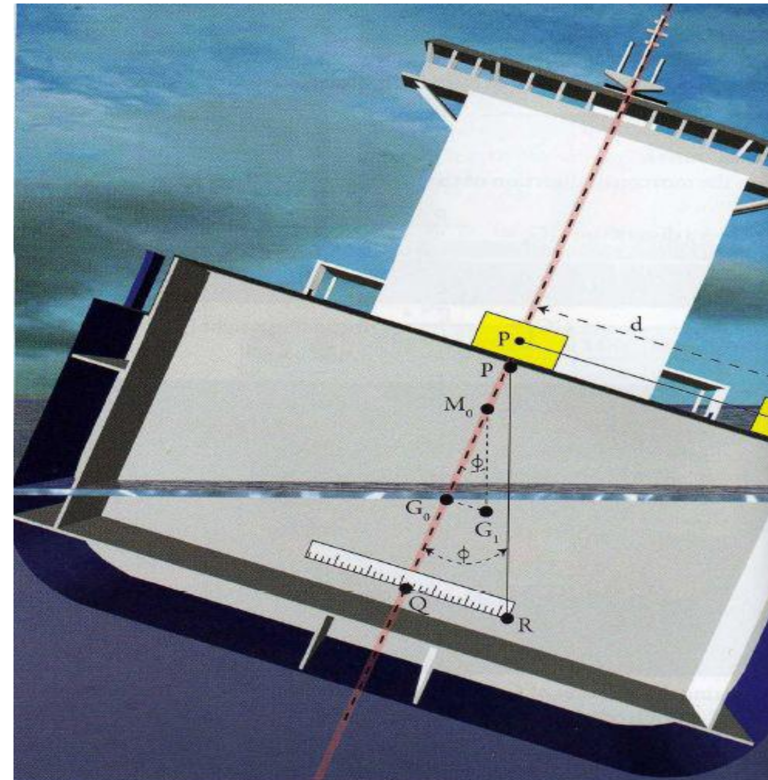
When the ship rolls, if one looks from the top, the entire water plane area seems to oscillate about its longitudinal centroidal axis. ***The area moment of inertia of this waterplane area about its centroidal axis is the transverse moment of inertia of waterplane at the corresponding draft.***



The inclining experiment

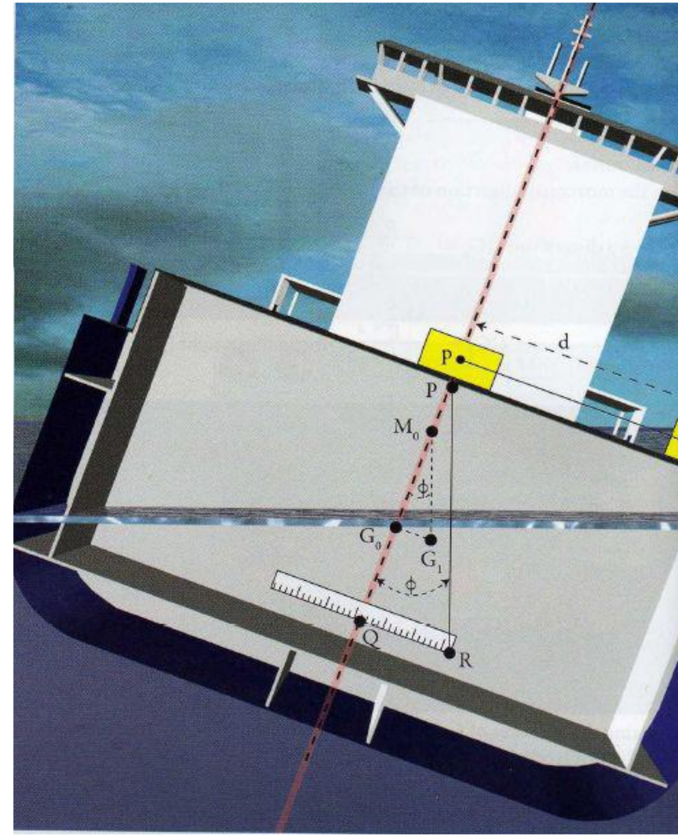
- The inclining experiment is essential to help us determine accurate values of KG and GM in new build vessels

- Test preparation
  - *Weight of the empty ship must be accurate*
  - *The ship must be free to roll*
  - *Sea must be calm with no wind*
  - *No disturbance waves*
  - *The test must be conducted on both starboard and port side with consistent outcome to ensure accuracy.*



Inclining Experiment

- Test set up
  - *A known weight (P) is moved transversely across distance (d) as a result of which the ship lists.*
  - *The weight must be so large that:*
    - ✓ The ship remains within an initial range of stability max list 9- 10º
    - ✓ Equal to about 2 % displacement
  - *(d) is approximately ½ the breadth*
  - *The ship's list due to relocating the weight is accurately measured (using pendulum)*
  - *KG and GM can be calculated.*



Ship Geometry

- ❑ The determination of a ship's geometry is a complex task that requires the consideration of multiple factors

  - *Volumes, dimensions of cargo holds*
  - *Seakeeping, i.e safety in waves*
  - *Resistance, i.e. hull performance and energy efficiency*
  - *Aesthetics*
  - *...*

