On the Ship's Trimming using Moments of Weight and Buoyancy Forces of High Order

By Lyuben D. Ivanov & John E. Kokarakis

ABSTRACT

The paper introduces a technique by which the hull girder shear forces and bending moments can be minimised, based on the equality of the moments of higher order of the weight and buoyancy forces. Cargo hold weights determined on the basis of these equalities can be shown to result to minimum bending moments and shear forces. The proposal is illustrated in the simplified case of a barge and a 24000 tdw bulk carrier.

Key words: bulker, optimisation, strength

1. Introduction

Traditionally, when trimming a ship, two equations are used: first, equality of ship weight and buoyancy forces (zero moment) and second, equality of the static moments of ship weight and displacement with respect to some point (first moment). Thus, the cargo load in two cargo holds or tanks can be determined. As there are more than two cargo holds or tanks in a ship, one should assume cargo load in the remaining cargo holds or tanks in order to have only two unknowns. This practice works relatively well, but can be improved using computers.

There will be no trim, hull girder bending and shear if the distributions of the weight and buoyancy forces are exactly the same. While this is not possible in real ships, the more high-order moments of the two forces can be made equal, the smaller the hull girder bending and shear will be. Thus one can increase the number of equations representing the equality of moments of the ship's weight and buoyancy forces up to the number of cargo holds and tanks to be loaded.

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2. Calculation of the Moments of Any Order

The mathematical base of the calculations is the integration by parts,

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx$$

which can be applied to moments of any order. For example, introducing elementary area $dA(y) = x(y)dy$, as well as elementary static moment $dS_x(y) = ydA(y) = yx(y)dy$ and elementary moment of inertia $dI_x(y) = y^2 dA(y) = ydS_x(y)$ with respect to x-axis, Fig. 1, we can calculate the area, the static moment and the moment of inertia respectively as

$$A(y) = \int_0^y x(y)dy = x(y)y - \int_0^y x'(y)ydy$$

$$S_x(y) = \int_0^y ydA(y) = yA(y) - \int_0^y A(y)dy$$

$$I_x(y) = \int_0^y ydS_x(y) = yS_x(y) - \int_0^y S_x(y)dy$$

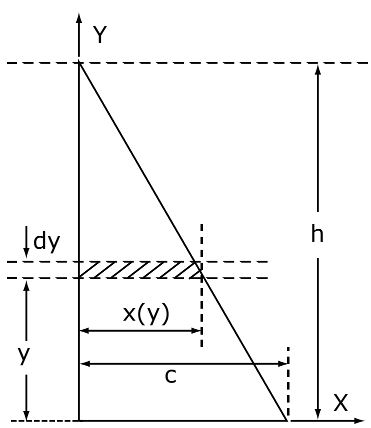

Fig. 1: Example for rectangular triangle.

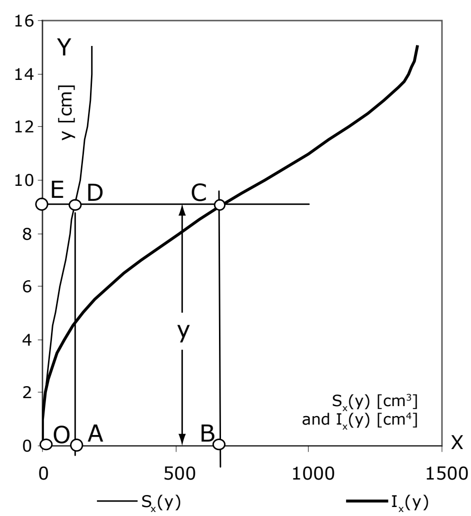

Fig. 2: Illustration of simplified procedure.

Geometric interpretation in Fig. 2: $I_x(y)$ is the area OAD, $yS_x(y)$ the area OADE, and $\int_0^y S_x(y)dy$ the area ODE. For example, if $x(y) = c(1 - y/h)$, $A = ch/2$, $S_x(h) = ch^2/6$ and $I_x(h) = ch^3/12$. The error of the trapezoidal rule of numerical integration for a rectangular triangle is 0.22%. The same principle can be applied to real ships, see an example in figures 3 to 5 for a 47000 tdw product tanker. The distance from point O to any transverse section is presented in dimensionless form, $\xi = x/L_{pp}$. The first dimensionless integral function of the lightweight $\eta(\xi)$ is defined as

$$\eta(\xi) = \frac{P(\xi)}{P_{LW}} = \frac{1}{P_{LW}} \int_0^\xi p(\xi)d\xi$$

$P(\xi)$ is the lightweight aft of ξ, P_{LW} is the total lightweight, $p(\xi)$ in Fig. 3 is the lightweight distribution, and $P(\xi)$ is the integral function of $p(\xi)$.