- ❑ Accurate representation of geometry is important for

  - *Safety and Performance assessment*
  - *Manufacturing*

- ❑ Challenge

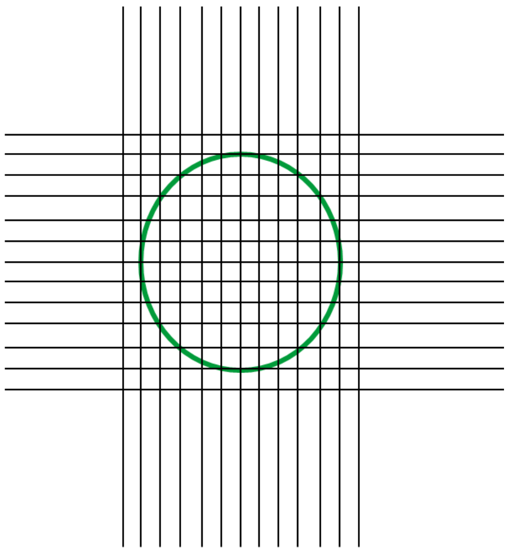
  - *Complex hull shapes*
    - Double curvature in many places
      - Bended beams, plates,..

Exact methods vs numerical aprox.

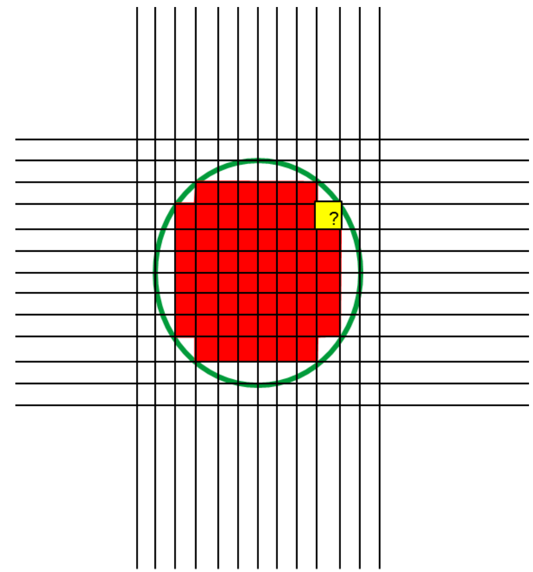
$$A = pr^2 = p(5)^2 = 25p^2 \text{ » } 78.5$$

$$A \approx whole\ \ squares + \frac{1}{2}\ part\ \ squares = 59 + \frac{29}{2} = 73.5$$

$$A \text{ » } 60 + \frac{28}{2} = 74$$



Radius = 10 cm



Difference between simplified and exact method aprox 6%

Numerical integration

- ❑Integration is nowadays carried out using computers, e.g.
  - ✓ *AutoCAD*
  - ✓ *SolidEdge*
  - ✓ *Plot Digitizer*
  - ✓ *NAPA*
  - ✓ *etc…*

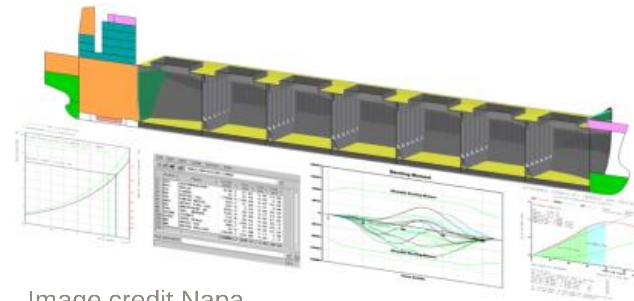


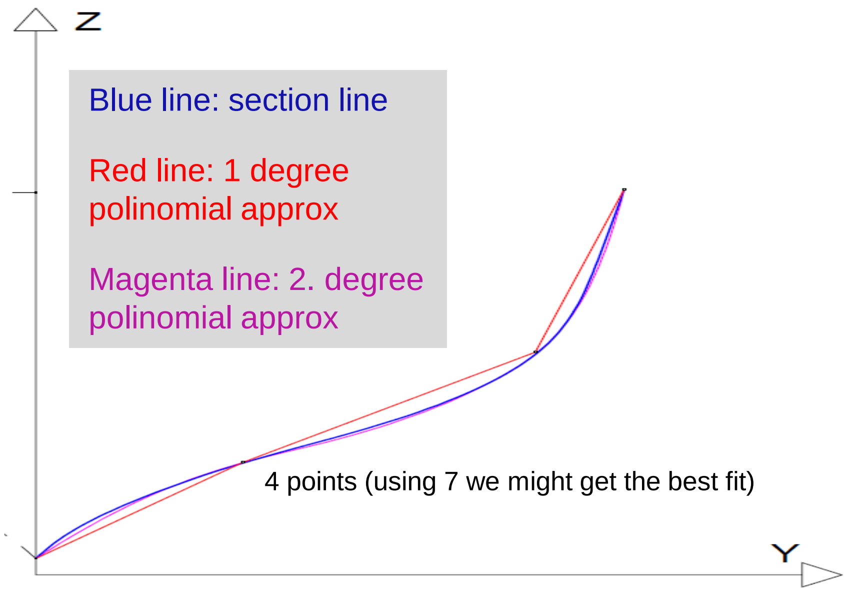
Image credit Napa

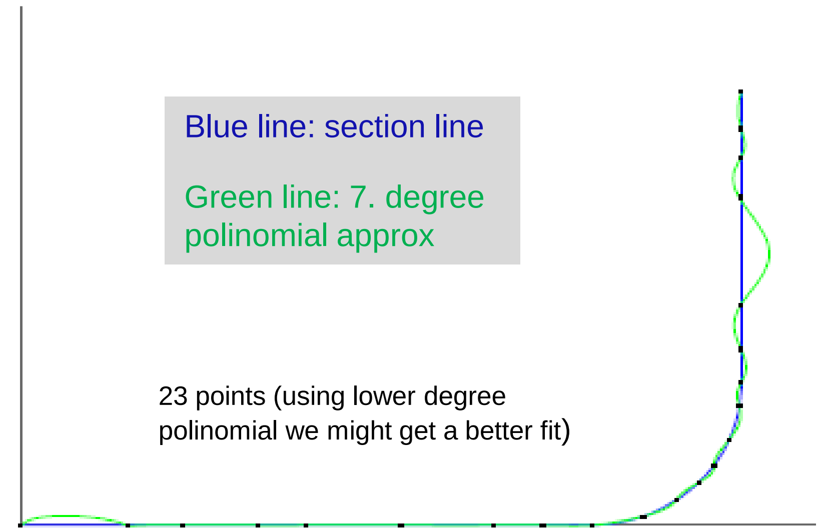
- ❑Numerical estimations do not always represent reality

- ❑Useful for any geometry (even geometries which do not have analytical solutions)

Numerical integration

How many points?...too many is not necessarily good!





Numerical integration

Area

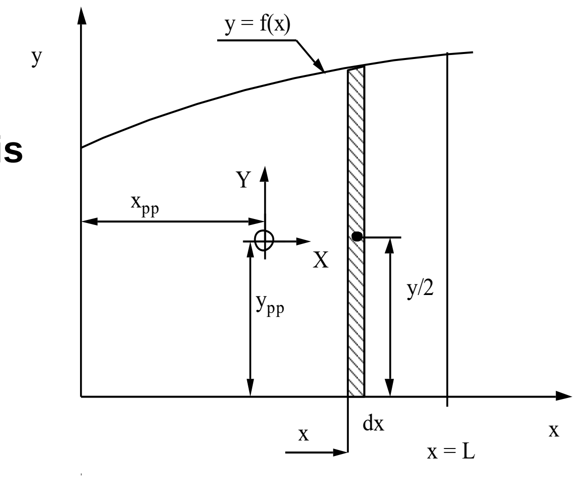
$$A = \int_0^A dA = \int_0^L y\,dx$$

Distance from y-axis

$$x_{pp} = \frac{M_y}{A}$$

Distance from x-axis

$$y_{pp} = \frac{M_x}{A}$$



Moment of area A with respect to y-axis M_y

$$M_y = \int_0^A \left(x + \frac{dx}{2}\right) dA = \int_0^L \left(x + \frac{dx}{2}\right) y\,dx = \int_0^L \left(xydx + \underbrace{\frac{y(dx)^2}{2}}_{\approx 0} \right) \approx \int_0^L xydx$$

Moment with respect to x-axis

$$M_x = \int_0^A \frac{y}{2}\,dA = \int_0^L \frac{y}{2}\,ydx = \frac{1}{2}\int_0^L y^2 dx$$

Parallel axis theorem

$$I_Y = I_y + x_{pp}^2 A$$

Second moment of area (A) with respect to x-axis

$$I_x = \int_0^A y^2 dA = \int_0^L \int_0^y y^2 dy\,dx = \frac{1}{3}\int_0^L y^3 dx$$

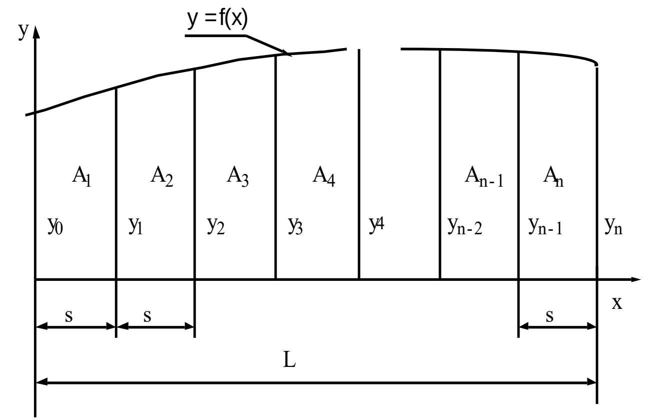
Second moment of area (A) with respect to y-axis

$$I_y = \int_0^A x^2 dA = \int_0^L x^2 y\,dx$$

Numerical integration

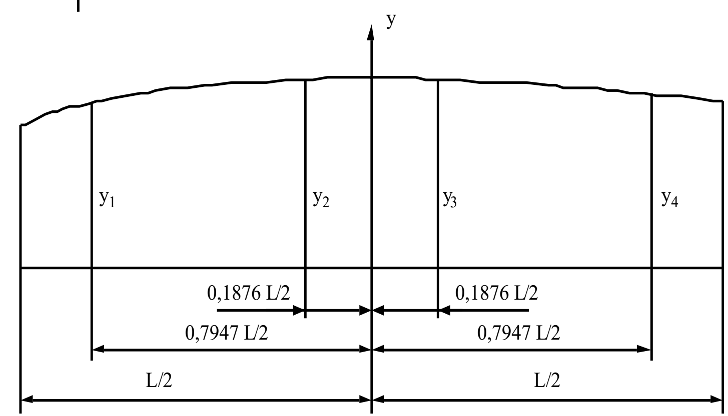
If evenly spaced ordinates

- ❑ Rectangle rule (Piecewise constant estimation of curve)
- ❑ Trapezoidal rule (Piecewise linear estimation of curve)
- ❑ Simpson I rule
- ❑ Simpson II rule