The second integral function is defined with respect to point O as

$$m(\xi) = \xi p(\xi) - \int_0^\xi \eta(\xi)d\xi$$

This corresponds to the area *OABC* less the area *OAB* in Fig. 3. Fig. 6 gives an example (note different scales used for η and m).

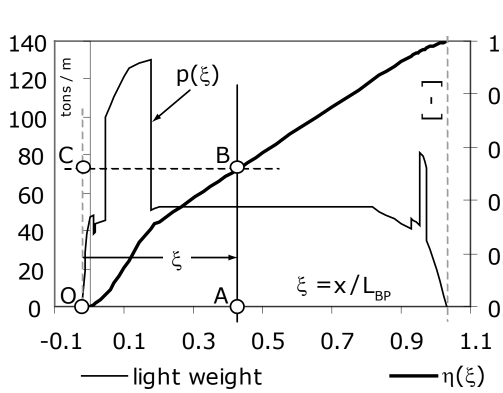

Fig. 3: Lightweight distribution and its first dimensionless integral function $\eta(\xi)$ of a 47000 tdw product tanker.

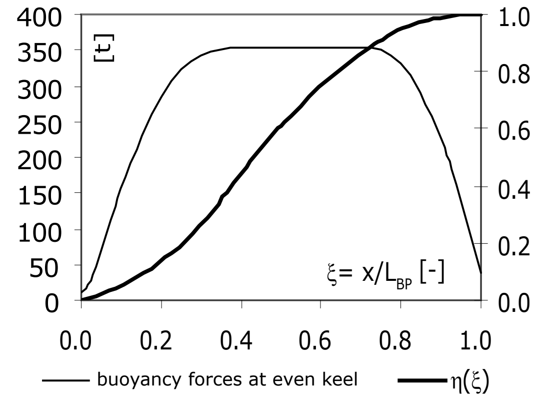

Fig. 4: Buoyancy force distribution and its first dimensionless integral function $\eta(\xi)$ of a 47000 tdw product tanker.

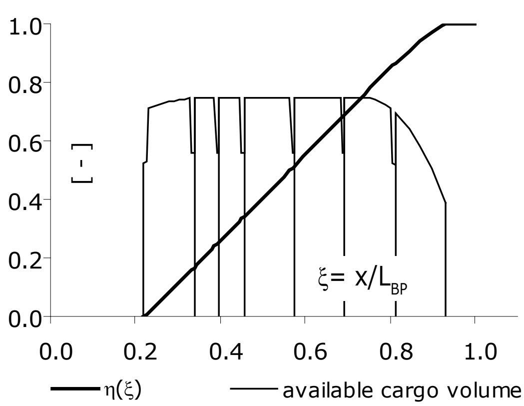

Fig. 5: Distribution of the available cargo volume and its first dimensionless integral function $\eta(\xi)$ of a 47000 tdw product tanker.

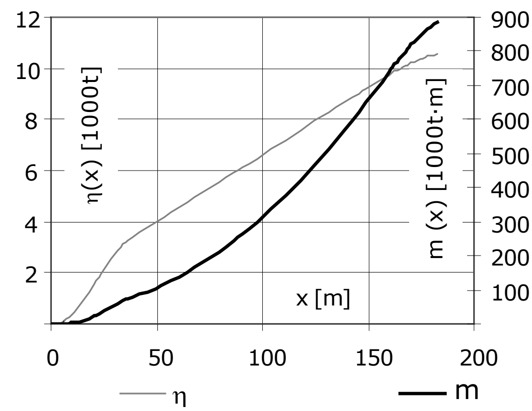

Fig. 6: Integral functions $\eta(x)$ and $m(x)$ for the lightweight of a 47000 tdw product tanker.

The ship's equilibrium is described by the following equations (n is the number of cargo holds to be loaded):

$$M_{P,0} = M_{B,0} \qquad M_{P,1} = M_{B,1} \qquad \ldots \qquad M_{P,n} = M_{B,n} \tag{1}$$

The subscripts P and B denote weight and buoyancy forces, respectively. The second subscript denotes the order of the corresponding moment (0 for zero moment, 1 for the first moment and n for the n^{th} moment). The 'zero' moment is equal to the buoyancy or ship's weight; the 'first' moment is equal to the static moment of the buoyancy or of the ship's weight etc.

If one assumes that the ship will be loaded up to its design draught, one can use the curve of the immersed cross-sectional areas for the calculation of the required moments of the buoyancy forces following the outlined procedure.