If unevenly spaced ordinates

- Tsebysev rule
- Gauss rule (used in FEM)



Numerical integration

$$A_i = \frac{1}{2} s\left(y_{i-1} + y_i\right)$$

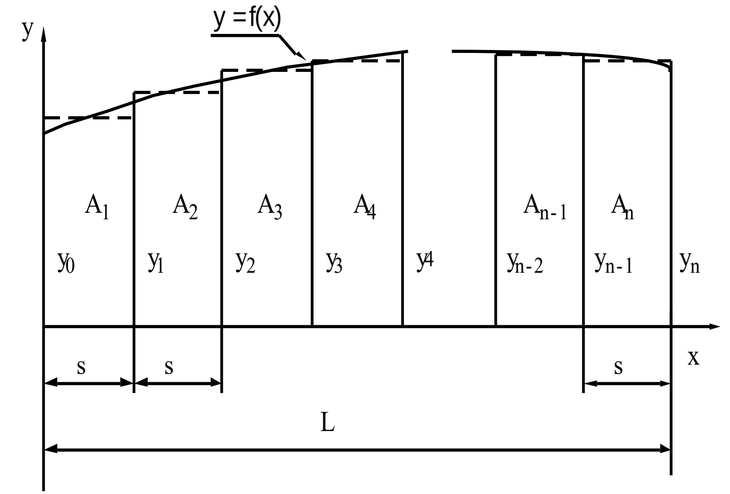
$$A = \sum_{i=1}^{n} A_i = A_1 + A_2 + A_3 + \ldots + A_n$$

$$= \frac{1}{2} s\left(y_{1-1} + y_1\right) + \frac{1}{2} s\left(y_{2-1} + y_2\right) + \frac{1}{2} s\left(y_{3-1} + y_3\right) + \ldots + \frac{1}{2} s\left(y_{n-2} + y_{n-1}\right) + \frac{1}{2} s\left(y_{n-1} + y_n\right)$$

$$= \frac{1}{2} s\left(y_0 + y_1\right) + \frac{1}{2} s\left(y_1 + y_2\right) + \frac{1}{2} s\left(y_2 + y_3\right) + \ldots + \frac{1}{2} s\left(y_{n-2} + y_{n-1}\right) + \frac{1}{2} s\left(y_{n-1} + y_n\right)$$

$$= s\left[\frac{1}{2}\left(y_0\right) + \frac{1}{2}\left(y_1 + y_1\right) + \frac{1}{2}\left(y_2 + y_2\right) + \frac{1}{2} y_3 + \ldots + \frac{1}{2}\left(y_{n-2}\right) + \frac{1}{2}\left(y_{n-1} + y_{n-1}\right) + \frac{1}{2} y_n\right]$$

$$= s\left[\frac{1}{2}\left(y_0\right) + y_1 + y_2 + \ldots + y_{n-1} + \frac{1}{2} y_n\right]$$



Numerical integration – Simpson I

The spacing needs to be even, because the solution of parabolic approximation requires solution of coefficients a, b and c based on three consecutive coordinates

$$\begin{cases} y_0 = a\cdot 0 + b\cdot 0 + c \\ y_1 = as^2 + bs + c \\ y_2 = 4as^2 + 2bs + c \end{cases} \Leftrightarrow \begin{cases} c = y_0 \\ b = \dfrac{4y_1 - y_2 - 3y_0}{2s} \\ a = \dfrac{y_2 - 2y_1 + y_0}{2s^2} \end{cases}$$

Equation for parabola

$$y = ax^2 + bx + c$$

Known values

x=0

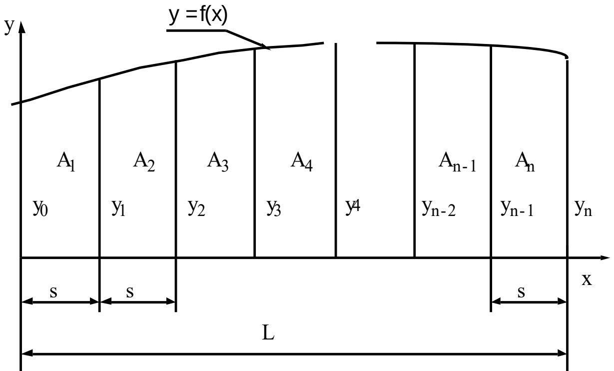
x=s

x=2s

So area for two areas is

$$A = A_1 + A_2 = \int_0^{2s} y\,dx = \int_0^{2s} \left(ax^2 + bx + c\right)dx = \frac{8}{3}as^3 + 2bs^2 + 2sc$$

$$= \frac{4}{3}\left(y_2 - 2y_1 + y_0\right)s + \left(4y_1 - y_2 - 3y_0\right)s + 2s\left(y_0\right) = \frac{s}{3}\left(y_0 + 4y_1 + y_2\right)$$



And the whole area investigated

$$\boxed{\begin{aligned} A &= \left(A_1 + A_2\right) + \left(A_3 + A_4\right) + \ldots + \left(A_{n-1} + A_n\right) \\ &= \left[\frac{s}{3}\left(y_0 + 4y_1 + y_2\right)\right] + \left[\frac{s}{3}\left(y_2 + 4y_3 + y_4\right)\right] + \ldots + \left[\frac{s}{3}\left(y_{n-2} + 4y_{n-1} + y_n\right)\right] \\ &= \frac{s}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right) \end{aligned}}$$

Reference : Kreyszig, Erwin. Advanced Engineering Mathematics. New York :Wiley, p. 832

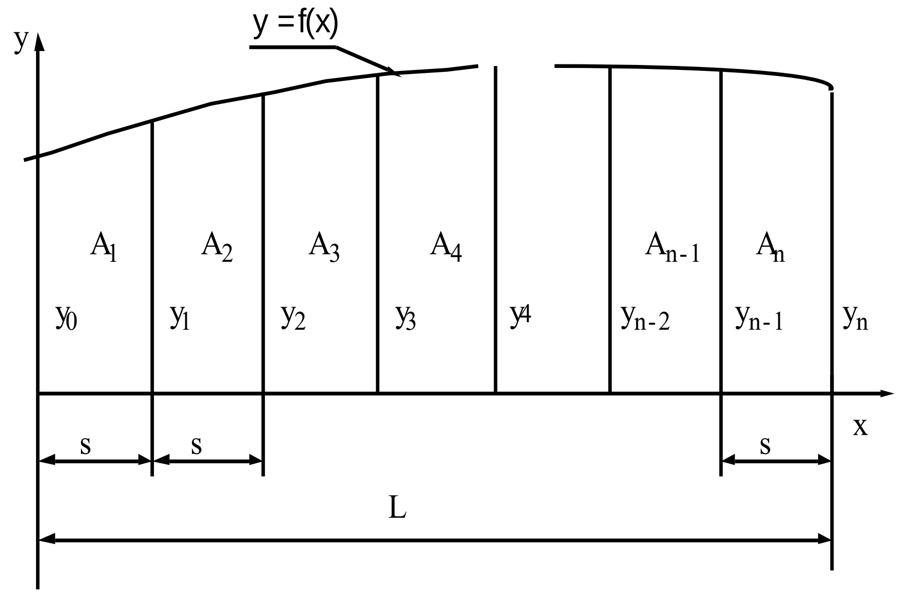
Numerical integration – Simpson II

- Third order polynomial calls for 4 ordinates

- Spacing n should be multiple of 3

$$A_1 + A_2 + A_3 = \frac{3}{8}s\left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

$$A = \frac{3}{8}s\left(\underbrace{y_0 + 3y_1 + 3y_2 + 2y_3}_{A_1 + A_2 + A_3} + \underbrace{3y_4 + 3y_5}_{A_4 + A_5 + A_6} + \ldots + 3y_{n-1} + y_n\right)$$



Numerical integration – example

Simpson's 1st Rule

Station	½ ordinates	Simpson's multiplier	Product for area
0	1.1	1	1.1
1	2.7	4	10.8
2	4	2	8
3	5.1	4	20.4
4	6.1	2	12.2
5	6.9	4	27.6
6	7.7	1	7.7

$$A = \frac{1}{3} \times \mathrm{h} \times \mathrm{sum} \times 2 = \mathbf{58.53h}$$

Simpson's 2nd Rule

Simpson's multiplier	Product for area
1	1.1
3	8.1
3	12
2	10.2
3	18.3
3	20.7
1	7.7

$$A = \frac{3}{8} \times h \times \mathrm{sum} \times 2 = \mathbf{58.575h}$$

Trapezoidal Rule

Simpson's multiplier	Product for area
1	1.1
2	5.4
2	8
2	10.2
2	12.2
2	13.8
1	7.7

$$A = \frac{1}{2} \times \mathrm{h} \times \mathrm{sum} \times 2 = \mathbf{58.4h}$$

Hydrostatic curves

- A series of graphs that give values such as the center of buoyancy, displacement, moment causing unit trim, and center of flotation

- This graph is used by the crew on-board to instantly obtain the value of a hydrostatic parameter of the ship for a given draft. However, one needs to be careful about the multi-scale horizontal axis that is used here, since multiple parameters with different units are plotted on the same graph.