The unknown parameters in the ship's weight are the cargo weights in the cargo holds or tanks envisaged for loading. For convenience, the moments of the ship's weight are presented in the following form:

$$M_{P,i} = M_{P,i,k} + M_{P,i,u}$$

The subscript k denotes all known weight components (lightweight, weight of the fuel oil, lubricants, potable water and water for the machinery, weight related to the crew etc.). The subscript u denotes the unknown moments of the cargo in the cargo holds or tanks envisaged for loading. We can write

$$M_{P,i,k} = M_{LW,i} + M_{f,i} + M_{w,i} + M_{L,i} + M_{c,i}$$

$M_{LW,i}$, $M_{f,i}$, $M_{w,i}$, $M_{L,i}$ and $M_{c,i}$ are the i^{th} order moments of the lightweight, fuel oil, water, lubricants and of the weight related to the crew, respectively.

It is convenient to express the unknown moments of the cargo weights in the cargo holds or tanks envisaged for loading through the moments of the corresponding volumes. These moments can be accurately calculated when the capacity plan is known. Another way to perform the calculations is to simplify the curve representing the cargo holds' (or tanks') volume distribution within the length of each cargo hold (or tank) e.g. as a trapezoid. For example, for three cargo holds (or tanks), one can write for $M_{P,i,u}$:

$$M_{P,i,u} = \lambda_1 M_{V1,i} + \lambda_2 M_{V2,i} + \lambda_3 M_{V3,i}$$

λ_1, λ_2 and λ_3 are unknown coefficients to be determined from eq. (1). They have the same dimension as the specific weight of the cargo, i.e. t/m^3. $M_{V1,i}$, $M_{V2,i}$ and $M_{V3,i}$ are the i^{th} moments of the total volume in the first, second and third cargo hold or tank envisaged for loading. Moments of higher order than one cannot be calculated as a product of the weight and the corresponding lever powered to 2, 3 etc. The correct calculation requires integration (the above given procedure can be applied). Some simplification is possible if the distribution of the volume available for cargo, fuel oil etc. is approximated as trapezoid, Fig. 7; then the equation for n^{th} moment M_n with respect to the axis of comparison is

$$M_n = \frac{1}{x_2 - x_1}\left[(y_1 x_2 - y_2 x_1)\frac{x_2^{n+1} - x_1^{n+1}}{n+1} + (y_2 - y_1)\frac{x_2^{n+2} - x_1^{n+2}}{n+2}\right] \tag{2}$$

$$y(x) = y_1 + (y_2 - y_1)(x - x_1)/(x_2 - x_1) \tag{3}$$

For $n = 0, 1, 2, 3$ etc. M_n are respectively the area, static moment, moment of inertia, moment of third order etc. of the trapezoid.

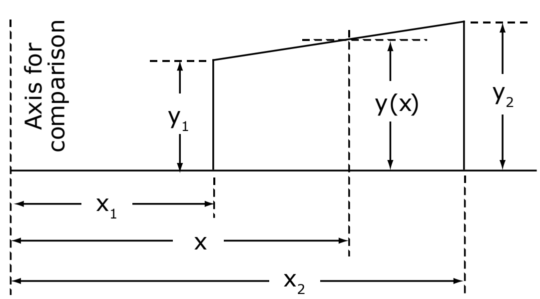

Fig. 7: Simplification of the volume distribution for cargo, fuel oil, lubricants etc.

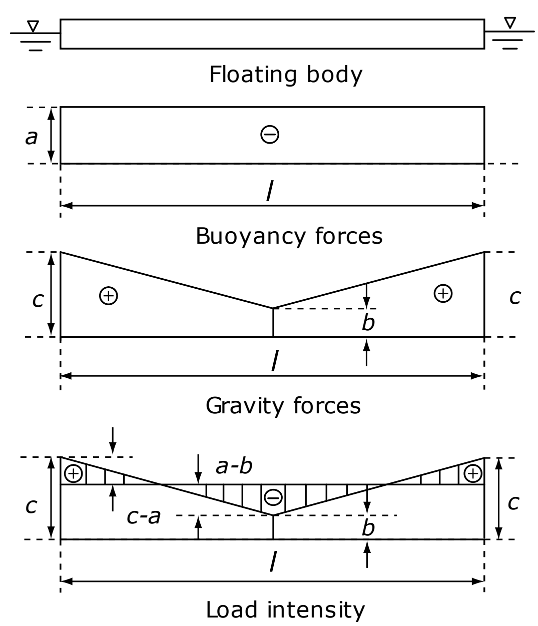

Fig. 8: Example for a floating homogeneous parallelepiped.

3. Simple Example

Before applying the proposed procedure to ship's trimming, a simple example for a homogeneous floating parallelepiped is considered. The weight is assumed equal to buoyancy forces and the centre of mass above the centre of buoyancy. A parametric study was performed by changing gradually the value b (Fig. 8) from 0 to a while keeping the weight and centroid constant. For each combination of b and c, the maximum bending moment acting on the parallelepiped was calculated together with the moments up to sixth order.