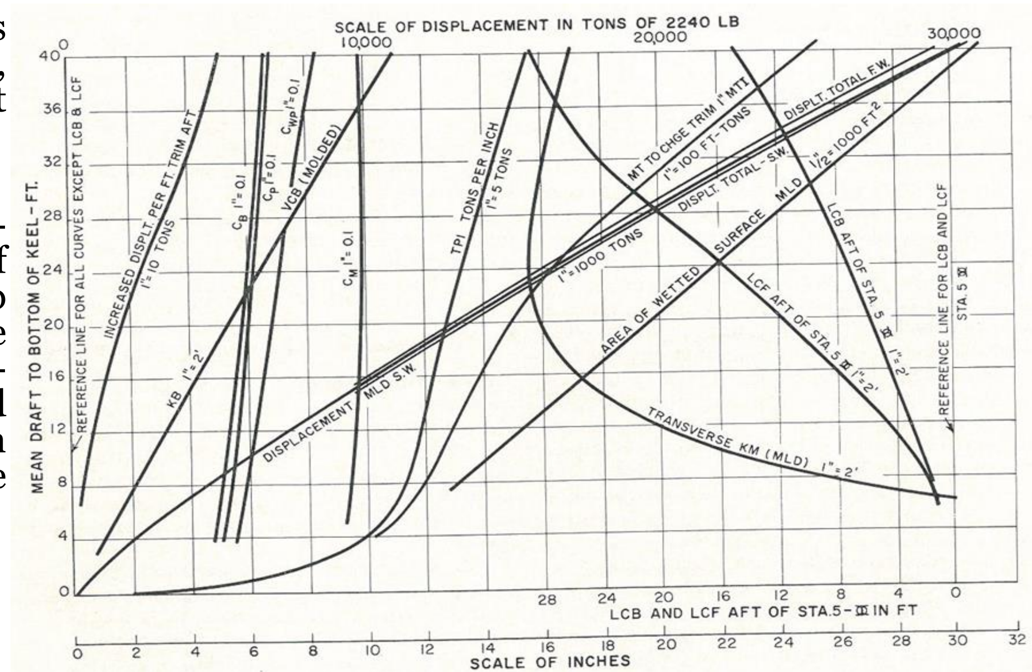
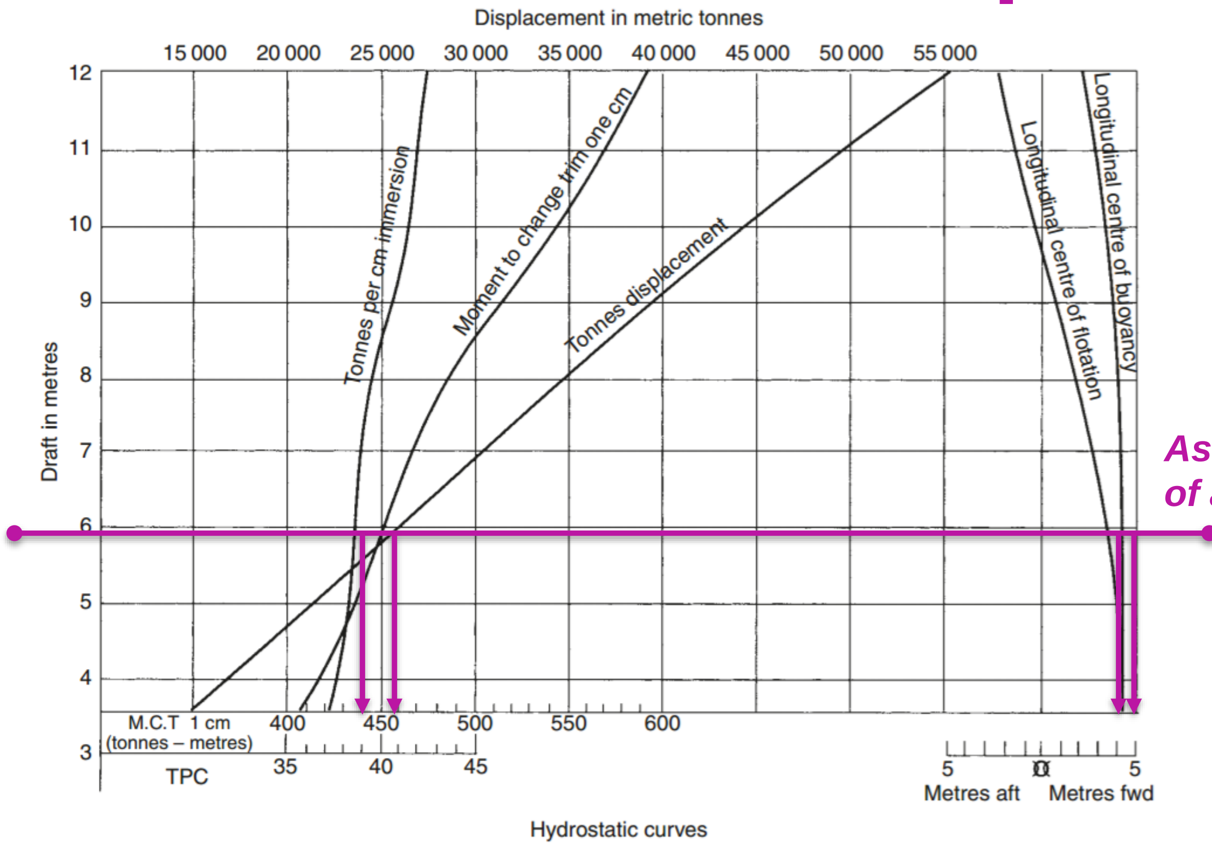