To avoid working with large numbers, present the n^{th} moment as $M_n = \rho_n^n P$, where P denotes weight, buoyancy force, volume etc. For comparison between different cases, it is convenient to present the gyradius r_n in dimensionless format as $r_n = \rho_n/\ell$, where ℓ is a characteristic length.

Fig. 9 shows the load intensity, shear force and bending moment distributions; the corresponding equations are:

$$q(x) = \begin{cases} (c-a)(1-4x/\ell) & \text{for } x \leq \ell/2 \\ (c-a)\left[\frac{2}{\ell}(2x-\ell)-1\right] & \text{for } x > \ell/2 \end{cases}$$

$$Q(x) = \begin{cases} (c-a)x(1-2x/\ell) & \text{for } x \leq \ell/2 \\ (c-a)\left(x-\frac{\ell}{2}\right)\left[\frac{2}{\ell}\left(x+\frac{\ell}{2}\right)-3\right] & \text{for } x > \ell/2 \end{cases}$$

$$M(x) = \begin{cases} (c-a)x^2\left(\frac{1}{2}-\frac{2}{3}\frac{x}{\ell}\right) & \text{for } x \leq \ell/2 \\ (c-a)\left[\frac{\ell^2}{24}+\left(x-\frac{\ell}{2}\right)\left(\frac{5\ell}{12}-\frac{7x}{6}+\frac{2}{3}\frac{x^2}{\ell}\right)\right] & \text{for } x > \ell/2 \end{cases}$$

The effect of the difference between the radii of gyration of weight and buoyancy forces of different order on the maximum of the bending moment is summarised in Fig. 10. One can observe the linear dependency of the bending moment on the difference between the second-order radii of gyration of weight and buoyancy forces, which coincides with the findings in *Ivanov* (2006). The dependency of the bending moment on the difference between higher-order radii of gyration of weight and buoyancy is nonlinear. The dependencies of the bending moment on the difference between the radii of gyration of weight and buoyancy for orders higher than three are very close. One can also observe that the smaller the difference between the moments, the smaller the bending moment. For example, if the difference between the dimensionless second-, third- and fourth-order radii of gyration of the weight and buoyancy forces is 0.03, the bending moment reduces by 15, 40 and 46%, respectively. Further reduction of the difference between higher order radii of gyration does not reduce bending moment significantly (the reduction of the bending moment is calculated relatively to the basic case $b = 0$, Fig. 8).

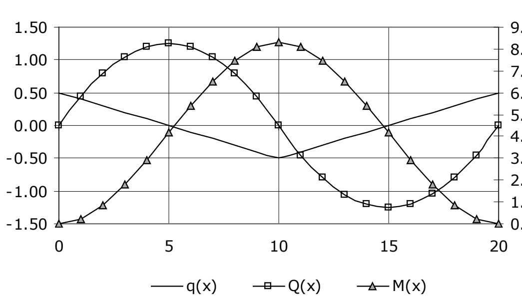

Fig. 9: Load intensity $q(x)$, shear force $Q(x)$ and bending moment $M(x)$ for the simple example.

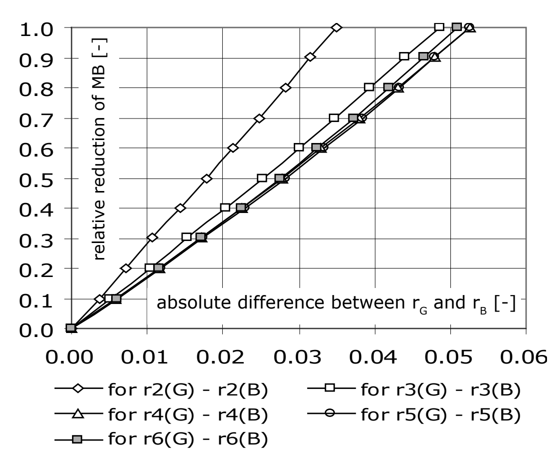

Fig. 10: Change of bending moment vs. difference between radii of gyration of different order. G and B denote weight and buoyancy forces, respectively.

4. Example for a 24000 tdw Bulk Carrier

The example refers to a 24000 tdw bulk carrier designed as a mathematical model to test the proposed methodology. Its main dimensions: $L_{pp} = 172.21$ m, $L_{oa} = 178.24$ m, $L_{WL} = 175.65$ m, $B_{WL} = 22.8$ m and $D = 14.11$ m. The ship has five cargo holds.