Image credit Principles of Naval Architecture, SNAME, USA

Hydrostatic curves - example

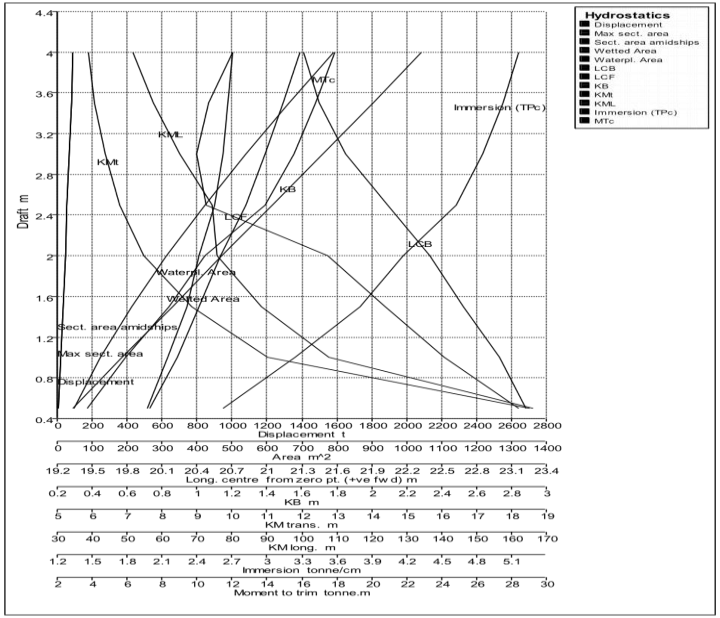


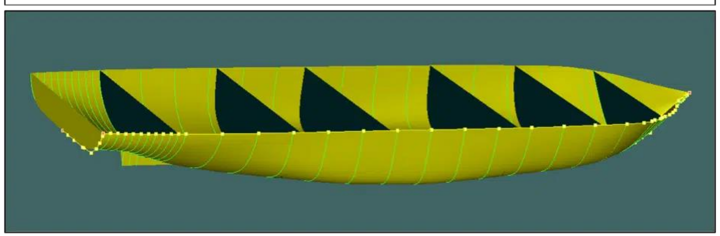
Assume mean draft of a ship is 6 m

Hydrostatic curves - discussion

- The only hydrostatic parameters that decrease with increase in draft are height of metacenter from the keel (KM), and longitudinal center of buoyancy (LCB).
- LCB is calculated from the forward perpendicular.
- Decreasing LCB with increasing draft implies, the LCB moves forward
- ***Does it hold true for all ships?***

Hydrostatic curves - discussion

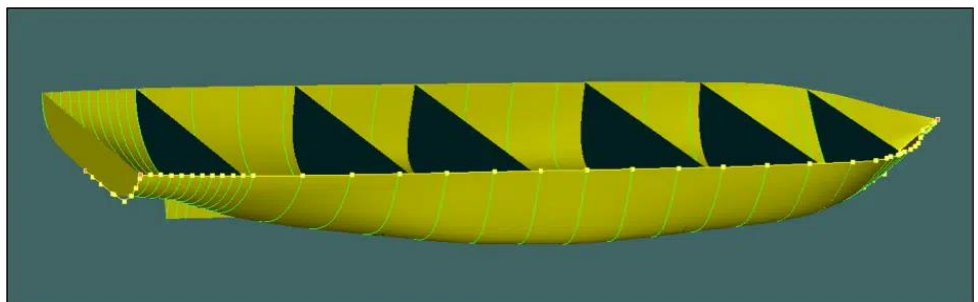




- While, the nature of KM is mostly the same, the nature of change of LCB with draft will vary according to the form of the hull.
- For example, the graph here is for a passenger ship with a fine stern
- A fine stern means, with increase in draft, the percentage of submerged volume towards the forward of the midship increases more rapidly than the submerged volume in the aft.
- Hence, at larger drafts, a majority of the submerged volume will be concentrated towards the forward of the midship.

Hydrostatic curves - discussion

- ❑ If this would have been a ship with finer bow and fuller stern, an increase in draft would have caused the LCB to shift towards the aft, thereby showing opposite nature on the hydrostatic curve.
- ❑ ***A ship designer can therefore predict the hullform of a ship just by looking at its LCB curve.***



Bonjean curves

They are used for the purpose of obtaining, for any given waterline, the areas of the immersed portion of each transverse section throughout the ship's length

Example

- Path KCL_1W_1K equals frame area A at point draught Q
- This can be used to estimate the buoyancy V=A*L of this section having length L