4.1. Calculation of Lightweight

The lightweight of the ship is $P_{LW} = 7092$ t with the centre of gravity at $-0.0745L_{pp}$ from midship. The dimensionless integral curves used to calculate the values of the zero moment (lightweight), first moment (static moment with respect to ship's end) and second moments (moment of inertia with respect to ship's end) m_0, m_1 and m_2 are given in Fig. 11. They were calculated following the above procedure. The dimensionless notation for the integral curves is introduced as

$$m_0(\xi) = \frac{M_0(\xi)}{P_{LW}} = \frac{P_{LW}(\xi)}{P_{LW}} \qquad m_1(\xi) = \frac{M_1(\xi)}{P_{LW}L_{oa}} \qquad m_2(\xi) = \frac{M_2(\xi)}{P_{LW}L_{oa}^2}$$

The function $\mu(\xi)$ in Fig. 11 is the moment of weight (area, force, etc.) aft of section ξ with respect to this section. The non-dimensional parameter α in Fig. 11 is the farthest point of the lightweight distribution curve from the aft perpendicular.

Table 1 gives the absolute values of the moments for the weight forces (lightweight) together with data for the buoyancy forces and the consumables (fuel oil, diesel oil, water, lubricants and weight related to crew).

Tab. 1: Data for M_n.

$M_{0,\Delta}$	$M_{1,\Delta}$	$M_{2,\Delta}$	$M_{0,DW1}$	$M_{1,DW1}$	$M_{2,DW1}$	$M_{0,LW}$	$M_{1,LW}$	$M_{2,LW}$
30913	2967065	336810433	2451	102122	4687704	7092	563080	64214181

The dimensionless function $m_0(\xi)$ is the same as the function $\eta(\xi)$ used for analytical calculation of shear forces in *Ivanov* (2006). The dimensionless integral function $\mu(\xi)$ used for analytical calculation of bending moments in *Ivanov* (2006) is related to the introduced dimensionless integral function $m_1(\xi)$ as

$$m_1(\xi) = \eta(\xi)[\xi - \alpha - \mu(\xi)/\eta(\xi)]$$

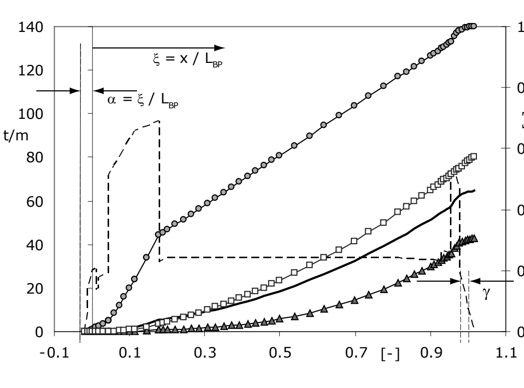

Fig. 11: Dimensionless integral functions $\mu(\xi)$, $m_0(\xi)$, $m_1(\xi)$ and $m_2(\xi)$ for the ship's lightweight.

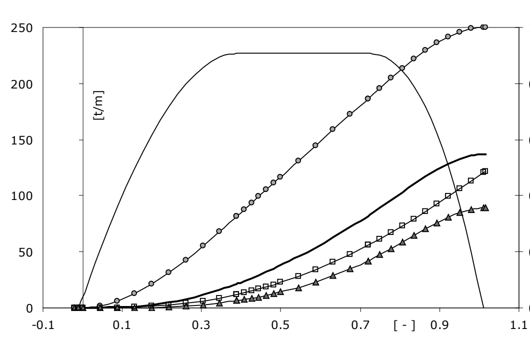

Fig. 12: Dimensionless integral functions $m_0(\xi)$, $m_1(\xi)$ and $m_2(\xi)$ for the buoyancy forces.

4.2. Calculation of Buoyancy Forces

The ship's displacement is $\Delta = 30913$ t with the centre $0.0224L_{pp}$ from midship. The dimensionless integral curves used to calculate the values of the moments m_0, m_1 and m_2 are given

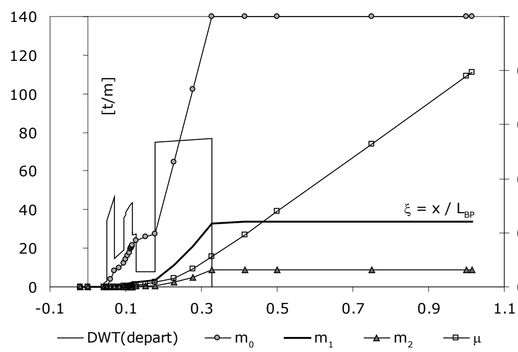

Fig. 13: Dimensionless integral functions $m_0(\xi)$, $m_1(\xi)$, $m_2(\xi)$ and $\mu(\xi)$ for the consumables and weight related to crew.

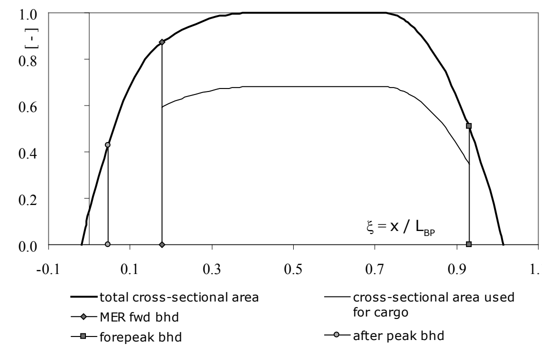

Fig. 14: Distribution of the total cross-sectional area and the area available for cargo loading.

in Fig. 12. They were calculated following the same procedure as for the weight forces. The absolute values of the moments for the buoyancy forces are given in Table 1. For comparison, the moments of weight and buoyancy forces are calculated with respect to different axes; they are recalculated with respect to the aft end of L_{oa} using the formulae of Appendix B.

4.3. Calculation of Deadweight

The deadweight is split into two parts, weight of consumables (fuel oil, diesel oil, water, lubricants, weight related to crew) and cargo weight. The weight of consumables consists of heavy fuel oil in main engine room (*104* t), heavy fuel oil outside main engine room (*1968* t), diesel oil (*127* t), lubricants (*104* t), water for machinery (*4.6* t) and weight related to crew (*145* t). The cargo weight is *21370* t. The dimensionless integral functions of the consumables are shown in Fig. 13 and the absolute values of the moments are given in Table 1.

The example considers the case when the ship is loaded in alternate cargo holds *1*, *3* and *5*. These cargo holds should be so loaded that the ship will be on even keel and the still-water shear forces and bending moments will be minimal. To determine the amount of cargo in each cargo hold, one should solve a system of three algebraic equations using moments of zero, first and second order. It is convenient to work with the volume of each cargo hold and use the specific weight of the cargo as the unknown parameter.

Once obtained, the product of the cargo hold volume and the corresponding 'virtual specific weight' will provide the absolute value of the cargo in the cargo hold. Then, using the stowage factor of the cargo, one can determine the required portion of the cargo hold volume needed. In other words, the mathematically derived 'virtual specific weight' consists of two components – specific weight of the real cargo and coefficient of cargo hold volume usage.

Following eq. (1) and using the numerical data for the ship's lightweight, displacement and first part of the deadweight, one can write the following equation:

$$\sum_{i=1}^{m} V_{n,i} x_i = C_n \tag{4}$$

where $C_n = M_{n,\Delta} - M_{n,DW1} - M_{n,LW}$.

If only three cargo holds are used for carrying the cargo, eq. (4) becomes

$$\begin{array}{ccccccc} V_{0,5}x_5 & + & V_{0,3}x_3 & + & V_{0,1}x_1 & = & C_0 \\ V_{1,5}x_5 & + & V_{1,3}x_3 & + & V_{1,1}x_1 & = & C_1 \\ V_{2,5}x_5 & + & V_{2,3}x_3 & + & V_{2,1}x_1 & = & C_2 \end{array} \tag{5}$$

Here the letter V is used to distinguish the moments for cargo volumes from the moments for weight and buoyancy forces. The first subscript denotes the order of the moment, the second subscript the number of the cargo hold. The roots x_1, x_3 and x_5 can be found by any specialised computer program; the input data and the results are shown in Tables 2 to 4.

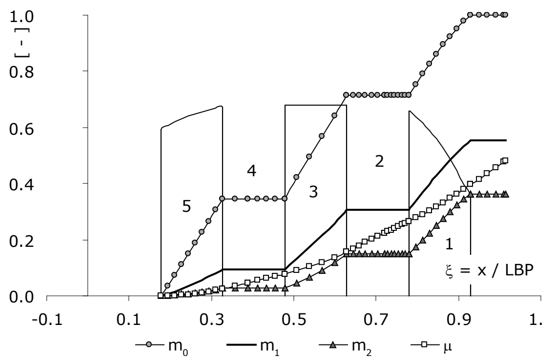

Fig. 15: Distribution of volume available for cargo and the dimensionless integral functions m_0, m_1, m_2 and μ.

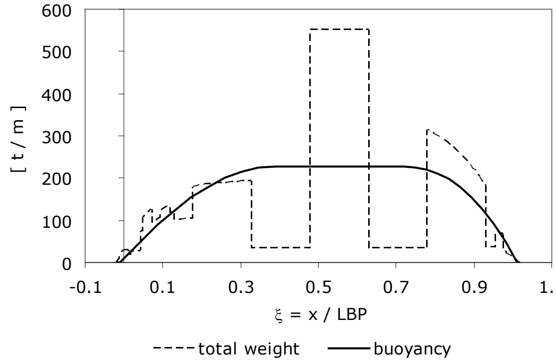

Fig. 16: Distribution of weight and buoyancy forces.