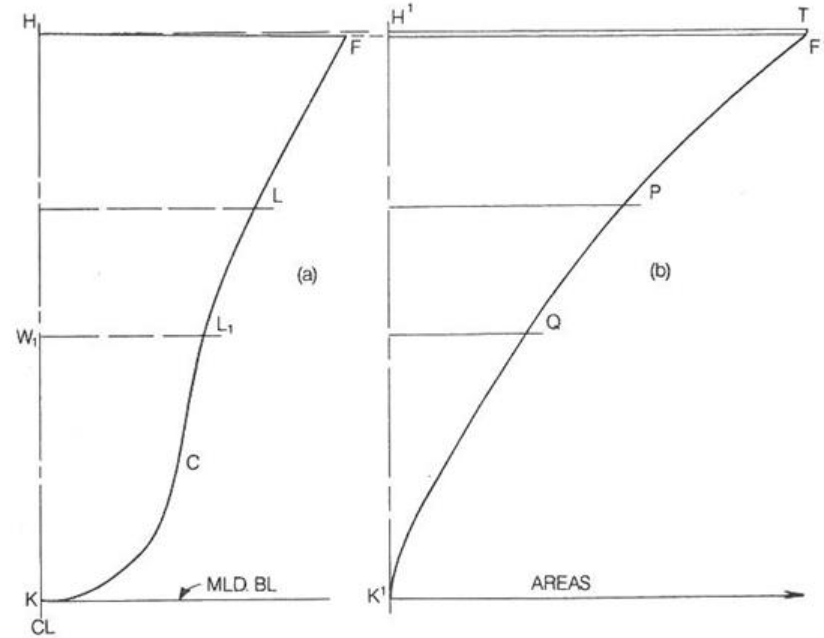
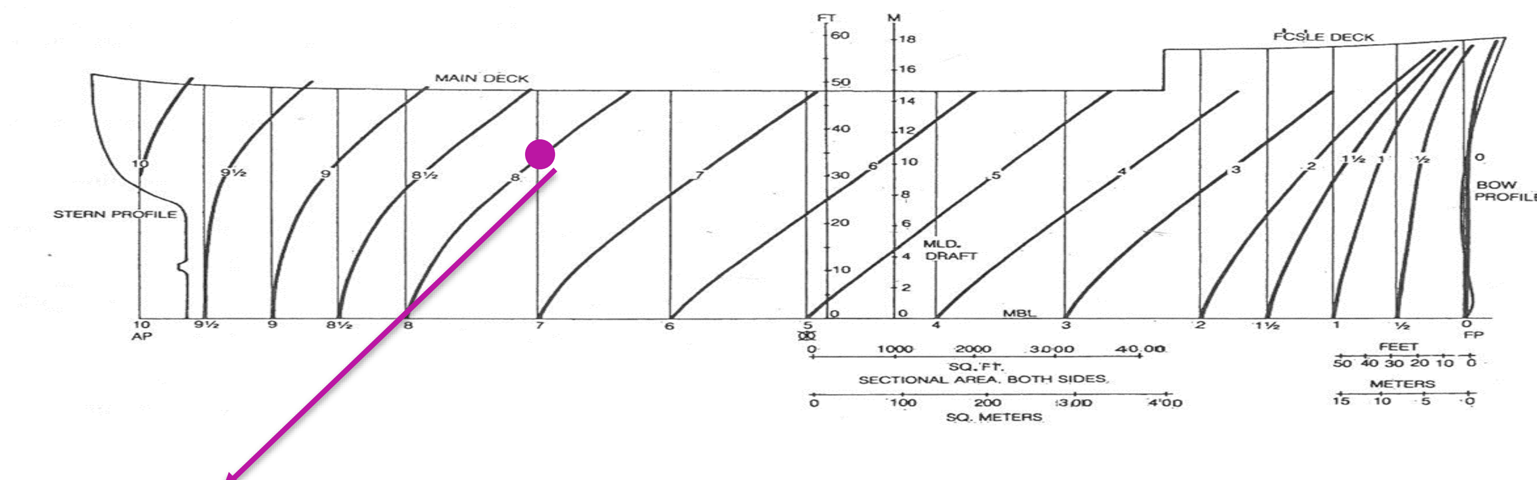


Fig. 32 Body plan section (a) and Bonjean Curve (b)

Bonjean curves example

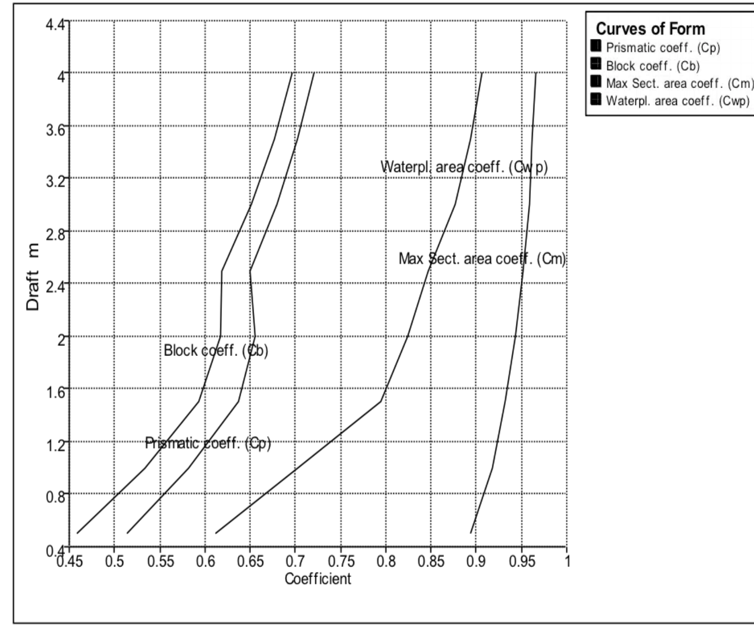


Point of immersion

- The immersed sectional area value for is the distance from the section base line to the intersection point of a Bonjean curve with the waterline.
- By using Bonjeans the displacement and Buoyancy can be calculated for inclined waterlines.
- This may be useful in trim analysis and ship launching calculations

Curves of form - discussion

- Play important role in optimizing the hull shape, and fairing the hull to a fine shape.
- The curves are not smooth.
- This implies that the hull at this stage of design, is not completely smooth, and would result in increased resistance.
- The same also applies to all the hydrostatic curves. Both these curves, along with the sectional area curve of a ship are simultaneously referred to, at each stage of hull modification, until a smooth set of curves are obtained.



Summary

- **Ship hydrostatics relate with complex hull shapes.**

- **Ship transverse stability for a ship subject to external load**
  - ✓ Stable if she returns to her upright condition
  - ✓ Neutral if she developes large angle of loll
  - ✓ Unstable if she continues to capsize

- **Numerical integration methods embedded in CAD packages help with the determination of areas, volumes, moments etc.** These are used to assess ship characteristics such as :
  - Buoyancy
  - Wetted surface
  - Center of buoyancy
  - ...etc.

Aalto University
School of Engineering

Thank you !