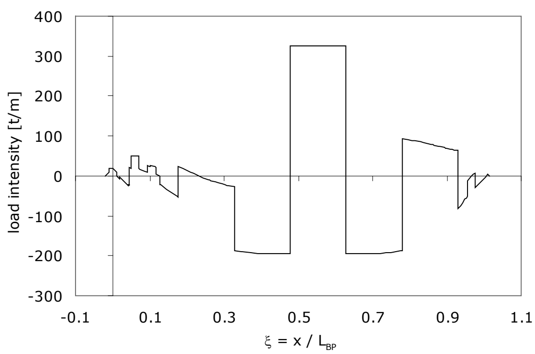

Fig. 17: Distribution of the load intensity.

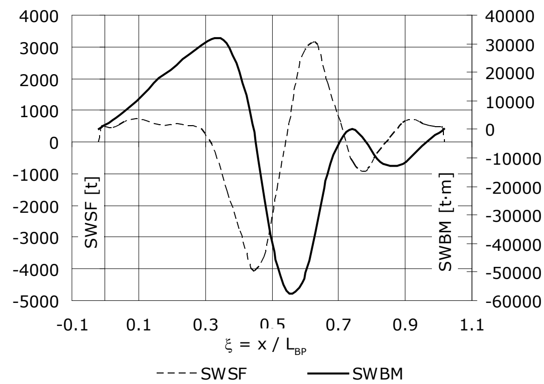

Fig. 18: Still-water shear force and bending moment.

The calculation of the n^{th} moments of cargo hold volumes requires a capacity curve, which provides information for the volume in each cargo hold and its longitudinal centre of gravity. For the sample ship, the volumes available for cargo are shown in figures 14 and 15 together with the volumes of the forepeak, after peak and the total volume below ship's deck. The volumes are non-dimensionalised with the total volume available for cargo in holds *1*, *3* and *5*, *15890* m^3.

The calculation of the n^{th} moments of the cargo hold volumes is performed by the trapezoidal rule following equations (2) and (3).

The example refers to a stowage factor of *0.40* m^3/t (iron ore). For this case, the weight of the cargo is distributed as *5869*, *13453* and *2048* t in holds *1*, *3* and *5*, respectively. Thus, the coefficients of usage (the ratio between the used volume and the total volume) are *0.52* (hold No. 1), *0.92* (No. 3) and *0.15* (No. 5). The load intensity and the still-water shear forces and bending moments are shown in figures 16 to 18.

The method is efficient when the stowage factor of the cargo is such that no cargo hold is fully used. If the stowage factor were *0.435* m^3/t, the volume of cargo hold No. 3 would be fully used. If the stowage factor were larger than *0.435* m^3/t, only two unknowns remain, the cargo in holds *1* and *5*. In this case, only two equations are needed to balance the ship and the traditional practice for

Tab. 2: n^{th} moments of the volumes of cargo holds.

$V_{0,5}$	$V_{0,3}$	$V_{0,1}$	$V_{1,5}$	$V_{1,3}$	$V_{1,1}$	$V_{2,5}$	$V_{2,3}$	$V_{2,1}$
5503	5847	4542	259473	577394	678443	12541850	57350086	101588519

Tab. 3: Parameters C_i.

C_0	C_1	C_2
21370	2301863	267908549

Tab. 4: Roots of Eq. (5).

x_5	x_3	x_1
0.3721	2.3010	1.2923

trimming the ship is to be followed. Nevertheless, the proposed methodology is still useful because it indicates the best location of the cargo even though this location may be impossible due to limited volumes of cargo holds. It might be difficult to implement the optimum solution in existing ships because of the limitation imposed by the maximum cargo per hold (in the design stage, the strength of the inner bottom and inner hull are already adapted to withstand the cargo as computed from the variety of loading patterns).

5. Conclusions

A method for ship's trimming and loading is proposed that uses moments of high order of the weight and buoyancy forces. The more higher order moments of the weight and buoyancy forces are equal, the smaller the hull-girder bending and shear will be. The existence of onshore and onboard ships computers facilitates the application of the method.

Appendix A. Distribution of Buoyancy Forces

If data for the buoyancy forces are not available, one can use the following formulae as a first approximation, Fig. 19:

$$y(x_r) = A(1 - x_r^2/r^2) + \delta(x_r^2)(x_r - r) \qquad y(x_e) = A(1 - x_e^2/e^2) + nx_e^2(x_e - e)$$

$$\delta = -\frac{12}{r^4}V - A[x_2 - x_1 + 2(e+r)/3] + e^4 n/12$$

$$n = -Ee^{-4}[(e+r)/20 + (x_2 - x_1)/12]^{-1}$$

$$\begin{aligned} E \;=\; & S_{mid} + A\left\{\tfrac{2}{3}\left[r\left(\tfrac{L}{2} - x_1\right) - e\left(x_2 - \tfrac{L}{2}\right)\right] - \tfrac{1}{2}\left[(x_2 - x_1)(x_2 + x_1 - L) + (e^2 - r^2)/2\right]\right\} \\ & + \left(\tfrac{27}{35}r + \tfrac{L}{2} - x_1\right)\left\{V - A\left[x_2 - x_1 + \tfrac{2}{3}(e+r)\right]\right\} \end{aligned}$$

V denotes buoyancy forces and S_{mid} the static moment of the buoyancy forces with respect to midship (positive when the centre of buoyancy is forward from midship). S_{mid} is the product of the buoyancy forces and the LCB from midship, $S_{mid} = V\zeta_{B,i}L_{pp}$, where $\zeta_{B,i}$ is the dimensionless LCB from midship. If LCB is not given, one can calculate it as a portion of L_{pp} based on data of *Moor et al.* (1961):

$$\begin{aligned} \zeta_{B,i} \;=\; & 4.366 - 11.619C_B + 7.771C_B^2 - 9.613\tfrac{T_i}{T} + 5.112\left(\tfrac{T_i}{T}\right)^2 + 25.12C_B\tfrac{T_i}{T} - 16.41C_B^2\tfrac{T_i}{T} \\ & -13.301C_B\left(\tfrac{T_i}{T}\right)^2 + 8.639C_B^2\left(\tfrac{T_i}{T}\right)^2 \end{aligned}$$

T_i is the draught. This equation is valid for a ship on even keel, which suits the aim of this work.

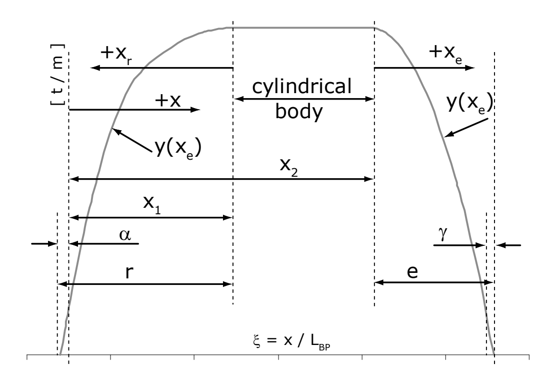

Fig. 19: Approximation of the buoyancy forces distribution.

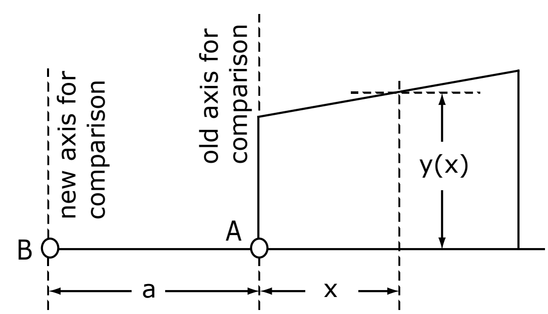

Fig. 20: Transfer of n^{th} moment to new axis.

Appendix B. Transfer of n^{th} Moment to Another Axis

Assume that n^{th} moment of a given figure is calculated with respect to an axis through point A (Fig. 20); the n^{th} moment with respect to another axis through point B can be calculated using the binomial formula

$$(a+x)^n = a^n + \binom{n}{1} a^{n-1}x + \binom{n}{2} a^{n-2}x^2 + \binom{n}{3} a^{n-3}x^3 + \cdots + \binom{n}{n} x^n$$

with the binomial coefficients

$$\binom{n}{k} = \frac{n(n-1)(n-2)...(n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$$

The equations up to $n = 5$ are

$$\begin{aligned}
M_{1,B}(x) &= a\ M_0(x) + M_{1,A}(x)\\
M_{2,B}(x) &= a^2 M_0(x) + 2a\ M_{1,A}(x) + M_{2,A}(x)\\
M_{3,B}(x) &= a^3 M_0(x) + 3a^2 M_{1,A}(x) + 3a\ M_{2,A}(x) + M_{3,A}(x)\\
M_{4,B}(x) &= a^4 M_0(x) + 4a^3 M_{1,A}(x) + 6a^2 M_{2,A}(x) + 4aM_{3,A}(x) + M_{4,A}(x)\\
M_{5,B}(x) &= a^5 M_0(x) + 5a^4 M_{1,A}(x) + 10a^3 M_{2,A}(x) + 10a^2 M_{3,A}(x) + 5aM_{4,A}(x) + M_{5,A}(x)
\end{aligned}$$

The subscripts A and B denote the n^{th} moment with respect to the axis through points A and B, respectively; the subscripts 1 to 5 denote the moment order.

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